Chapterse-wise 15 Years Solved Papers

With Video Solutions

CLASS 10th

MATHEMATICS

Basic and Standard

2020 -2021

With Reduced Syllabus

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7. Find the HCF and LCM of 90 and 144 by the method of prime factorization. [Board Term-1 2012]

Ans :

We have $90 = 9 \times 10 = 9 \times 2 \times 5$

$= 2 \times 3^2 \times 5$

and $144 = 16 \times 9$

$= 2^4 \times 3^2$

HCF $= 2 \times 3^2 = 18$

LCM $= 2^4 \times 3^2 \times 5 = 720$

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CHAPTER 1

REAL NUMBERS

SUMMARY

1. **Algorithm :** An algorithm means a series of well defined step which gives a procedure for solving a type of problem.
2. **Lemma :** A lemma is a proven statement used for proving another statement.
3. **Fundamental Theorem of Arithmetic :** Every composite number can be expressed (factorised) as a product of primes and this factorisation is unique apart from the order in which the prime factors occur.
4. If p is prime number and p divides a^2, then p divides a, where a is a positive integer.
5. If x be any rational number whose decimal expansion terminates, then we can express x in the form $\frac{p}{q}$, where p and q are co-prime and the prime factorisation of q is of the form $2^n \times 5^m$, where n and m are non-negative integers.
6. Let $x = \frac{p}{q}$ be a rational number such that the prime factorisation of q is not of the form $2^n \times 5^m$, where n and m are non-negative integers, then x has a decimal expansion which terminates.
7. Let $x = \frac{p}{q}$ be a rational number such that the prime factorisation of q is not of the form $2^n \times 5^m$, where n and m are non-negative integers, then x has a decimal expansion which is non-terminating repeating (recurring).
8. For any two positive integers p and q, HCF $(p, q) \times$ LCM $(p, q) = p \times q$.
9. For any three positive integers p, q and r,

$$\text{LCM } (p, q, r) = \frac{p \times q \times r \times \text{HCF}(p, q, r)}{\text{HCF}(p, q) \times \text{HCF}(q, r) \times \text{HCF}(p, r)}$$

$$\text{HCF } (p, q, r) = \frac{p \times q \times r \times \text{LCM}(p, q, r)}{\text{LCM}(p, q) \times \text{LCM}(q, r) \times \text{LCM}(p, r)}$$

ONE MARK QUESTIONS

MULTIPLE CHOICE QUESTIONS

1. The sum of exponents of prime factors in the prime-factorisation of 196 is

(a) 3 (b) 4

(c) 5 (d) 2

Ans : [Board 2020 OD Standard]

Prime factors of 196,

$$196 = 4 \times 49$$
$$= 2^2 \times 7^2$$

The sum of exponents of prime factor is $2 + 2 = 4$.

Thus (b) is correct option.

2. The total number of factors of prime number is

(a) 1 (b) 0

(c) 2 (d) 3

Ans : [Board 2020 Delhi Standard]

There are only two factors (1 and number itself) of any prime number.

Thus (c) is correct option.

3. The HCF and the LCM of 12, 21, 15 respectively are

(a) 3, 140 (b) 12, 420

(c) 3, 420 (d) 420, 3

Ans : [Board 2020 Delhi Standard]

We have
$$12 = 2 \times 2 \times 3$$
$$21 = 3 \times 7$$
$$15 = 3 \times 5$$
$$\text{HCF}(12, 21, 15) = 3$$
$$\text{LCM } (12, 21, 15) = 2 \times 2 \times 3 \times 5 \times 7 = 420$$

Thus (c) is correct option.

4. The decimal representation of $\frac{11}{2^3 \times 5}$ will

(a) terminate after 1 decimal place
(b) terminate after 2 decimal place
(c) terminate after 3 decimal places
(d) not terminate

Ans : [Board 2020 SQP Standard]

We have $\frac{11}{2^3 \times 5} = \frac{11}{2^3 \times 5^1}$

Denominator of $\frac{11}{2^3 \times 5}$ is of the form $2^m \times 5^n$, where m, n are non- negative integers. Hence, $\frac{11}{2^3 \times 5}$ has terminating decimal expansion.

Now
$$\frac{11}{2^3 \times 5} = \frac{11}{2^3 \times 5} \times \frac{5^2}{5^2}$$
$$= \frac{11 \times 5^2}{2^3 \times 5^3} = \frac{11 \times 25}{10^3} = 0.275$$

So , it will terminate after 3 decimal places.

Thus (c) is correct option.

5. The LCM of smallest two digit composite number and smallest composite number is

(a) 12 (b) 4
(c) 20 (d) 44

Ans : [Board 2020 SQP Standard]

Smallest two digit composite number is 10 and smallest composite number is 4.

$$\text{LCM}\ (10, 4) = 20$$

Thus (c) is correct option.

6. HCF of two numbers is 27 and their LCM is 162. If one of the numbers is 54, then the other number is

(a) 36 (b) 35
(c) 9 (d) 81

Ans : [Board 2020 OD Basic]

Let y be the second number.

Since, product of two numbers is equal to product of LCM and HCM,

$$54 \times y = \text{LCM} \times \text{HCF}$$
$$54 \times y = 162 \times 27$$
$$y = \frac{162 \times 27}{54} = 81$$

Thus (b) is correct option.

7. HCF of 144 and 198 is

(a) 9 (b) 18
(c) 6 (d) 12

Ans : [Board 2020 Delhi Basic]

Using prime factorization method,

$$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$$
$$= 2^4 \times 3^2$$

and
$$198 = 2 \times 3 \times 3 \times 11$$
$$= 2 \times 3^2 \times 11$$
$$\text{HCF}(144, 198) = 2 \times 3^2 = 2 \times 9 = 18$$

Thus (b) is correct option.

8. 225 can be expressed as

(a) 5×3^2 (b) $5^2 \times 3$
(c) $5^2 \times 3^2$ (d) $5^3 \times 3$

Ans : [Board 2020 Delhi Basic]

By prime factorization of 225, we have

$$225 = 3 \times 3 \times 5 \times 5$$
$$= 3^2 \times 5^2 \text{ or } 5^2 \times 3^2$$

Thus (c) is correct option.

9. The decimal expansion of $\frac{23}{2^5 \times 5^2}$ will terminate after how many places of decimal?

(a) 2 (b) 4
(c) 5 (d) 1

Ans : [Board 2020 OD Basic]

$$\frac{23}{2^5 \times 5^2} = \frac{23 \times 5^3}{2^5 \times 5^2 \times 5^3}$$
$$= \frac{23 \times 125}{2^5 \times 5^5} = \frac{2875}{(10)^5}$$
$$= \frac{2875}{100000} = 0.02875$$

Hence, $\frac{23}{2^5 \times 5^2}$ will terminate after 5 five decimal places.

Thus (c) is correct option.

10. The decimal expansion of the rational number $\frac{14587}{1250}$ will terminate after

(a) one decimal place (b) two decimal places
(c) three decimal places (d) four decimal places

Ans : [Board 2020 Delhi Standard]

Rational number,

$$\frac{14587}{1250} = \frac{14587}{2^1 \times 5^4} = \frac{14587}{2^1 \times 5^4} \times \frac{2^3}{2^3}$$

$$= \frac{14587 \times 8}{2^4 \times 5^4} = \frac{116696}{(10)^4}$$

$$= 11.6696$$

Hence, given rational number will terminate after four decimal places.

Thus (d) is correct option.

11. $2.\overline{35}$ is

(a) an integer (b) a rational number

(c) an irrational number (d) a natural number

Ans : [Board 2020 Delhi Basic]

$2.\overline{35}$ is a rational number because it is a non terminating repeating decimal.

Thus (b) is correct option.

12. $2\sqrt{3}$ is

(a) an integer (b) a rational number

(c) an irrational number (d) a whole number

Ans : [Board 2020 OD Basic]

Let us assume that $2\sqrt{3}$ is a rational number.

Now $2\sqrt{3} = r$ where r is rational number

or $\sqrt{3} = \frac{r}{2}$

Now, we know that $\sqrt{3}$ is an irrational number, So, $\frac{r}{2}$ has to be irrational to make the equation true. This is a contradiction to our assumption. Thus, our assumption is wrong and $2\sqrt{3}$ is an irrational number.

Thus (c) is correct option.

13. The product of a non-zero rational and an irrational number is

(a) always irrational (b) always rational

(c) rational or irrational (d) one

Ans :

Product of a non-zero rational and an irrational number is always irrational i.e., $\frac{3}{4} \times \sqrt{2} = \frac{3\sqrt{2}}{4}$ which is irrational.

a185

Thus (a) is correct option.

14. For some integer m, every even integer is of the form

(a) m (b) $m+1$

(c) $2m$ (d) $2m+1$

Ans :

We know that even integers are 2, 4, 6, ...

So, it can be written in the form of $2m$ where m is a integer.

$$m = ..., -1, 0, 1, 2, 3, ...$$

$$2m = ..., -2, 0, 2, 4, 6, ...$$

Thus (c) is correct option.

15. For some integer q, every odd integer is of the form

(a) q (b) $q+1$

(c) $2q$ (d) $2q+1$

Ans :

We know that odd integers are 1, 3, 5, ...

So, it can be written in the form of $2q+1$ where q is integer.

$$q = ..., -2, -1, 0, 1, 2, 3, ...$$

$$2q+1 = ..., -3, -1, 1, 3, 5, 7, ...$$

Thus (d) is correct option.

16. If two positive integers a and b are written as $a = x^3y^2$ and $b = xy^3$, where x, y are prime numbers, then HCF (a, b) is

(a) xy (b) xy^2

(c) x^3y^3 (d) x^2y^2

Ans :

We have $a = x^3y^2 = x \times x \times x \times y \times y$

$$b = xy^3 = x \times y \times y \times y$$

$$\text{HCF}(a, b) = \text{HCF}(x^3y^3, xy^3)$$

$$= x \times y \times y = xy^2$$

HCF is the product of the smallest power of each common prime factor involved in the numbers.

Thus (b) is correct option.

17. If two positive integers p and q can be expressed as $p = ab^2$ and $q = a^3b$; where a, b being prime numbers, then LCM (p, q) is equal to

(a) ab (b) a^2b^2

(c) a^3b^2 (d) a^3b^3

Ans :

We have $p = ab^2 = a \times b \times b$

and $q = a^3b = a \times a \times a \times b$

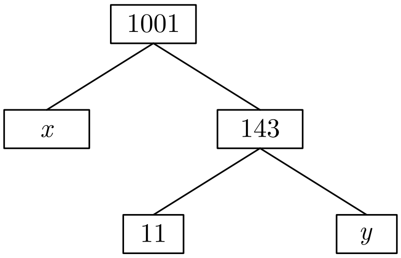
$$\text{LCM}(p, q) = \text{LCM}(ab^2, a^3b)$$

$$= a \times b \times b \times a \times a = a^3 b^2$$

LCM is the product of the greatest power of each prime factor involved in the numbers.

Thus (c) is correct option.

18. The values of x and y in the given figure are



(a) 7, 13 (b) 13, 7

(c) 9, 12 (d) 12, 9

Ans :

$$1001 = x \times 143 \Rightarrow x = 7$$

$$143 = y \times 11 \Rightarrow y = 13$$

Hence $x = 7,\ y = 13$

Thus (a) is correct option.

19. The least number that is divisible by all the numbers from 1 to 10 (both inclusive)

(a) 10 (b) 100

(c) 504 (d) 2520

Ans :

Factor of 1 to 10 numbers

$$1 = 1$$

$$2 = 1 \times 2$$

$$3 = 1 \times 3$$

$$4 = 1 \times 2 \times 2$$

$$5 = 1 \times 5$$

$$6 = 1 \times 2 \times 3$$

$$7 = 1 \times 7$$

$$8 = 1 \times 2 \times 2 \times 2$$

$$9 = 1 \times 3 \times 3$$

$$10 = 1 \times 2 \times 5$$

$$\text{LCM}(1 \text{ to } 10) = \text{LCM}\,(1,2,3,4,5,6,7,8,9,10)$$

$$= 1 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7$$

$$= 2520$$

Thus (d) is correct option.

20. If p_1 and p_2 are two odd prime numbers such that $p_1 > p_2$, then $p_1^2 - p_2^2$ is

(a) an even number (b) an odd number

(c) an odd prime number (d) a prime number

Ans :

$p_1^2 - p_2^2$ is an even number.

Let us take $p_1 = 5$

and $p_2 = 3$

Then, $p_1^2 - p_2^2 = 25 - 9 = 16$

16 is an even number.

Thus (a) is correct option.

21. The rational form of $0.2\overline{54}$ is in the form of $\frac{p}{q}$ then $(p+q)$ is

(a) 14 (b) 55

(c) 69 (d) 79

Ans :

Let, $x = 0.2\overline{54}$, then

$$x = 0.2545454 \text{ ....} \qquad \text{...(1)}$$

Multiplying equation (1) by 100, we get

$$100x = 25.4545. \text{ ....} \qquad \text{...(2)}$$

Subtracting equation (1) from equation. (2), we get

$$99x = 25.2 \Rightarrow x = \frac{252}{990} = \frac{14}{55}$$

Comparing with $\frac{p}{q}$, we get

$$p = 14$$

and $q = 55$

Hence, $p + q = 14 + 55 = 69$

Alternative :

$$0.2\overline{54} = \frac{254 - 2}{990} = \frac{252}{990} = \frac{14}{55}$$

Thus (c) is correct option.

22. The rational number of the form $\frac{p}{q}$, $q \neq 0$, p and q are positive integers, which represents $0.1\overline{34}$ i.e., (0.1343434 ....) is

(a) $\frac{134}{999}$ (b) $\frac{134}{990}$

(c) $\frac{133}{999}$ (d) $\frac{133}{990}$

Ans :

$$0.1\overline{34} = \frac{134-1}{990} = \frac{133}{990}$$

Thus (d) is correct option.

23. Which of the following will have a terminating decimal expansion?

(a) $\frac{77}{210}$ (b) $\frac{23}{30}$

(c) $\frac{125}{441}$ (d) $\frac{23}{8}$

Ans :

For terminating decimal expansion, denominator must the form of $2^m \times 5^n$ where n, m are non-negative integers.

Here, $\frac{23}{8} = \frac{23}{2^3}$

Here only 2 is factor of denominator so terminating.
Thus (d) is correct option.

24. If $x = 0.\overline{7}$, then $2x$ is

(a) $1.\overline{4}$ (b) $1.\overline{5}$

(c) $1.\overline{54}$ (d) $1.\overline{45}$

Ans :

We have $x = 0.\overline{7}$

$10x = 7.\overline{7}$

Subtracting, $9x = 7$

$x = \frac{7}{9}$

$2x = \frac{14}{9} = 1.555$ ....

$= 1.\overline{5}$

25. Which of the following rational number have non-terminating repeating decimal expansion?

(a) $\frac{31}{3125}$ (b) $\frac{71}{512}$

(c) $\frac{23}{200}$ (d) None of these

Ans :

$3125 = 5^5 = 5^5 \times 2^0$

$512 = 2^9 = 2^9 \times 5^0$

$200 = 2^3 \times 5^2$

Thus 3125, 512 and 200 has factorization of the form $2^m \times 5^n$ (where m and n are whole numbers). So given fractions has terminating decimal expansion.
Thus (d) is correct option.

26. The number $3^{13} - 3^{10}$ is divisible by

(a) 2 and 3 (b) 3 and 10

(c) 2, 3 and 10 (d) 2, 3 and 13

Ans :

$$3^{13} - 3^{10} = 3^{10}(3^3 - 1) = 3^{10}(26)$$

$$= 2 \times 13 \times 3^{10}$$

Hence, $3^{13} - 3^{10}$ is divisible by 2, 3 and 13.
Thus (d) is correct option.

27. 1. The L.C.M. of x and 18 is 36.
2. The H.C.F. of x and 18 is 2.
What is the number x?

(a) 1 (b) 2

(c) 3 (d) 4

Ans :

LCM $\times$ HCF = First number $\times$ second number

Hence, required number $= \frac{36 \times 2}{18} = 4$

Thus (d) is correct option.

28. If $a = 2^3 \times 3$, $b = 2 \times 3 \times 5$, $c = 3^n \times 5$ and $\text{LCM}(a, b, c) = 2^3 \times 3^2 \times 5$, then n is

(a) 1 (b) 2

(c) 3 (d) 4

Ans :

Value of n must be 2.
Thus (b) is correct option.

29. The least number which is a perfect square and is divisible by each of 16, 20 and 24 is

(a) 240 (b) 1600

(c) 2400 (d) 3600

Ans :

The LCM of 16, 20 and 24 is 240. The least multiple of 240 that is a perfect square is 3600 and also we can easily eliminate choices (a) and (c) since they are not perfect square number. 1600 is not multiple of 240.
Thus (d) is correct option.

30. n^2-1 is divisible by 8, if n is

(a) an integer (b) a natural number

(c) an odd integer (d) an even integer

Ans :

Let, $a = n^2-1$

For n^2-1 to be divisible by 8 (even number), n^2-1 should be even. It means n^2 should be odd i.e. n should be odd.

If n is odd, $n = 2k+1$ where k is an integer

$$a = (2k+1)^2-1$$
$$= 4k^2+4k+1-1$$
$$= 4k^2+4k$$
$$a = 4k(k+1)$$

At $k=-1$, $a = 4(-1)(-1+1)=0$

which is divisible by 8.

At $k=0$, $a = 4(0)+(0+1)=0$

which is divisible by 8.

Hence, we can conclude from above two cases, if n is odd, then n^2-1 is divisible by 8.

Thus (c) is correct option.

31. When 2^{256} is divided by 17 the remainder would be

(a) 1 (b) 16

(c) 14 (d) None of these

Ans : (a) 1

When 2^{256} is divided by 17 then,

$$\frac{2^{256}}{2^4+1} = \frac{(2^4)^{64}}{(2^4+1)}$$

By remainder theorem when $f(x)$ is divided by $x+a$ the remainder is $f(-a)$.

Here, $f(x) = (2^4)^{64}$ and $x=2^4$ and $a=1$

Hence, remainder $f(-1) = (-1)^{64} = 1$

Thus (a) is correct option.

32. Assertion : $\frac{13}{3125}$ is a terminating decimal fraction.

Reason : If $q=2^m5^n$ where m, n are non-negative integers, then $\frac{p}{q}$ is a terminating decimal fraction.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

We have $3125 = 5^5 = 5^5\times 2^0$

Since the factors of the denominator 3125 is of the form $2^0\times 5^5$, $\frac{13}{3125}$ is a terminating decimal

Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

Thus (a) is correct option.

33. Assertion : 34.12345 is a terminating decimal fraction.

Reason : Denominator of 34.12345, when expressed in the form $\frac{p}{q}$, $q\neq 0$, is of the form $2^m\times 5^n$, where m and n are non-negative integers.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

$$34.12345 = \frac{3412345}{100000} = \frac{682469}{20000} = \frac{682469}{2^5\times 5^4}$$

Its denominator is of the form $2^m\times 5^n$, where $m=5$ and $n=4$ which are non-negative integers.

Thus both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Thus (a) is correct option.

34. Assertion : The HCF of two numbers is 5 and their product is 150, then their LCM is 30

Reason : For any two positive integers a and b, HCF (a,b) +LCM (a,b) $= a\times b$.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans : (c) Assertion (A) is true but reason (R) is false.

We have,

$$\text{LCM}(a,b)\times\text{HCF}(a,b) = a\times b$$

$$\text{LCM}\times 5 = 150$$

$$\text{LCM} = \frac{150}{5} = 30$$

Thus (c) is correct option.

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Fill in the Blank Questions

35. If every positive even integer is of the form $2q$, then every positive odd integer is of the form .... where q is some integer.

Ans :

$2q+1$

VIDEO Click Here

36. The exponent of 2 in the prime factorisation of 144, is ....

Ans :

4

a212

37. $\sqrt{2},\sqrt{3},\sqrt{7}$, etc. are .... numbers.

Ans :

Irrational

VIDEO Click Here

38. Every point on the number line corresponds to a .... number.

Ans :

Real

VIDEO Click Here

39. The product of three numbers is .... to the product of their HCF and LCM.

Ans :

Not equal

a217

40. If p is a prime number and it divides a^2 then it also divides ...., where a is a positive integer.

Ans :

a

a218

41. Every real number is either a .... number or an .... number.

Ans :

Rational, irrational

a219

42. Numbers having non-terminating, non-repeating decimal expansion are known as ....

Ans :

Irrational numbers

a221

Very Short Answer Questions

43. What is the HCF of smallest primer number and the smallest composite number?

Ans : [Board 2018]

Smallest prime number is 2 and smallest composite number is 4. HCF of 2 and 4 is 2.

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44. Write one rational and one irrational number lying between 0.25 and 0.32.

Ans : [Board 2020 SQP Standard]

Given numbers are 0.25 and 0.32.

Clearly $$0.30 = \frac{30}{100} = \frac{3}{10}$$

Thus 0.30 is a rational number lying between 0.25 and 0.32. Also 0.280280028000.....has non-terminating non-repeating decimal expansion. It is an irrational number lying between 0.25 and 0.32.

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45. If HCF(336, 54) $=6$, find LCM(336, 54).

Ans : [Board 2019 OD]

$$\text{HCF}\times\text{LCM} = \text{Product of number}$$

$$6\times\text{LCM} = 336\times 54$$

$$\text{LCM} = \frac{336\times 54}{6}$$

$$= 56\times 54 = 3024$$

Thus LCM of 336 and 54 is 3024.

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46. Explain why 13233343563715 is a composite number?

Ans : [Board Term-1 2016]

The number 13233343563715 ends in 5. Hence it is a multiple of 5. Therefore it is a composite number.

a101

47. a and b are two positive integers such that the least prime factor of a is 3 and the least prime factor of b is 5. Then calculate the least prime factor of $(a+b)$.

Ans : [Board Term-1 2014]

Here a and b are two positive integers such that the least prime factor of a is 3 and the least prime factor of b is 5. The least prime factor of $(a+b)$ would be 2.

a102

48. What is the HCF of the smallest composite number and the smallest prime number?

Ans : [Board Term-

a103

The smallest prime number is 2 and the smallest composite number is $4 = 2^2$.

Hence, required HCF is $(2^2,\ 2) = 2$.

49. Calculate the HCF of $3^3 \times 5$ and $3^2 \times 5^2$.

Ans : [Board 2007]

We have
$$3^3 \times 5 = 3^2 \times 5 \times 3$$
$$3^2 \times 5^2 = 3^2 \times 5 \times 5$$
$$\text{HCF }(3^3 \times 5,\ 3^2 \times 5^2) = 3^2 \times 5$$
$$= 9 \times 5 = 45$$

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50. If HCF $(a, b) = 12$ and $a \times b = 1{,}800$, then find LCM (a, b).

Ans :

VIDEO Click Here

We know that
$$\text{HCF}\,(a, b) \times \text{LCM}\,(a, b) = a \times b$$

Substituting the values we have
$$12 \times \text{LCM}\,(a, b) = 1800$$

or,
$$\text{LCM}\,(a,\ b) = \frac{1{,}800}{12} = 150$$

51. What is the condition for the decimal expansion of a rational number to terminate? Explain with the help of an example.

Ans : [Board Term-1 2016]

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The decimal expansion of a rational number terminates, if the denominator of rational number can be expressed as $2^m 5^n$ where m and n are non negative integers and p and q both co-primes.

e.g.
$$\frac{3}{10} = \frac{3}{2^1 \times 5^1} = 0.3$$

52. Find the smallest positive rational number by which $\frac{1}{7}$ should be multiplied so that its decimal expansion terminates after 2 places of decimal.

Ans : [Board Term-1 2016]

Since
$$\frac{1}{7} \times \frac{7}{100} = \frac{1}{100} = 0.01.$$

a149

Thus smallest rational number is $\frac{7}{100}$

53. What type of decimal expansion does a rational number has? How can you distinguish it from decimal expansion of irrational numbers?

Ans : [Board Term-1 2016]

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A rational number has its decimal expansion either terminating or non-terminating, repeating An irrational numbers has its decimal expansion non-repeating and non-terminating.

54. Calculate $\frac{3}{8}$ in the decimal form.

Ans : [Board 2008]

VIDEO Click Here

We have
$$\frac{3}{8} = \frac{3}{2^3} = \frac{2 \times 5^3}{2^3 \times 5^3}$$
$$= \frac{375}{10^3} = \frac{375}{1{,}000}$$
$$= 0.375$$

55. The decimal representation of $\frac{6}{1250}$ will terminate after how many places of decimal?

Ans : [Board 2009]

We have
$$\frac{6}{1250} = \frac{6}{2 \times 5^4} = \frac{6 \times 2^3}{2 \times 2^3 \times 5^4}$$
$$= \frac{6 \times 2^3}{2^4 \times 5^4} = \frac{6 \times 2^3}{(10)^4}$$
$$= \frac{48}{10000} = 0.0048$$

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Thus $\frac{6}{1250}$ will terminate after 4 decimal places.

56. Find the least number that is divisible by all numbers between 1 and 10 (both inclusive).

Ans : [Board 2010]

The required number is the LCM of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10,
$$\text{LCM} = 2 \times 2 \times 3 \times 2 \times 3 \times 5 \times 7$$
$$= 2520$$

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57. Write whether rational number $\frac{7}{75}$ will have terminating decimal expansion or a non-terminating decimal.

Ans : [Board Term-1 2017, SQP]

We have
$$\frac{7}{75} = \frac{7}{3 \times 5^2}$$

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Since denominator of given rational number is not of form $2^m \times 5^n$, Hence, It is non-terminating decimal expansion.

TWO MARKS QUESTIONS

58. If HCF of 144 and 180 is expressed in the form $13m-16$. Find the value of m.

Ans : [Board 2020 SQP Standard]

According to Euclid's algorithm any number a can be written in the form,

$$a = bq+r \text{ where } 0 \le r < b$$

Applying Euclid's division lemma on 144 and 180 we have

$$180 = 144 \times 1 + 36$$

$$144 = 36 \times 4 + 0$$

Here, remainder is 0 and divisor is 36. Thus HCF of 144 and 180 is 36.

Now $\quad 36 = 13m - 16$

$$36 + 16 = 13m$$

$$52 = 13m \Rightarrow m = 4$$

59. Find HCF and LCM of 404 and 96 and verify that HCF $\times$ LCM $=$ Product of the two given numbers.

Ans : [Board 2018]

We have $\quad 404 = 2 \times 2 \times 101$

$$= 2^2 \times 101$$

$$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$$

$$= 2^5 \times 3$$

$$\text{HCF}(404, 96) = 2^2 = 4$$

$$\text{LCM}(404, 96) = 101 \times 2^5 \times 3 = 9696$$

$$\text{HCF} \times \text{LCM} = 4 \times 9696 = 38784$$

Also, $\quad 404 \times 96 = 38784$

Hence, HCF $\times$ LCM $=$ Product of 404 and 96

60. Find HCF of the numbers given below:
$k, 2k, 3k, 4k$ and $5k$,, where k is a positive integer.

Ans : [Board Term-1 2015, Set-FHN8MGD]

Here we can see easily that k is common factor between all and this is highest factor Thus HCF of k,$2k$,$3k$,$4k$ and $5k$, is k.

61. Find the HCF and LCM of 90 and 144 by the method of prime factorization.

Ans : [Board Term-1 2012]

We have $\quad 90 = 9 \times 10 = 9 \times 2 \times 5$

$$= 2 \times 3^2 \times 5$$

and $\quad 144 = 16 \times 9$

$$= 2^4 \times 3^2$$

$$\text{HCF} = 2 \times 3^2 = 18$$

$$\text{LCM} = 2^4 \times 3^2 \times 5 = 720$$

62. Given that HCF $(306, 1314) = 18$. Find LCM $(306, 1314)$

Ans : [Board Term-1 2013]

We have $\quad \text{HCF}(306, 1314) = 18$

$$\text{LCM}(306, 1314) = ?$$

Let $a = 306$ and $b = 1314$, then we have

$$\text{LCM}(a, b) \times \text{HCF}(a, b) = a \times b$$

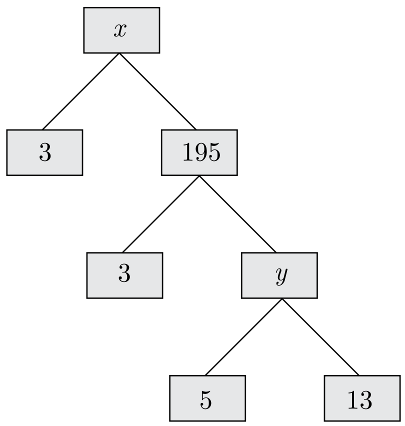
Substituting values we have

$$\text{LCM}(a, b) \times 18 = 306 \times 1314$$

$$\text{LCM}(a, b) = \frac{306 \times 1314}{18}$$

$$\text{LCM}(306, 1314) = 22,338$$

63. Complete the following factor tree and find the composite number x.



Ans : [Board Term-1 2015]

We have $\quad y = 5 \times 13 = 65$

and $\quad x = 3 \times 195 = 585$

64. Explain why $(7 \times 13 \times 11) + 11$ and $(7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1) + 3$ are composite numbers.

Ans : [Board Term-1 2012, Set-64]

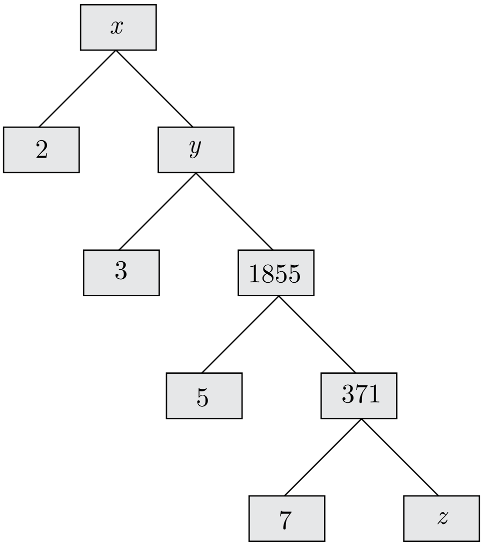
$$\begin{aligned}(7 \times 13 \times 11) + 11 &= 11 \times (7 \times 13 + 1)\\ &= 11 \times (91 + 1)\\ &= 11 \times 92\end{aligned}$$

and

$$\begin{aligned}(7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1) + 3\\ &= 3(7 \times 6 \times 5 \times 4 \times 2 \times 1 + 1)\\ &= 3 \times (1681) = 3 \times 41 \times 41\end{aligned}$$

Since given numbers have more than two prime factors, both number are composite.

65. Complete the following factor tree and find the composite number x



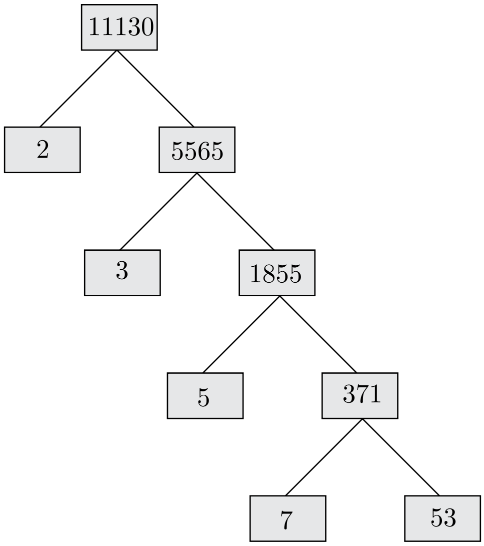
Ans : [Board Term-1 2015, Set DDE-M]

We have $z = \dfrac{371}{7} = 53$

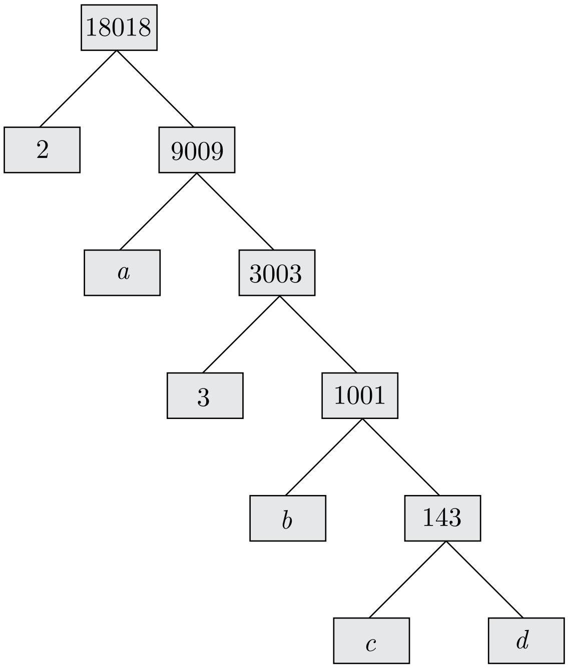
$y = 1855 \times 3 = 5565$

$x = 2 \times y = 2 \times 5565 = 11130$

Thus complete factor tree is as given below.



66. Find the missing numbers a, b, c and d in the given factor tree:



Ans : [Board Term-1 2012]

We have $a = \dfrac{9009}{3003} = 3$

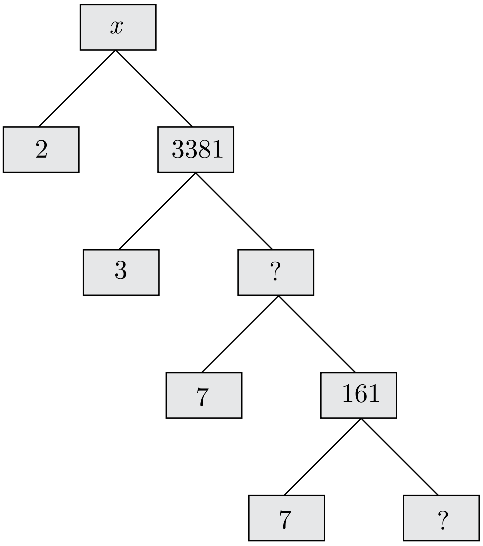
$b = \dfrac{1001}{143} = 7$

a113

Since $143 = 11 \times 13$,

Thus $c = 11$ and $d = 13$ or $c = 13$ and $d = 11$

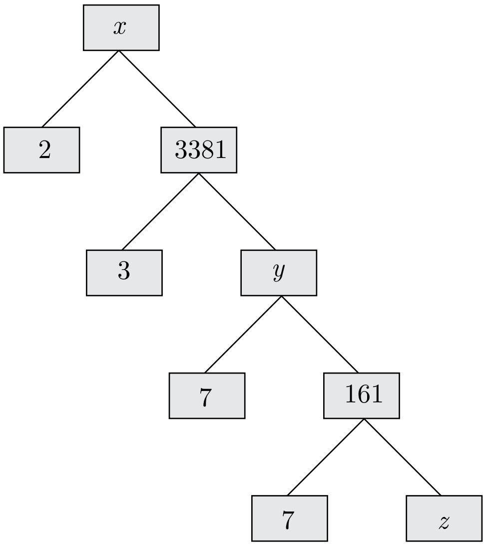
67. Complete the following factor tree and find the composite number x.



Ans : [Board Term-1 2015, 2014]

We complete the given factor tree writing variable y and z as following.

a114



We have $$z = \frac{161}{7} = 23$$

$$y = 7 \times 161 = 1127$$

Composite number, $x = 2 \times 3381 = 6762$

68. Explain whether $3 \times 12 \times 101 + 4$ is a prime number or a composite number.

Ans : [Board Term-1 2016-17 Set; 193RQTQ, 2015, DDE-E]

A prime number (or a prime) is a natural number greater than 1 that cannot be formed by multiplying two smaller natural numbers. A natural number greater than 1 that is not prime is called a composite number. For example, 5 is prime because the only ways of writing it as a product, 1×5 or 5×1, involve 5 itself. However, 6 is composite because it is the product of two numbers (2×3) that are both smaller than 6. Every composite number can be written as the product of two or more (not necessarily distinct) primes.

$$\begin{aligned} 3 \times 12 \times 101 + 4 &= 4(3 \times 3 \times 101 + 1) \\ &= 4(909 + 1) \\ &= 4(910) \\ &= 2 \times 2 \times (10 \times 7 \times 13) \\ &= 2 \times 2 \times 2 \times 5 \times 7 \times 13 \\ &= \text{a composite number} \end{aligned}$$

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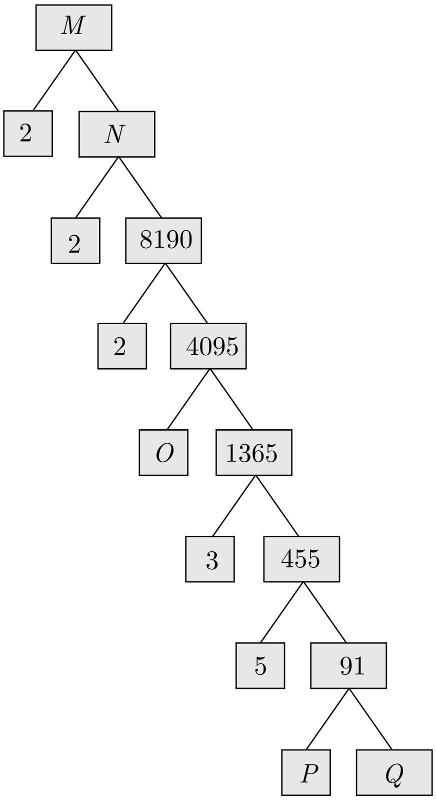
69. Complete the factor-tree and find the composite number M.



Ans : [Board Term-1 2013]

We have $$91 = P \times Q = 7 \times 13$$

So $P = 7, Q = 13$ or $P = 13, Q = 7$

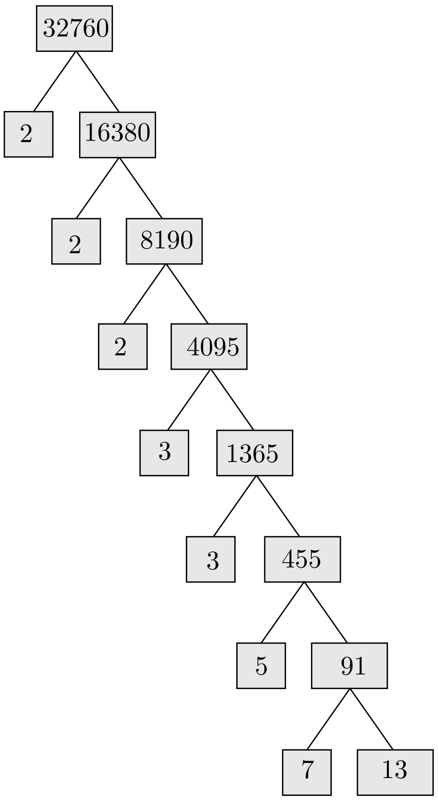
$$O = \frac{4095}{1365} = 3$$

$$N = 2 \times 8190 = 16380$$

Composite number,

$$M = 16380 \times 2 = 32760$$

Thus complete factor tree is shown below.



70. Find the smallest natural number by which 1200 should be multiplied so that the square root of the product is a rational number.

Ans : [Board Term-1 2016, 2015]

We have $1200 = 12 \times 100$

$$= 4 \times 3 \times 4 \times 25$$

$$= 4^2 \times 3 \times 5^2$$

Here if we multiply by 3, then its square root will be $4 \times 3 \times 5$ which is a rational number. Thus the required smallest natural number is 3.

71. Can two numbers have 15 as their HCF and their LCM? Give reasons.

Ans : [Board Te

LCM of two numbers should be exactly divisible by their HCF. Since, 15 does not divide 175, two numbers cannot have their HCF as 15 and LCM as 175.

72. Check whether 4^n can end with the digit 0 for any natural number n.

Ans : [Board Term-1 2015, Set-FHN8MGD; NCERT]

If the number 4^n, for any n, were to end with the digit zero, then it would be divisible by 5 and 2.

That is, the prime factorization of 4^n would contain the prime 5 and 2. This is not possible because the only prime in the factorization of $4^n = 2^{2n}$ is 2. So, the uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes in the factorization of 4^n. So, there is no natural number n for which 4^n ends with the digit zero. Hence 4^n cannot end with the digit zero.

73. Show that 7^n cannot end with the digit zero, for any natural number n.

Ans : [Board Term-1 2012, Set-63]

If the number 7^n, for any n, were to end with the digit zero, then it would be divisible by 5 and 2.

That is, the prime factorization of 7^n would contain the prime 5 and 2. This is not possible because the only prime in the factorization of $7^n = (1 \times 7)^n$ is 7. So, the uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes in the factorization of 7^n. So, there is no natural number n for which 7^n ends with the digit zero. Hence

7^n cannot end with the digit zero.

74. Check whether $(15)^n$ can end with digit 0 for any $n \in N$.

Ans : [Board Term-1 2012]

If the number $(15)^n$, for any n, were to end with the digit zero, then it would be divisible by 5 and 2.

That is, the prime factorization of $(15)^n$ would contain the prime 5 and 2. This is not possible because the only prime in the factorization of $(15)^n = (3 \times 5)^n$ are 3 and 5. The uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes in the factorization of $(15)^n$. Since there is no prime factor 2, $(15)^n$ cannot end with the digit zero.

75. The length, breadth and height of a room are 8 m 50 cm, 6 m 25 cm and 4 m 75 cm respectively. Find the length of the longest rod that can measure the dimensions of the room exactly.

Ans : [Board Term-1 2016]

Here we have to determine the HCF of all length which can measure all dimension.

$$\text{Length,} \quad l = 8 \text{ m } 50 \text{ cm} = 850 \text{ cm}$$
$$= 50 \times 17 = 2 \times 5^2 \times 17$$
$$\text{Breadth,} \quad b = 6 \text{ m } 25 \text{ cm} = 625 \text{ cm}$$
$$= 25 \times 25 = 5^2 \times 5^2$$
$$\text{Height,} \quad h = 4 \text{ m } 75 \text{ cm} = 475 \text{ cm}$$
$$= 25 \times 19 = 5^2 \times 19$$
$$\text{HCF}(l, b, h) = \text{HCF } (850, 625, 475)$$
$$= \text{HCF } (2 \times 5^2 \times 17, \; 5^2, \; 5^2 \times 19)$$
$$= 5^2 = 25 \text{ cm}$$

Thus 25 cm rod can measure the dimensions of the room exactly. This is longest rod that can measure exactly.

76. Show that $5\sqrt{6}$ is an irrational number.

Ans : [Board Term-1 2015]

Let $5\sqrt{6}$ be a rational number, which can be expressed as $\frac{a}{b}$, where $b \neq 0$; a and b are co-primes.

$$\text{Now} \quad 5\sqrt{6} = \frac{a}{b}$$
$$\sqrt{6} = \frac{a}{5b}$$
$$\text{or,} \quad \sqrt{6} = \text{rational}$$

But, $\sqrt{6}$ is an irrational number. Thus, our assumption is wrong. Hence, $5\sqrt{6}$ is an irrational number.

77. Write the denominator of the rational number $\frac{257}{500}$ in the form $2^m \times 5^n$, where m and n are non-negative integers. Hence write its decimal expansion without actual division.

Ans : [Board Term-1 2012, NCERT Exemplar]

$$\text{We have} \quad 500 = 25 \times 20$$
$$= 5^2 \times 5 \times 4$$
$$= 5^3 \times 2^2$$

Here denominator is 500 which can be written as $2^2 \times 5^3$.

Now decimal expansion,

$$\frac{257}{500} = \frac{257 \times 2}{2 \times 2^2 \times 5^3} = \frac{514}{10^3}$$
$$= 0.514$$

78. Write a rational number between $\sqrt{2}$ and $\sqrt{3}$.

Ans : [K.V.S.]

We have $\sqrt{2} = \sqrt{\frac{200}{100}}$ and $\sqrt{3} = \sqrt{\frac{300}{100}}$

We need to find a rational number x such that

$$\frac{1}{10}\sqrt{200} < x < \frac{1}{10}\sqrt{300}$$

Choosing any perfect square such as 225 or 256 in between 200 and 300, we have

$$x = \sqrt{\frac{225}{100}} = \frac{15}{10} = \frac{5}{3}$$

Similarly if we choose 256, then we have

$$x = \sqrt{\frac{256}{100}} = \frac{16}{10} = \frac{8}{5}$$

79. Write the rational number $\frac{7}{75}$ will have a terminating decimal expansion. or a non-terminating repeating decimal.

Ans : [Board 2018 SQP]

$$\text{We have} \quad \frac{7}{75} = \frac{7}{3 \times 5^2}$$

The denominator of rational number $\frac{7}{75}$ can not be written in form $2^m 5^n$ So it is non-terminating repeating decimal expansion.

80. Show that 571 is a prime number.

Ans :

$$\text{Let} \quad x = 571$$
$$\sqrt{x} = \sqrt{571}$$

Now 571 lies between the perfect squares of $(23)^2 = 529$ and $(24)^2 = 576$. Prime numbers less than 24 are 2, 3, 5, 7, 11, 13, 17, 19, 23. Here 571 is not divisible by any of the above numbers, thus 571 is a prime number.

81. If two positive integers p and q are written as $p = a^2 b^3$ and $q = a^3 b$, where a and b are prime numbers than verify $\text{LCM}(p, q) \times \text{HCF}(q, q) = pq$

Ans : [Sample Paper 2017]

We have $p = a^2 b^3 = a \times a \times b \times b \times b$

and $q = a^3 b = a \times a \times a \times b$

Now $\text{LCM}(p, q) = a \times a \times a \times b \times b \times b$

$= a^3 b^3$

and $\text{HCF}(p, q) = a \times a \times b$

$= a^2 b$

$$\begin{aligned}\text{LCM}(p, q) \times \text{HCF}(p, q) &= a^3 b^3 \times a^2 b \\ &= a^5 b^4 \\ &= a^2 b^3 \times a^3 b \\ &= pq\end{aligned}$$

THREE MARKS QUESTIONS

82. An army contingent of 612 members is to march behind an army band of 48 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Ans : [Board 2020 Delhi Basic]

Let the number of columns be x which is the largest number, which should divide both 612 and 48. It means x should be HCF of 612 and 48.

We can write 612 and 48 as follows

$$\begin{aligned}612 &= 2 \times 2 \times 3 \times 3 \times 5 \times 17 \\ 48 &= 2 \times 2 \times 2 \times 2 \times 3 \\ \text{HCF}(612,\ 28) &= 2 \times 2 \times 3 = 12\end{aligned}$$

a230

Thus HCF of 104 and 96 is 12 i.e. 12 columns are required.

Here we have solved using Euclid's algorithm but you can solve this problem by simple mehtod of HCF.

83. Given that $\sqrt{5}$ is irrational, prove that $2\sqrt{5} - 3$ is an irrational number.

Ans : [Board 2020 SQP Standard]

Assume that $2\sqrt{5} - 3$ is a rational number. Therefore, we can write it in the form of $\frac{p}{q}$ where p and q are co-prime integers and $q \neq 0$.

Now $2\sqrt{5} - 3 = \dfrac{p}{q}$

where $q \neq 0$ and p and q are co-prime integers.

Rewriting the above expression as,

$$2\sqrt{5} = \frac{p}{q} + 3$$

$$\sqrt{5} = \frac{p + 3q}{2q}$$

Here $\frac{p+3q}{2q}$ is rational because p and q are co-prime integers, thus $\sqrt{5}$ should be a rational number. But $\sqrt{5}$ is irrational. This contradicts the given fact that $\sqrt{5}$ is irrational. Hence $2\sqrt{5} - 3$ is an irrational number.

84. Prove that $\frac{2+\sqrt{3}}{5}$ is an irrational number, given that $\sqrt{3}$ is an irrational number.

Ans : [Board 2019 Delhi]

Assume that $\frac{2+\sqrt{3}}{5}$ is a rational number. Therefore, we can write it in the form of $\frac{p}{q}$ where p and q are co-prime integers and $q \neq 0$.

$$\begin{aligned}\frac{2+\sqrt{3}}{5} &= \frac{p}{q} \\ 2 + \sqrt{3} &= \frac{5p}{q} \\ \sqrt{3} &= \frac{5p}{q} - 2 \\ \sqrt{3} &= \frac{5p - 2q}{q}\end{aligned}$$

Since, p and q are co-prime integers, then $\frac{5p-2q}{q}$ is a rational number. But this contradicts the fact that $\sqrt{3}$ is an irrational number. So, our assumption is wrong. Therefore $\frac{2+\sqrt{3}}{5}$ is an irrational number.

85. Given that $\sqrt{3}$ is irrational, prove that $(5 + 2\sqrt{3})$ is an irrational number.

Ans : [Board 2020 Delhi Basic]

Assume that $(5 + 2\sqrt{3})$ is a rational number. Therefore, we can write it in the form of $\frac{p}{q}$ where p

and q are co-prime integers and $q \neq 0$.

Now $$5 + 2\sqrt{3} = \frac{p}{q}$$

where $q \neq 0$ and p and q are integers.

Rewriting the above expression as,

a235

$$2\sqrt{3} = \frac{p}{q} - 5$$

$$\sqrt{3} = \frac{p - 5q}{2q}$$

Here $\frac{p-5q}{2q}$ is rational because p and q are co-prime integers, thus $\sqrt{3}$ should be a rational number. But $\sqrt{3}$ is irrational. This contradicts the given fact that $\sqrt{3}$ is irrational. Hence $(5 + 2\sqrt{3})$ is an irrational number.

86. Prove that $2 + 5\sqrt{3}$ is an irrational number, given that $\sqrt{3}$ is an irrational number.

Ans : [Board 2019 OD]

Assume that $2 + 5\sqrt{3}$ is a rational number. Therefore, we can write it in the form of $\frac{p}{q}$ where p and q are co-prime integers and $q \neq 0$.

a236

$$2 + 5\sqrt{3} = \frac{p}{q}, \quad q \neq 0$$

$$5\sqrt{3} = \frac{p}{q} - 2$$

$$5\sqrt{3} = \frac{p - 2q}{q}$$

$$\sqrt{3} = \frac{p - 2q}{5q}$$

Here $\sqrt{3}$ is irrational and $\frac{p-2q}{5q}$ is rational because p and q are co-prime integers. But rational number cannot be equal to an irrational number. Hence $2 + 5\sqrt{3}$ is an irrational number.

87. Given that $\sqrt{2}$ is irrational, prove that $(5 + 3\sqrt{2})$ is an irrational number.

Ans : [Board 2018]

Assume that $(5 + 3\sqrt{2})$ is a rational number. Therefore, we can write it in the form of $\frac{p}{q}$ where p and q are co-prime integers and $q \neq 0$.

Now $$5 + 3\sqrt{2} = \frac{p}{q}$$

where $q \neq 0$ and p and q are integers.

Rewriting the above expression as,

a237

$$3\sqrt{2} = \frac{p}{q} - 5$$

$$\sqrt{2} = \frac{p - 5q}{3q}$$

Here $\frac{p-5q}{3q}$ is rational because p and q are co-prime integers, thus $\sqrt{2}$ should be a rational number. But $\sqrt{2}$ is irrational. This contradicts the given fact that $\sqrt{2}$ is irrational. Hence $(5 + 3\sqrt{2})$ is an irrational number.

88. Write the smallest number which is divisible by both 306 and 657.

Ans : [Board 2019 OD]

The smallest number that is divisible by two numbers is obtained by finding the LCM of these numbers Here, the given numbers are 306 and 657.

$$306 = 6 \times 51 = 3 \times 2 \times 3 \times 17$$

$$657 = 9 \times 73 = 3 \times 3 \times 73$$

$$\text{LCM}(306, 657) = 2 \times 3 \times 3 \times 17 \times 73$$

$$= 22338$$

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Hence, the smallest number which is divisible by 306 and 657 is 22338.

89. Show that numbers 8^n can never end with digit 0 of any natural number n.

Ans : [Board Term-1 2015, NCERT]

a128

If the number 8^n, for any n, were to end with the digit zero, then it would be divisible by 5 and 2. That is, the prime factorization of 8^n would contain the prime 5 and 2. This is not possible because the only prime in the factorization of $(8)^n = (2^3)^n = 2^{3n}$ is 2. The uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes in the factorization of $(8)^n$. Since there is no prime factor 5, $(8)^n$ cannot end with the digit zero.

90. 144 cartons of Coke cans and 90 cartons of Pepsi cans are to be stacked in a canteen. If each stack is of the same height and if it equal contain cartons of the same drink, what would be the greatest number of cartons each stack would have?

Ans : [Board Term-1 2011]

The required answer will be HCF of 144 and 90.

a130

$$144 = 2^4 \times 3^2$$

$$90 = 2 \times 3^2 \times 5$$

$$\text{HCF}(144, 90) = 2 \times 3^2 = 18$$

Thus each stack would have 18 cartons.

91. Three bells toll at intervals of 9, 12, 15 minutes respectively. If they start tolling together, after what time will they next toll together?

Ans : [Board Term-1 2011, Set-44]

The required answer is the LCM of 9, 12, and 15 minutes.

Finding prime factor of given number we have,

$$9 = 3 \times 3 = 3^2$$

$$12 = 2 \times 2 \times 3 = 2^2 \times 3$$

$$15 = 3 \times 5$$

$$\text{LCM}(9, 12, 15) = 2^2 \times 3^2 \times 5$$

$$= 150 \text{ minutes}$$

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The bells will toll next together after 180 minutes.

92. Find HCF and LCM of 16 and 36 by prime factorization and check your answer.

Ans :

Finding prime factor of given number we have,

$$16 = 2 \times 2 \times 2 \times 2 = 2^4$$

$$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$$

$$\text{HCF}(16, 36) = 2 \times 2 = 4$$

$$\text{LCM}(16, 36) = 2^4 \times 3^2$$

$$= 16 \times 9 = 144$$

a132

Check :

$$\text{HCF}(a, b) \times \text{LCM}(a, b) = a \times b$$

or, $$4 \times 144 = 16 \times 36$$

$$576 = 576$$

Thus $$\text{LHS} = \text{RHS}$$

93. Find the HCF and LCM of 510 and 92 and verify that HCF × LCM = Product of two given numbers.

Ans : [Board Term-1 2011]

Finding prime factor of given number we have,

$$92 = 2^2 \times 23$$

$$510 = 30 \times 17 = 2 \times 3 \times 5 \times 17$$

$$\text{HCF}(510, 92) = 2$$

$$\text{LCM}(510, 92 = 2^2 \times 23 \times 3 \times 5 \times 14$$

$$= 23460$$

$$\text{HCF}(510, 92) \times \text{LCM}(510, 92)$$

$$= 2 \times 23460 = 46920$$

a133

Product of two numbers $= 510 \times 92 = 46920$

Hence, $\text{HCF} \times \text{LCM} = \text{Product of two numbers}$

94. The HCF of 65 and 117 is expressible in the form $65m–117$. Find the value of m. Also find the LCM of 65 and 117 using prime factorization method.

Ans : [Board Term-1 2011, Set-40]

Finding prime factor of given number we have.

$$117 = 13 \times 2 \times 3$$

$$65 = 13 \times 5$$

VIDEO Click Here

$$\text{HCF}(117, 65) = 13$$

$$\text{LCM}(117, 65) = 13 \times 5 \times 3 \times 3 = 585$$

$$\text{HCF} = 65m - 117$$

$$13 = 65m–117$$

$$65m = 117 + 13 = 130$$

$$m = \frac{130}{65} = 2$$

95. Express $\left(\frac{15}{4} + \frac{5}{40}\right)$ as a decimal fraction without actual division.

Ans : [Board Term-1 2011, Set-74]

We have $$\frac{15}{4} + \frac{5}{40} = \frac{15}{4} \times \frac{25}{25} + \frac{5}{40} \times \frac{25}{25}$$

$$= \frac{375}{100} + \frac{125}{1000}$$

a158

$$= 3.75 + 0.125 = 3.875$$

96. Express the number $0.3\overline{178}$ in the form of rational number $\frac{a}{b}$.

Ans : [Board Term-1 2011, Set-A1]

Let $$x = 0.3\overline{178}$$

$$x = 0.3178178178$$

$$10,000x = 3178.178178...$$

a159

$$10x = 3.178178....$$

Subtracting, $$9990x = 3175$$

or, $$x = \frac{3175}{9990} = \frac{635}{1998}$$

97. Prove that $\sqrt{2}$ is an irrational number.

Ans : [Board Term-1 2011, NCERT]

Let $\sqrt{2}$ be a rational number.

Then $\sqrt{2} = \frac{p}{q},$

where p and q are co-prime integers and $q \neq 0$ On squaring both the sides we have,

$$2 = \frac{p^2}{q^2}$$

or, $p^2 = 2p^2$

Since p^2 is divisible by 2, thus p is also divisible by 2.

Let $p = 2r$ for some positive integer r, then we have

$$p^2 = 4r^2$$

$$2q^2 = 4r^2$$

or, $q^2 = 2r^2$

Since q^2 is divisible by 2, thus q is also divisible by 2.

We have seen that p and q are divisible by 2, which contradicts the fact that p and q are co-primes. Hence, our assumption is false and $\sqrt{2}$ is irrational.

98. If p is prime number, then prove that $\sqrt{p}$ is an irrational.

Ans : [Board Term-1 2013]

Let p be a prime number and if possible, let $\sqrt{p}$ be rational

Thus $\sqrt{p} = \frac{m}{n},$

where m and n are co-primes and $n \neq 0$.
Squaring on both sides, we get

$$p = \frac{m^2}{n^2}$$

or, $pn^2 = m^2$ ...(1)

Here p divides pn^2. Thus p divides m^2 and in result p also divides m.

Let $m = pq$ for some integer q and putting $m = pq$ in eq. (1), we have

$$pn^2 = p^2q^2$$

or, $n^2 = pq^2$

Here p divides pq^2.Thus p divides n^2 and in result p also divides n.

[$\because$ p is prime and p divides $n^2 \Rightarrow p$ divides n]

Thus p is a common factor of m and n but this contradicts the fact that m and n are primes. The contradiction arises by assuming that $\sqrt{p}$ is rational.

Hence, $\sqrt{p}$ is irrational.

99. Prove that $3+\sqrt{5}$ is an irrational number.

Ans :

Assume that $3+\sqrt{5}$ is a rational number, then we have

$$3+\sqrt{5} = \frac{p}{q}, \quad q \neq 0$$

$$\sqrt{5} = \frac{p}{q} - 3$$

$$\sqrt{5} = \frac{p-3q}{q}$$

Here $\sqrt{5}$ is irrational and $\frac{p-3q}{q}$ is rational. But rational number cannot be equal to an irrational number. Hence $3+\sqrt{5}$ is an irrational number.

100. Prove that $\sqrt{5}$ is an irrational number and hence show that $2-\sqrt{5}$ is also an irrational number.

Ans : [Board Term-1 2011]

Assume that $\sqrt{5}$ be a rational number then we have

$$\sqrt{5} = \frac{a}{b}, \qquad (a, b \text{ are co-primes and } b \neq 0)$$

$$a = b\sqrt{5}$$

Squaring both the sides, we have

$$a^2 = 5b^2$$

Thus 5 is a factor of a^2 and in result 5 is also a factor of a.

Let $a = 5c$ where c is some integer, then we have

$$a^2 = 25c^2$$

Substituting $a^2 = 5b^2$ we have

$$5b^2 = 25c^2$$

$$b^2 = 5c^2$$

Thus 5 is a factor of b^2 and in result 5 is also a factor of b.

Thus 5 is a common factor of a and b. But this contradicts the fact that a and b are co-primes. Thus, our assumption that $\sqrt{5}$ is rational number is wrong. Hence $\sqrt{5}$ is irrational.

Let us assume that $2-\sqrt{5}$ be rational equal to a, then we have

$$2-\sqrt{5} = a$$

$$2-a = \sqrt{5}$$

Since we have assume $2-a$ is rational, but $\sqrt{5}$ is not rational. Rational number cannot be equal to an irrational number. Thus $2-\sqrt{5}$ is irrational.

101. Show that exactly one of the number $n, n+2$ or $n+4$ is divisible by 3.

Ans : [Sample Paper 2017]

a136

If n is divisible by 3, clearly $n+2$ and $n+4$ is not divisible by 3.

If n is not divisible by 3, then two case arise as given below.

Case 1: $n = 3k+1$

$$n+2 = 3k+1+2 = 3k+3 = 3(k+1)$$

and $$n+4 = 3k+1+4 = 3k+5 = 3(k+1)+2$$

We can clearly see that in this case $n+2$ is divisible by 3 and $n+4$ is not divisible by 3. Thus in this case only $n+2$ is divisible by 3.

Case 1: $n = 3k+2$

$$n+2 = 3k+2+2 = 3k+4 = 3(k+1)+1$$

and $$n+4 = 3k+2+4 = 3k+6 = 3(k+2)$$

We can clearly see that in this case $n+4$ is divisible by 3 and $n+2$ is not divisible by 3. Thus in this case only $n+4$ is divisible by 3.

Hence, exactly one of the numbers $n, n+2, n+4$ is divisible by 3.

FOUR MARKS QUESTIONS

102. Prove that $\sqrt{3}$ is an irrational number.

Ans : [Board 2020 OD Basic]

Assume that $\sqrt{3}$ is a rational number. Therefore, we can write it in the form of $\frac{a}{b}$ where a and b are co-prime integers and $q \neq 0$.

Assume that $\sqrt{3}$ be a rational number then we have

$$\sqrt{3} = \frac{a}{b},$$

where a and b are co-primes and $b \neq 0$.

a231

Now $$a = b\sqrt{3}$$

Squaring both the sides, we have

$$a^2 = 3b^2$$

Thus 3 is a factor of a^2 and in result 3 is also a factor of a.

Let $a = 3c$ where c is some integer, then we have

$$a^2 = 9c^2$$

Substituting $a^2 = 3b^2$ we have

$$3b^2 = 9c^2$$

$$b^2 = 3c^2$$

Thus 3 is a factor of b^2 and in result 3 is also a factor of b.

Thus 3 is a common factor of a and b. But this contradicts the fact that a and b are co-primes. Thus, our assumption that $\sqrt{3}$ is rational number is wrong. Hence $\sqrt{3}$ is irrational.

103. Prove that $\sqrt{5}$ is an irrational number.

Ans : [Board 2020 OD Standard]

Assume that $\sqrt{5}$ be a rational number then we have

$$\sqrt{5} = \frac{a}{b},$$

where a and b are co-primes and $b \neq 0$.

$$a = b\sqrt{5}$$

Squaring both the sides, we have

$$a^2 = 5b^2$$

Thus 5 is a factor of a^2 and in result 5 is also a factor of a.

Let $a = 5c$ where c is some integer, then we have

$$a^2 = 25c^2$$

Substituting $a^2 = 5b^2$ we have

$$5b^2 = 25c^2$$

$$b^2 = 5c^2$$

Thus 5 is a factor of b^2 and in result 5 is also a factor of b.

Thus 5 is a common factor of a and b. But this contradicts the fact that a and b are co-primes. Thus, our assumption that $\sqrt{5}$ is rational number is wrong. Hence $\sqrt{5}$ is irrational.

104. Find HCF and LCM of 378, 180 and 420 by prime factorization method. Is HCF × LCM of these numbers equal to the product of the given three numbers?

Ans :

Finding prime factor of given number we have,

$$378 = 2 \times 3^3 \times 7$$

$$180 = 2^2 \times 3^2 \times 5$$

$$420 = 2^2 \times 3 \times 7 \times 5$$

a137

$$\text{HCF}(378,\ 180,\ 420) = 2 \times 3 = 6$$

$$\text{LCM}(378, 180, 420) = 2^2 \times 3^3 \times 5 \times 7$$
$$= 2^2 \times 3^3 \times 5 \times 7 = 3780$$
$$\text{HCF}\times\text{LCM} = 6\times 3780 = 22680$$

Product of given numbers

$$= 378 \times 180 \times 420$$
$$= 28576800$$

Hence, HCF $\times$ LCM $\neq$ Product of three numbers.

105. State Fundamental theorem of Arithmetic. Find LCM of numbers 2520 and 10530 by prime factorization by 3.

Ans : [Board Term-1 2016]

The fundamental theorem of arithmetic (FTA), also called the unique factorization theorem or the unique-prime-factorization theorem, states that every integer greater than 1 either is prime itself or is the product of a unique combination of prime numbers.

a138

OR

Every composite number can be expressed as the product powers of primes and this factorization is unique.

Finding prime factor of given number we have,

$$2520 = 20 \times 126 = 20 \times 6 \times 21$$
$$= 2^3 \times 3^2 \times 5 \times 7$$
$$10530 = 30 \times 351 = 30 \times 9 \times 39$$
$$= 30 \times 9 \times 3 \times 13$$
$$= 2 \times 3^4 \times 5 \times 13$$
$$\text{LCM}(2520, 10530) = 2^3 \times 3^4 \times 5 \times 7 \times 13$$
$$= 294840$$

106. Can the number 6^n, n being a natural number, end with the digit 5 ? Give reasons.

Ans : [Board Term-1 2015]

If the number 6^n for any n, were to end with the digit five, then it would be divisible by 5. That is, the prime factorization of 6^n would contain the prime 5. This is not possible because the only prime in the factorization of $6^n = (2 \times 3)^n$ are 2 and 3. The uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes in the factorization of 6^n. Since there is no prime factor 5, 6^n cannot end with the digit five.

a139

107. State Fundamental theorem of Arithmetic. Is it possible that HCF and LCM of two numbers be 24 and 540 respectively. Justify your answer.

Ans : [Board Term-1 2015]

Fundamental theorem of Arithmetic : Every integer greater than one ither is prime itself or is the product of prime numbers and that this product is unique. Up to the order of the factors.

a140

LCM of two numbers should be exactly divisible by their HCF. In other words LCM is always a multiple of HCF. Since, 24 does not divide 540 two numbers cannot have their HCF as 24 and LCM as 540.

$$\text{HCF} = 24$$
$$\text{LCM} = 540$$
$$\frac{LCM}{HCF} = \frac{540}{24} = 22.5 \text{ not an integer}$$

108. For any positive integer n, prove that $n^3 - n$ is divisible by 6.

Ans : [Board Term-1 2015, 2012]

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We have $\quad n^3 - n = n(n^2 - 1)$
$$= (n-1)n(n+1)$$
$$= (n-1)n(n+1)$$

Thus $n^3 - n$ is product of three consecutive positive integers.

Since, any positive integers a is of the form $3q, 3q+1$ or $3q+2$ for some integer q.

Let $a, a+1, a+2$ be any three consecutive integers.

Case I : $a = 3q$

If $a = 3q$ then,

$$a(a+1)(a+2) = 3q(3q+1)(3q+2)$$

Product of two consecutive integers $(3q+1)$ and $(3q+2)$ is an even integer, say $2r$.

Thus $a(a+1)(a+2) = 3q(2r)$
$$= 6qr, \text{ which is divisible by 6.}$$

Case II : $a = 3q+1$

If $a = 3q+1$ then

$$\begin{aligned} a(a+1)(a+2) &= (3q+1)(3q+2)(3q+3) \\ &= (2r)(3)(q+1) \\ &= 6r(q+1) \end{aligned}$$

which is divisible by 6.

Case III : $a = 3q+2$

If $a = 3q+2$ then

$$\begin{aligned} a(a+1)(a+2) &= (3q+2)(3q+3)(3q+4) \\ &= 3(3q+2)(q+1)(3q+4) \end{aligned}$$

Here $(3q+2)$ and $= 3(3q+2)(q+1)(3q+4)$

$= $ multiple of 6 every q

$= 6r$ (say)

which is divisible by 6. Hence, the product of three consecutive integers is divisible by 6 and $n^3 - n$ is also divisible by 3.

109. Prove that $n^2 - n$ is divisible by 2 for every positive integer n.

Ans : [Board Term-1 2012 Set-25]

We have $\quad n^2 - n = n(n-1)$

Thus $n^2 - n$ is product of two consecutive positive integers.

Any positive integer is of the form $2q$ or $2q+1$, for some integer q.

a144

Case 1 : $n = 2q$

If $n = 2q$ we have

$$\begin{aligned} n(n-1) &= 2q(2q-1) \\ &= 2m, \end{aligned}$$

where $m = q(2q-1)$ which is divisible by 2.

Case 1 : $n = 2q+1$

If $n = 2q+1$, we have

$$\begin{aligned} n(n-1) &= (2q+1)(2q+1-1) \\ &= 2q(2q+1) \\ &= 2m \end{aligned}$$

where $m = q(2q+1)$ which is divisible by 2.

Hence, $n^2 - n$ is divisible by 2 for every positive integer n.

110. Prove that $\sqrt{3}$ is an irrational number. Hence, show that $7+2\sqrt{3}$ is also an irrational number.

Ans : [Board Term-1 2012]

Assume that $\sqrt{3}$ be a rational number then we have

$$\sqrt{3} = \frac{a}{b}, \qquad (a,b \text{ are co-primes and } b \neq 0)$$

$$a = b\sqrt{3}$$

Squaring both the sides, we have

$$a^2 = 3b^2$$

Thus 3 is a factor of a^2 and in result 3 is also a factor of a.

Let $a = 3c$ where c is some integer, then we have

$$a^2 = 9c^2$$

Substituting $a^2 = 9b^2$ we have

VIDEO Click Here

$$\begin{aligned} 3b^2 &= 9c^2 \\ b^2 &= 3c^2 \end{aligned}$$

Thus 3 is a factor of b^2 and in result 3 is also a factor of b.

Thus 3 is a common factor of a and b. But this contradicts the fact that a and b are co-primes. Thus, our assumption that $\sqrt{3}$ is rational number is wrong. Hence $\sqrt{3}$ is irrational.

Let us assume that $7+2\sqrt{3}$ be rational equal to a, then we have

$$7+2\sqrt{3} = \frac{p}{q} \qquad q \neq 0 \text{ and } p \text{ and } q \text{ are co-primes}$$

$$2\sqrt{3} = \frac{p}{q} - 7 = \frac{p-7q}{q}$$

or $$\sqrt{3} = \frac{p-7q}{2q}$$

Here $p-7q$ and $2q$ both are integers, hence $\sqrt{3}$ should be a rational number. But this contradicts the fact that $\sqrt{3}$ is an irrational number. Hence our assumption is not correct and $7+2\sqrt{3}$ is irrational.

111. Show that there is no positive integer n, for which $\sqrt{n-1}+\sqrt{n-1}$ is rational.

Ans : [Board Term-1 2012]

Let us assume that there is a positive integer n for which $\sqrt{n-1}+\sqrt{n-1}$ is rational and equal to $\frac{p}{q}$, where p and q are positive integers and $(q \neq 0)$.

a165

$$\sqrt{n-1}+\sqrt{n-1} = \frac{p}{q} \qquad \ldots(1)$$

or, $$\frac{q}{p} = \frac{1}{\sqrt{n-1}+\sqrt{n+1}}$$

$$= \frac{\sqrt{n-1}-\sqrt{n+1}}{(\sqrt{n-1}+\sqrt{n+1})(\sqrt{n-1}-\sqrt{n+1})}$$

$$= \frac{\sqrt{n-1}-\sqrt{n+1}}{(n-1)-(n+1)}$$

or $$\frac{q}{p} = \frac{\sqrt{n-1}-\sqrt{n+1}}{-2}$$

$$\sqrt{n+1}-\sqrt{n-1} = \frac{2q}{p} \quad ...(2)$$

Adding (1) and (2), we get

$$2\sqrt{n+1} = \frac{p}{q}+\frac{2q}{p} = \frac{p^2+2q^2}{pq} \quad ...(3)$$

Subtracting (2) from (1) we have

$$2\sqrt{n-1} = \frac{p^2-2q^2}{pq} \quad ...(4)$$

From (3) and (4), we observe that $\sqrt{n+1}$ and $\sqrt{n-1}$ both are rational because p and q both are rational. But it possible only when $(n+1)$ and $(n-1)$ both are perfect squares. But they differ by 2 and two perfect squares never differ by 2. So both $(n+1)$ and $(n-1)$ cannot be perfect squares, hence there is no positive integer n for which $\sqrt{n-1}+\sqrt{n+1}$ is rational.

CHAPTER 2

POLYNOMIALS

ONE MARK QUESTIONS

MULTIPLE CHOICE QUESTIONS

1. If one zero of a quadratic polynomial $(kx^2 + 3x + k)$ is 2, then the value of k is

(a) $\frac{5}{6}$ (b) $-\frac{5}{6}$

(c) $\frac{6}{5}$ (d) $-\frac{6}{5}$

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Ans : [Board 2020 Delhi Basic]

We have $p(x) = kx^2 + 3x + k$

Since, 2 is a zero of the quadratic polynomial

$$p(2) = 0$$

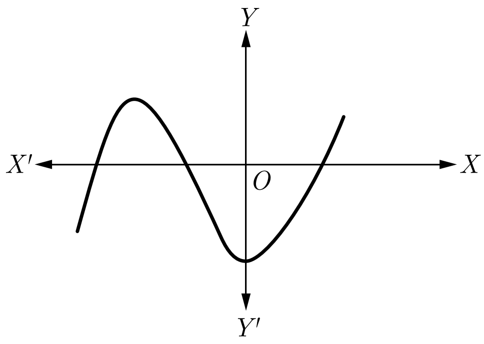
$$k(2)^2 + 3(2) + k = 0$$

$$4k + 6 + k = 0$$

$$5k = -6 \Rightarrow k = -\frac{6}{5}$$

Thus (d) is correct option.

2. The graph of a polynomial is shown in Figure, then the number of its zeroes is



b173

(a) 3 (b) 1

(c) 2 (d) 4

Ans : [Board 2020 Delhi Basic]

Since, the graph cuts the x-axis at 3 points, the number of zeroes of polynomial $p(x)$ is 3.

Thus (a) is correct option.

3. The maximum number of zeroes a cubic polynomial can have, is

(a) 1 (b) 4

(c) 2 (d) 3

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Ans : [Board 2020 OD Basic]

A cubic polynomial has maximum 3 zeroes because its degree is 3.

Thus (d) is correct option.

4. If one zero of the quadratic polynomial $x^2 + 3x + k$ is 2, then the value of k is

(a) 10 (b) -10

(c) -7 (d) -2

Ans : [Board 2020 Delhi Standard]

We have $p(x) = x^2 + 3x + k$

b175

If 2 is a zero of $p(x)$, then we have

$$p(2) = 0$$

$$(2)^2 + 3(2) + k = 0$$

$$4 + 6 + k = 0$$

$$10 + k = 0 \Rightarrow k = -10$$

Thus (b) is correct option.

5. The quadratic polynomial, the sum of whose zeroes is -5 and their product is 6, is

(a) $x^2 + 5x + 6$ (b) $x^2 - 5x + 6$

(c) $x^2 - 5x - 6$ (d) $-x^2 + 5x + 6$

Ans : [Board 2020 Delhi Standard]

Let α and β be the zeroes of the quadratic polynomial, then we have

$$\alpha + \beta = -5$$

and $$\alpha\beta = 6$$

b176

Now $$p(x) = x^2 - (\alpha + \beta)x + \alpha\beta$$

$$= x^2 - (-5)x + 6$$

$$= x^2 + 5x + 6$$

Thus (a) is correct option.

6. If one zero of the polynomial $(3x^2 + 8x + k)$ is the reciprocal of the other, then value of k is

(a) 3 (b) -3

(c) $\frac{1}{3}$ (d) $-\frac{1}{3}$

b177

Ans : [Board 2020 OD Basic]

Let the zeroes be α and $\frac{1}{\alpha}$.

Product of zeroes, $\alpha \cdot \frac{1}{\alpha} = \frac{\text{constant}}{\text{coefficient of } x^2}$

$$1 = \frac{k}{3} \Rightarrow k = 3$$

Thus (a) is correct option.

7. The zeroes of the polynomial $x^2 - 3x - m(m+3)$ are

(a) $m, m+3$ (b) $-m, m+3$

(c) $m, -(m+3)$ (d) $-m, -(m+3)$

Ans : [Board 2020 OD Standard]

b178

We have $p(x) = x^2 - 3x - m(m+3)$

Substituting $x = -m$ in $p(x)$ we have

$$p(-m) = (-m)^2 - 3(-m) - m(m+3)$$
$$= m^2 + 3m - m^2 - 3m = 0$$

Thus $x = -m$ is a zero of given polynomial.

Now substituting $x = m+3$ in given polynomial we have

$$p(x) = (m+3)^2 - 3(m+3) - m(m+3)$$
$$= (m+3)[m+3-3-m]$$
$$= (m+3)[0] = 0$$

Thus $x = m+3$ is also a zero of given polynomial.

Hence, $-m$ and $m+3$ are the zeroes of given polynomial.

Thus (b) is correct option.

8. The value of x, for which the polynomials $x^2 - 1$ and $x^2 - 2x + 1$ vanish simultaneously, is

(a) 2 (b) -2

(c) -1 (d) 1

b179

Ans :

Both expression $(x-1)(x+1)$ and $(x-1)(x-1)$ have 1 as zero. This both vanish if $x = 1$.

Thus (d) is correct option.

9. If α and β are zeroes and the quadratic polynomial $f(x) = x^2 - x - 4$, then the value of $\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta$ is

(a) $\frac{15}{4}$ (b) $\frac{-15}{4}$

b180

(c) 4 (d) 15

Ans :

We have $f(x) = x^2 - x - 4$

$$\alpha + \beta = -\frac{-1}{1} = 1 \text{ and } \alpha\beta = \frac{-4}{1} - 4$$

Now $\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta = \frac{\alpha+\beta}{\alpha\beta} - \alpha\beta$

$$= -\frac{1}{4} + 4 = \frac{15}{4}$$

Thus (a) is correct option.

10. The value of the polynomial $x^8 - x^5 + x^2 - x + 1$ is

(a) positive for all the real numbers

(b) negative for all the real numbers

(c) 0

(d) depends on value of x

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Ans :

We have $f(x) = \mathrm{x}^8 - \mathrm{x}^5 + \mathrm{x}^2 - \mathrm{x} + 1$

$f(x)$ is always positive for all $x > 1$

For $x = 1$ or 0, $\mathrm{f(x)} = 1 > 0$

For $x < 0$ each term of $f(x)$ is positive, thus $f(x) > 0$. Hence, $f(x)$ is positive for all real x.

Thus (a) is correct option.

11. Lowest value of $x^2 + 4x + 2$ is

(a) 0 (b) -2

(c) 2 (d) 4

b182

Ans :

$$x^2 + 4x + 2 = (x^2 + 4x + 4) - 2$$
$$= (x+2)^2 - 2$$

Here $(x+2)^2$ is always positive and its lowest value is zero. Thus lowest value of $(x+2)^2 - 2$ is -2 when $x + 2 = 0$.

Thus (b) is correct option.

12. If the sum of the zeroes of the polynomial $f(x) = 2x^3 - 3kx^2 + 4x - 5$ is 6, then the value of k is

(a) 2 (b) -2

(c) 4 (d) -4

b183

Ans :

Sum of the zeroes, $6 = \frac{3k}{2}$

$$k = \frac{12}{3} = 4$$

Thus (c) is correct option.

13. If the square of difference of the zeroes of the quadratic polynomial $x^2 + px + 45$ is equal to 144, then the value of p is

(a) ± 9 (b) ± 12

(c) ± 15 (d) ± 18

b184

Ans :

We have $f(x) = x^2 + px + 45$

Then, $\alpha + \beta = \frac{-p}{1} = -p$

and $\alpha\beta = \frac{45}{1} = 45$

According to given condition, we have

$$(\alpha - \beta)^2 = 144$$
$$(\alpha + \beta)^2 - 4\alpha\beta = 144$$
$$(-p)^2 - 4(45) = 144$$
$$p^2 = 144 + 180 = 324 \Rightarrow p = \pm 18$$

14. If one of the zeroes of the quadratic polynomial $(k-1)x^2 + kx + 1$ is -3, then the value of k is

(a) $\frac{4}{3}$ (b) $\frac{-4}{3}$

(c) $\frac{2}{3}$ (d) $-\frac{2}{3}$

b185

Ans :

If a is zero of quadratic polynomial $f(x)$, then

$$f(a) = 0$$

So, $f(-3) = (k-1)(-3)^2 + (-3)k + 1$

$$0 = (k-1)(9) - 3k + 1$$
$$0 = 9k - 9 - 3k + 1$$
$$0 = 6k - 8$$
$$k = \frac{8}{6} = \frac{4}{3}$$

Thus (a) is correct option.

15. A quadratic polynomial, whose zeroes are -3 and 4, is

(a) $x^2 - x + 12$ (b) $x^2 + x + 12$

(c) $\frac{x^2}{2} - \frac{x}{2} - 6$ (d) $2x^2 + 2x - 24$

Ans :

We have $\alpha = -3$ and $\beta = 4$.

b186

Sum of zeros $\alpha + \beta = -3 + 4 = 1$

Product of zeros, $\alpha \cdot \beta = -3 \times 4 = -12$

So, the quadratic polynomial is

$$x^2 - (\alpha + \beta)x + \alpha\beta = x^2 - 1 \times x + (-12)$$
$$= x^2 - x - 12$$
$$= \frac{x^2}{2} - \frac{x}{2} - 6$$

Thus (c) is correct option.

16. If the zeroes of the quadratic polynomial $x^2 + (a+1)x + b$ are 2 and -3, then

(a) $a = -7, b = -1$

(b) $a = 5, b = -1$

(c) $a = 2, b = -6$

(d) $a = 0, b = -6$

b187

Ans :

If a is zero of the polynomial, then $f(a) = 0$.

Here, 2 and -3 are zeroes of the polynomial $x^2 + (a+1)x + b$

So, $f(2) = (2)^2 + (a+1)(-3) + b = 0$

$$4 + 2a + 2 + b = 0$$
$$6 + 2a + b = 0$$
$$2a + b = -6 \quad ...(1)$$

Again, $f(-3) = (-3)^2 + (a+1)2 + b = 0$

$$9 - 3(a+1) + b = 0$$
$$9 - 3a - 3 + b = 0$$
$$6 - 3a + b = 0$$
$$-3a + b = -6$$
$$3a - b = 6 \quad ...(2)$$

Adding equations (1) and (2), we get

$$5a = 0 \Rightarrow a = 0$$

Substituting value of a in equation (1), we get

$$b = -6$$

Hence, $a = 0$ and $b = -6$.

Thus (d) is correct option.

17. The zeroes of the quadratic polynomial $x^2+99x+127$ are

(a) both positive

(b) both negative

(c) one positive and one negative

(d) both equal

b188

Ans :

Let $f(x) = x^2+99x+127$

Comparing the given polynomial with ax^2+bx+c, we get $a=1$, $b=99$ and $c=127$.

Sum of zeroes $\alpha+\beta = \frac{-b}{a} = -99$

Product of zeroes $\alpha\beta = \frac{c}{a} = 127$

Now, product is positive and the sum is negative, so both of the numbers must be negative.

Alternative Method :

Let $f(x) = x^2+99x+127$

Comparing the given polynomial with ax^2+bx+c, we get $a=1$, $b=99$ and $c=127$.

Now by discriminant rule,

$$D = \sqrt{b^2-4ac}$$

$$= \sqrt{(99)^2-4\times1\times127}$$

$$= \sqrt{9801-508} = \sqrt{9293}$$

$$= 96.4$$

So, the zeroes of given polynomial,

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$= \frac{-99\pm\sqrt{96.4}}{2}$$

Now,as $99 > 96.4$

So, both zeroes are negative.

Thus (b) is correct option.

18. The zeroes of the quadratic polynomial x^2+kx+k where $k\neq0$,

(a) cannot both be positive

(b) cannot both be negative

(c) are always unequal

(d) are always equal

b189

Ans :

Let $f(x) = x^2+kx+k$, $k\neq0$

Comparing the given polynomial with ax^2+bx+c, we get $a=1$, $b=k$ and $c=k$.

Again, let if α,β be the zeroes of given polynomial then,

$$\alpha+\beta = -k$$

$$\alpha\beta = k$$

Case 1: If k is negative, then $\alpha\beta$ is negative. It means α and β are of opposite sign.

Case 2: If k is positive, then $\alpha+\beta$ must be negative and $\alpha\beta$ must be positive and α and β both negative.

Hence, α and β cannot both positive.

Thus (a) is correct option.

19. If the zeroes of the quadratic polynomial ax^2+bx+c, where $c\neq0$, are equal, then

(a) c and a have opposite signs

(b) c and b have opposite signs

(c) c and a have same sign

(d) c and b have the same sign

b190

Ans :

Let $f(x) = ax^2+bx+c$

Let α and β are zeroes of this polynomial

Then, $\alpha+\beta = -\frac{b}{a}$

and $\alpha\beta = \frac{c}{a}$

Since $\alpha=\beta$, then α and β must be of same sign i.e. either both are positive or both are negative. In both case

$$\alpha\beta > 0$$

$$\frac{c}{a} > 0$$

Both c and a are of same sign.

Thus (c) is correct option.

20. If one of the zeroes of a quadratic polynomial of the form x^2+ax+b is the negative of the other, then it

(a) has no linear term and the constant term is negative.

(b) has no linear term and the constant term is positive.

(c) can have a linear term but the constant term is negative.

(d) can have a linear term but the constant term is positive.

Ans :

Let $f(x) = x^2 + ax + b$

and let the zeroes of $f(x)$ are α and β,

As one of zeroes is negative of other,

sum of zeroes $\alpha + \beta = \alpha + (-\alpha) = 0$ ...(1)

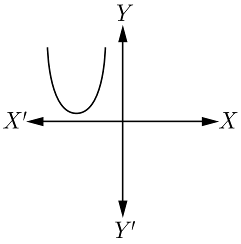
and $\alpha\beta = \alpha \cdot (-\alpha) = -\alpha^2$ ...(2)

Hence, the given quadratic polynomial has no linear term and the constant term is negative.

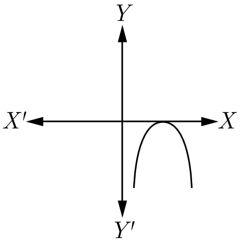
Thus (a) is correct option.

21. Which of the following is not the graph of a quadratic polynomial?

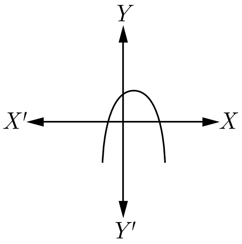
(a)



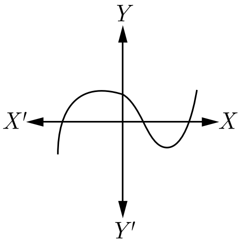
(b)



(c)



(d)



Ans :

As the graph of option (d) cuts x-axis at three points. So, it does not represent the graph of quadratic polynomial.

Thus (d) is correct option.

22. Assertion : $(2 - \sqrt{3})$ is one zero of the quadratic polynomial then other zero will be $(2 + \sqrt{3})$.

Reason : Irrational zeros (roots) always occurs in pairs.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

As irrational roots/zeros always occurs in pairs therefore, when one zero is $(2 - \sqrt{3})$ then other will be $2 + \sqrt{3}$. So, both A and R are correct and R explains A.

Thus (a) is correct option.

23. Assertion : If one zero of poly-nominal $p(x) = (k^2 + 4)x^2 + 13x + 4k$ is reciprocal of other, then $k = 2$.

Reason : If $(x - \alpha)$ is a factor of $p(x)$, then $p(\alpha) = 0$ i.e. α is a zero of $p(x)$.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

Let $\alpha, \frac{1}{\alpha}$ be the zeroes of $p(x)$, then

$$\alpha \cdot \frac{1}{\alpha} = \frac{4k}{k^2 + 4}$$

$$1 = \frac{4k}{k^2 + 4}$$

$$k^2 - 4k + 4 = 0$$

$$(k - 2)^2 = 0 \Rightarrow k = 2$$

Assertion is true Since, Reason is not correct for Assertion.

Thus (b) is correct option.

24. Assertion : $p(x) = 14x^3 - 2x^2 + 8x^4 + 7x - 8$ is a polynomial of degree 3.

Reason : The highest power of x in the polynomial $p(x)$ is the degree of the polynomial.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

The highest power of x in the polynomial $p(x) = 14x^3 - 2x^2 + 8x^4 + 7x - 8$ is 4. Degree is 4. So, A is incorrect but R is correct.

Thus (d) is correct option.

b191 b192 b193 b194 b195

25. **Assertion :** $x^3 + x$ has only one real zero.

Reason : A polynomial of nth degree must have n real zeroes.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

A polynomial of nth degree at most can have n real zeroes. Thus reason is not true.

b196

Again, $x^3 + x = x(x^2 + 1)$

which has only one real zero because $x^2 + 1 \neq 0$ for all $x \in R$.

Assertion (A) is true but reason (R) is false.

Thus (c) is correct option.

26. **Assertion :** If both zeros of the quadratic polynomial $x^2 - 2kx + 2$ are equal in magnitude but opposite in sign then value of k is ½.

Reason : Sum of zeros of a quadratic polynomial $ax^2 + bx + c$ is $\frac{-b}{a}$

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

As the polynomial is $x^2 - 2kx + 2$ and its zeros are equal but opposition sign, sum of zeroes must be zero.

$$\text{sum of zeros} = 0$$

$$\frac{-(-2k)}{1} = 0 \Rightarrow k = 0$$

b197

Assertion (A) is false but reason (R) is true.

Thus (d) is correct option.

Fill in the Blank Questions

27. A .... polynomial is of degree one.

Ans :

b198

Linear

28. A cubic polynomial is of degree..........

Ans :

Three

b199

29. Degree of remainder is always .... than degree of divisor.

Ans :

Smaller/less

b200

30. Polynomials of degrees 1, 2 and 3 are called .... , ... and .... polynomials respectively.

Ans :

linear, quadratic, cubic

b201

31. .... is not equal to zero when the divisor is not a factor of dividend.

Ans :

Remainder

b202

32. The zeroes of a polynomial $p(x)$ are precisely the $x-$coordinates of the points, where the graph of $y = p(x)$ intersects the .... axis.

Ans :

x

b203

33. The algebraic expression in which the variable has non-negative integral exponents only is called ....

Ans :

Polynomial

b204

34. A quadratic polynomial can have at most 2 zeroes and a cubic polynomial can have at most .... zeroes.

Ans :

3

b205

35. A .... is a polynomial of degree 0.

Ans :

Constant

b206

36. The highest power of a variable in a polynomial is called its ....

Ans :

Degree

b207

37. A polynomial of degree n has at the most .... zeroes.

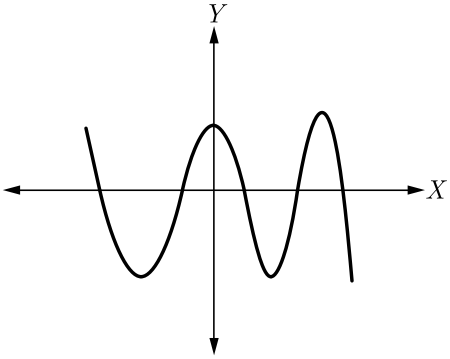
Ans :

n

b208

38. The graph of $y = p(x)$, where $p(x)$ is a polynomial in variable x, is as follows.

b209



The number of zeroes of $p(x)$ is ....

Ans : [Board 2020 SQP Standard]

The graph of the given polynomial $p(x)$ crosses the x-axis at 5 points. So, number of zeroes of $p(x)$ is 5.

39. If one root of the equation $(k-1)x^2 - 10x + 3 = 0$ is the reciprocal of the other then the value of k is ....

Ans : [Board 2020 SQP Standard]

We have $(k-1)x^2 - 10x + 3 = 0$

Let one root be α, then another root will be $\frac{1}{\alpha}$

Now $$\alpha \cdot \frac{1}{\alpha} = \frac{c}{a} = \frac{3}{(k-1)}$$

$$1 = \frac{3}{(k-1)}$$

b210

$$k - 1 = 3 \Rightarrow k = 4$$

Very Short Answer Questions

40. If α and β are the roots of $ax^2 - bx + c = 0 \, (a \neq 0)$, then calculate $\alpha + \beta$.

Ans : [Board Term-1 2014]

We know that

$$\text{Sum of the roots} = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$$

b101

Thus $$\alpha + \beta = -\left(\frac{-b}{a}\right) = \frac{b}{a}$$

41. Calculate the zeroes of the polynomial $p(x) = 4x^2 - 12x + 9$.

Ans : [Board Term-1 2010]

$$p(x) = 4x^2 - 12x + 9$$

b103

$$= 4x^2 - 6x - 6x + 9$$

$$= 2x(2x-3) - 3(2x-3)$$

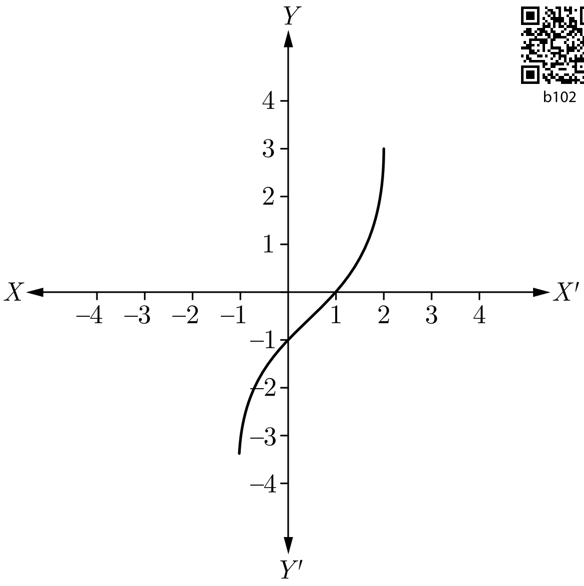
$$= (2x-3)(2x-3)$$

Substituting $p(x) = 0$, and solving we get $x = \frac{3}{2}, \frac{3}{2}$

$$x = \frac{3}{2}, \frac{3}{2}$$

Hence, zeroes of the polynomial are $\frac{3}{2}, \frac{3}{2}$.

42. In given figure, the graph of a polynomial $p(x)$ is shown. Calculate the number of zeroes of $p(x)$.



b102

Ans : [Board Term-1 2013]

The graph intersects x-axis at one point $x = 1$. Thus the number of zeroes of $p(x)$ is 1.

43. If sum of the zeroes of the quadratic polynomial $3x^2 - kx + 6$ is 3, then find the value of k.

Ans : [Board 2009]

We have $$p(x) = 3x^2 - kx - 6$$

b104

$$\text{Sum of the zeroes} = 3 = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$$

Thus $$3 = -\frac{(-k)}{3} \Rightarrow k = 9$$

44. If -1 is a zero of the polynomial $f(x) = x^2 - 7x - 8$, then calculate the other zero.

Ans :

We have $$f(x) = x^2 - 7x - 8$$

b105

Let other zero be k, then we have

Sum of zeroes, $$-1 + k = -\left(\frac{-7}{1}\right) = 7$$

or $$k = 8$$

TWO MARKS QUESTIONS

45. If zeroes of the polynomial $x^2+4x+2a$ are a and $\frac{2}{a}$, then find the value of a.

Ans : [Board Term-1 2016]

Product of (zeroes) roots,

$$\frac{c}{a} = \frac{2a}{1} = \alpha \times \frac{2}{\alpha} = 2$$

or, $2a = 2$

Thus $a = 1$

b106

46. Find all the zeroes of $f(x) = x^2 - 2x$.

Ans : [Board Term-1 2013]

We have $f(x) = x^2 - 2x$

$= x(x-2)$

b107

Substituting $f(x) = 0$, and solving we get $x = 0, 2$

Hence, zeroes are 0 and 2.

47. Find the zeroes of the quadratic polynomial $\sqrt{3}x^2 - 8x + 4\sqrt{3}$.

Ans : [Board Term-1 2013]

We have $p(x) = \sqrt{3}x^2 - 8x + 4\sqrt{3}$

$= \sqrt{3}x^2 - 6x - 2x + 4\sqrt{3}$

$= \sqrt{3}x(x - 2\sqrt{3}) - 2(x - 2\sqrt{3})$

$= (\sqrt{3}x - 2)(x - 2\sqrt{3})$

Substituting $p(x) = 0$, we have

$$(\sqrt{3}x - 2)(x - 2\sqrt{3})\ p(x) = 0$$

Solving we get $x = \frac{2}{\sqrt{3}},\ 2\sqrt{3}$

b108

Hence, zeroes are $\frac{2}{\sqrt{3}}$ and $2\sqrt{3}$.

48. Find a quadratic polynomial, the sum and product of whose zeroes are 6 and 9 respectively. Hence find the zeroes.

Ans : [Board Term-1 2016]

Sum of zeroes, $\alpha + \beta = 6$

Product of zeroes $\alpha\beta = 9$

b109

Now $p(x) = x^2 - (\alpha+\beta)x + \alpha\beta$

Thus $= x^2 - 6x + 9$

Thus quadratic polynomial is $x^2 - 6x + 9$.

Now $p(x) = x^2 - 6x + 9$

$= (x-3)(x-3)$

Substituting $p(x) = 0$, we get $x = 3, 3$

Hence zeroes are 3, 3

49. Find the quadratic polynomial whose sum and product of the zeroes are $\frac{21}{8}$ and $\frac{5}{16}$ respectively.

Ans : [Board Term-1 2012, Set-35]

Sum of zeroes, $\alpha + \beta = \frac{21}{8}$

Product of zeroes $\alpha\beta = \frac{5}{16}$

b110

Now $p(x)\ x^2 - (\alpha+\beta)x + \alpha\beta$

$= x^2 - \frac{21}{8}x + \frac{5}{16}$

or $p(x) = \frac{1}{16}(16x^2 - 42x + 5)$

50. Form a quadratic polynomial $p(x)$ with 3 and $-\frac{2}{5}$ as sum and product of its zeroes, respectively.

Ans : [Board Term-1 2012]

Sum of zeroes, $\alpha + \beta = 3$

Product of zeroes $\alpha\beta = -\frac{2}{5}$

b111

Now $p(x)\ x^2 - (\alpha+\beta)x + \alpha\beta$

$= x^2 - 3x - \frac{2}{5}$

$= \frac{1}{5}(5x^2 - 15x - 2)$

The required quadratic polynomial is $\frac{1}{5}(5x^2 - 15x - 2)$

51. If m and n are the zeroes of the polynomial $3x^2 + 11x - 4$, find the value of $\frac{m}{n} + \frac{n}{m}$.

Ans : [Board Term-1 2012]

We have $\frac{m}{n} + \frac{n}{m} = \frac{m^2+n^2}{mn} = \frac{(m+n)^2 - 2mn}{mn}$ (1)

Sum of zeroes $m + n = -\frac{11}{3}$

b113

Product of zeroes $mn = \frac{-4}{3}$

Substituting in (1) we have

$$\frac{m}{n}+\frac{n}{m} = \frac{(m+n)^2-2mn}{mn}$$

$$= \frac{(-\frac{11}{3})^2-\frac{-4}{3}\times 2}{\frac{-4}{3}}$$

$$= \frac{121+4\times 3\times 2}{-4\times 3}$$

or $$\frac{m}{n}+\frac{n}{m} = \frac{-145}{12}$$

52. If p and q are the zeroes of polynomial $f(x)=2x^2-7x+3$, find the value of p^2+q^2.

Ans : [Board Term-1 2012]

We have $f(x) = 2x^2-7x+3$

Sum of zeroes $p+q = -\frac{b}{a} = -\left(\frac{-7}{2}\right) = \frac{7}{2}$

Product of zeroes $pq = \frac{c}{a} = \frac{3}{2}$

b114

Since, $(p+q)^2 = p^2+q^2+2pq$

so, $p^2+q^2 = (p+q)^2-2pq$

$$= \left(\frac{7}{2}\right)^2-3 = \frac{49}{4}-\frac{3}{1} = \frac{37}{4}$$

Hence $p^2+q^2 = \frac{37}{4}$.

53. Find the condition that zeroes of polynomial $p(x)=ax^2+bx+c$ are reciprocal of each other.

Ans : [Board Term-1 2012]

We have $p(x) = ax^2+bx+c$

b115

Let α and $\frac{1}{\alpha}$ be the zeroes of $p(x)$, then

Product of zeroes,

$$\frac{c}{a} = \alpha\times\frac{1}{\alpha} = 1 \text{ or } \frac{c}{a} = 1$$

So, required condition is, $c = a$

54. Find the value of k if -1 is a zero of the polynomial $p(x)=kx^2-4x+k$.

Ans : [Board Term-1 2012]

We have $p(x) = kx^2-4x+k$

Since, -1 is a zero of the polynomial, then

$$p(-1) = 0$$

b116

$$k(-1)^2-4(-1)+k = 0$$

$$k+4+k = 0$$

$$2k+4 = 0$$

$$2k = -4$$

Hence, $k = -2$

55. If α and β are the zeroes of a polynomial $x^2-4\sqrt{3}\,x+3$, then find the value of $\alpha+\beta-\alpha\beta$.

Ans : [Board Term-1 2015]

We have $p(x) = x^2-4\sqrt{3}\,x+3$

If α and β are the zeroes of $x^2-4\sqrt{3}\,x+3$, then

Sum of zeroes, $\alpha+\beta = -\frac{b}{a} = -\frac{(-4\sqrt{3})}{1}$

or, $\alpha+\beta = 4\sqrt{3}$

b117

Product of zeroes $\alpha\beta = \frac{c}{a} = \frac{3}{1}$

or, $\alpha\beta = 3$

Now $\alpha+\beta-\alpha\beta = 4\sqrt{3}-3$.

56. Find the values of a and b, if they are the zeroes of polynomial x^2+ax+b.

Ans : [Board Term-1 2013]

We have $p(x) = x^2+ax+b$

Since a and b, are the zeroes of polynomial, we get,

Product of zeroes, $ab = b \Rightarrow a = 1$

Sum of zeroes, $a+b = -a \Rightarrow b = -2a = -2$

57. If α and β are the zeroes of the polynomial $f(x)=x^2-6x+k$, find the value of k, such that $\alpha^2+\beta^2=40$.

Ans : [Board Term-1 2015]

We have $f(x) = x^2-6x+k$

Sum of zeroes, $\alpha+\beta = -\frac{b}{a} = \frac{-(-6)}{1} = 6$

Product of zeroes, $\alpha\beta = \frac{c}{a} = \frac{k}{1} = k$

Now $\alpha^2+\beta^2 = (\alpha+\beta)^2-2\alpha\beta = 40$

$$(6)^2-2k = 40$$

$$36-2k = 40$$

b119

$$-2k = 4$$

Thus $\quad k = -2$

58. If one of the zeroes of the quadratic polynomial $f(x) = 14x^2 - 42k^2x - 9$ is negative of the other, find the value of 'k'.

Ans : [Board Term-1 2012]

We have $\quad f(x) = 14x^2 - 42k^2x - 9$

Let one zero be α, then other zero will be $-\alpha$.

Sum of zeroes $\alpha + (-\alpha) = 0$.

Thus sum of zero will be 0.

Sum of zeroes $\quad 0 = -\dfrac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

$$0 = -\frac{42k^2}{14} = -3k^2$$

Thus $k = 0$.

59. If one zero of the polynomial $2x^2 + 3x + \lambda$ is $\frac{1}{2}$, find the value of λ and the other zero.

Ans : [Board Term-1 2012]

Let, the zero of $2x^2 + 3x + \lambda$ be $\dfrac{1}{2}$ and β.

Product of zeroes $\frac{c}{a}$, $\quad \dfrac{1}{2}\beta = \dfrac{\lambda}{2}$

or, $\quad \beta = \lambda$

and sum of zeroes $-\frac{b}{a}$, $\dfrac{1}{2} + \beta = -\dfrac{3}{2}$

or $\quad \beta = -\dfrac{3}{2} - \dfrac{1}{2} = -2$

Hence $\quad \lambda = \beta = -2$

Thus other zero is -2.

60. If α and β are zeroes of the polynomial $f(x) = x^2 - x - k$, such that $\alpha - \beta = 9$, find k.

Ans : [Board Term-1 2013, Set FFC]

We have $\quad f(x) = x^2 - x - k$

Since α and β are the zeroes of the polynomial, then

Sum of zeroes, $\alpha + \beta = -\dfrac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

$$= -\left(\frac{-1}{1}\right) = 1$$

$$\alpha + \beta = 1 \quad \text{...(1)}$$

Given $\quad \alpha - \beta = 9 \quad$ ...(2)

Solving (1) and (2) we get $\alpha = 5$ and $\beta = -4$

$$\alpha\beta = \frac{\text{Constan term}}{\text{Coefficient of } x^2}$$

or $\quad \alpha\beta = -k$

Substituting $\alpha = 5$ and $\beta = -4$ we have

$$(5)(-4) = -k$$

Thus $\quad k = 20$

61. If the zeroes of the polynomial $x^2 + px + q$ are double in value to the zeroes of $2x^2 - 5x - 3$, find the value of p and q.

Ans : [Board Term-1 2012, Set-39]

We have $\quad f(x) = 2x^2 - 5x - 3$

Let the zeroes of polynomial be α and β, then

Sum of zeroes $\quad \alpha + \beta = \dfrac{5}{2}$

Product of zeroes $\quad \alpha\beta = -\dfrac{3}{2}$

According to the question, zeroes of $x^2 + px + q$ are 2α and 2β.

Sum of zeros, $\quad 2\alpha + 2\beta = \dfrac{-p}{1}$

$$2(\alpha + \beta) = -p$$

Substituting $\alpha + \beta = \frac{5}{2}$ we have

$$2 \times \frac{5}{2} = -p$$

or $\quad p = -5$

Product of zeroes, $\quad 2\alpha 2\beta = \dfrac{q}{1}$

$$4\alpha\beta = q$$

Substituting $\alpha\beta = -\frac{3}{2}$ we have

$$4 \times \frac{-3}{2} = q$$

$$-6 = q$$

Thus $p = -5$ and $q = -6$.

62. If α and β are zeroes of $x^2 - (k-6)x + 2(2k-1)$, find the value of k if $\alpha + \beta = \dfrac{1}{2}\alpha\beta$.

Ans : [KVS Practice Test 2017]

We have $\quad p(x) = x^2 - (k-6)x + 2(2k-1)$

Since α, β are the zeroes of polynomial $p(x)$, we get

$$\alpha+\beta = -[-(k-6)] = k-6$$

$$\alpha\beta = 2(2k-1)$$

b124

Now $\alpha+\beta = \frac{1}{2}\alpha\beta$

Thus $k+6 = \frac{2(2k-1)}{2}$

or, $k-6 = 2k-1$

$$k = -5$$

Hence the value of k is -5.

THREE MARKS QUESTIONS

63. Find a quadratic polynomial whose zeroes are reciprocals of the zeroes of the polynomial $f(x) = ax^2 + bx + c$, $a \neq 0$, $c \neq 0$.

[Board 2020 Delhi Standard]

Ans :

Let α and β be zeros of the given polynomial $ax^2 + bx + c$.

$$\alpha+\beta = -\frac{b}{a} \text{ and } \alpha\beta = \frac{c}{a}$$

Let $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ be the zeros of new polynomial then we have

b216

Sum of zeros, $s = \frac{1}{\alpha}+\frac{1}{\beta} = \frac{\beta+\alpha}{\alpha\beta}$

$$= \frac{-\frac{b}{a}}{\frac{c}{a}} = \frac{-b}{c}$$

Product of zeros, $p = \frac{1}{\alpha}\cdot\frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{a}{c}$

Required polynomial,

$$g(x) = x^2 - sx + p$$

$$g(x) = x^2 + \frac{b}{c}x + \frac{a}{c}$$

$$cg(x) = cx^2 + bx + a$$

$$g'(x) = cx^2 + bx + a$$

64. Verify whether 2, 3 and $\frac{1}{2}$ are the zeroes of the polynomial $p(x) = 2x^3 - 11x^2 + 17x - 6$.

[Board Term-1 2013, LK-59]

Ans :

If 2, 3 and $\frac{1}{2}$ are the zeroes of the polynomial $p(x)$, then these must satisfy $p(x) = 0$

(1) 2, $p(x) = 2x^2 - 11x^2 + 17x - 6$

$$p(2) = 2(2)^3 - 11(2)^2 + 17(2) - 6$$

$$= 16 - 44 + 34 - 6$$

$$= 50 - 50$$

b125

or $p(2) = 0$

(2) 3, $p(3) = 2(3)^3 - 11(3)^2 + 17(3) - 6$

$$= 54 - 99 + 51 - 6$$

$$= 105 - 105$$

or $p(3) = 0$

(3) $\frac{1}{2}$ $p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 - 11\left(\frac{1}{2}\right)^2 + 17\left(\frac{1}{2}\right) - 6$

$$= \frac{1}{4} - \frac{11}{4} + \frac{17}{2} - 6$$

or $p\left(\frac{1}{2}\right) = 0$

Hence, 2, 3, and $\frac{1}{2}$ are the zeroes of $p(x)$.

65. If the sum and product of the zeroes of the polynomial $ax^2 - 5x + c$ are equal to 10 each, find the value of 'a' and 'c'.

[Board Term-1 2011, Set-25]

Ans :

We have $f(x) = ax^2 - 5x + c$

Let the zeroes of $f(x)$ be α and β, then,

b126

Sum of zeroes $\alpha+\beta = -\frac{-5}{a} = \frac{5}{a}$

Product of zeroes $\alpha\beta = \frac{c}{a}$

According to question, the sum and product of the zeroes of the polynomial $f(x)$ are equal to 10 each.

Thus $\frac{5}{a} = 10$ ...(1)

and $\frac{c}{a} = 10$ ...(2)

Dividing (2) by eq. (1) we have

$$\frac{c}{5} = 1 \Rightarrow c = 5$$

Substituting $c = 5$ in (2) we get $a = \frac{1}{2}$

Hence $a = \frac{1}{2}$ and $c = 5$.

66. If one the zero of a polynomial $3x^2 - 8x + 2k + 1$ is seven times the other, find the value of k.

Ans : [Board Term-1 2011, Set-40]

We have $f(x) = 3x^2 - 8x + 2k + 1$

Let α and β be the zeroes of the polynomial, then

$$\beta = 7\alpha$$

Sum of zeroes, $$\alpha + \beta = -\left(-\frac{8}{3}\right)$$

b127

$$\alpha + 7\alpha = 8\alpha = \frac{8}{3}$$

So $$\alpha = \frac{1}{3}$$

Product of zeroes, $$\alpha \times 7\alpha = \frac{2k+1}{3}$$

$$7\alpha^2 = \frac{2k+1}{3}$$

$$7\left(\frac{1}{3}\right)^2 = \frac{2k+1}{3}$$

$$7 \times \frac{1}{9} = \frac{2k+1}{1}$$

$$\frac{7}{3} - 1 = 2k$$

$$\frac{4}{3} = 2k \Rightarrow k = \frac{2}{3}$$

67. Quadratic polynomial $2x^2 - 3x + 1$ has zeroes as α and β. Now form a quadratic polynomial whose zeroes are 3α and 3β.

Ans : [Board Term-2 2015]

We have $f(x) = 2x^2 - 3x + 1$

If α and β are the zeroes of $2x^2 - 3x + 1$, then

Sum of zeroes $$\alpha + \beta = \frac{-b}{a} = \frac{3}{2}$$

b128

Product of zeroes $$\alpha\beta = \frac{c}{a} = \frac{1}{2}$$

New quadratic polynomial whose zeroes are 3α and 3β is,

$$p(x) = x^2 - (3\alpha + 3\beta)x + 3\alpha \times 3\beta$$

$$= x^2 - 3(\alpha + \beta)x + 9\alpha\beta$$

$$= x^2 - 3\left(\frac{3}{2}\right)x + 9\left(\frac{1}{2}\right)$$

$$= x^2 - \frac{9}{2}x + \frac{9}{2}$$

$$= \frac{1}{2}(2x^2 - 9x + 9)$$

Hence, required quadratic polynomial is $\frac{1}{2}(2x^2 - 9x + 9)$

68. If α and β are the zeroes of the polynomial $6y^2 - 7y + 2$, find a quadratic polynomial whose zeroes are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

Ans : [Board Term-1 2011]

We have $p(y) = 6y^2 - 7y + 2$

Sum of zeroes $$\alpha + \beta = -\left(-\frac{7}{6}\right) = \frac{7}{6}$$

Product of zeroes $$\alpha\beta = \frac{2}{6} = \frac{1}{3}$$

b129

Sum of zeroes of new polynomial $g(y)$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{7/6}{2/6} = \frac{7}{2}$$

and product of zeroes of new polynomial $g(y)$,

$$\frac{1}{\alpha} \times \frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{1}{1/3} = 3$$

The required polynomial is

$$g(x) = y^2 - \frac{7}{2}y + 3$$

$$= \frac{1}{2}[2y^2 - 7y + 6]$$

69. Show that $\frac{1}{2}$ and $\frac{-3}{2}$ are the zeroes of the polynomial $4x^2 + 4x - 3$ and verify relationship between zeroes and coefficients of the polynomial.

Ans : [Board Term-1 2011]

We have $p(x) = 4x^2 + 4x - 3$

If $\frac{1}{2}$ and $\frac{-3}{2}$ are the zeroes of the polynomial $p(x)$, then these must satisfy $p(x) = 0$

$$p\left(\frac{1}{2}\right) = 4\left(\frac{1}{4}\right) + 4\left(\frac{1}{2}\right) - 3$$

b130

$$= 1 + 2 - 3 = 0$$

and $$p\left(-\frac{3}{2}\right) = 4\left(\frac{9}{2}\right) + 4\left(-\frac{3}{2}\right) - 3$$

$$= 9 - 6 - 3 = 0$$

Thus $\frac{1}{2}, -\frac{3}{2}$ are zeroes of polynomial $4x^2 + 4x - 3$.

Sum of zeroes $$= \frac{1}{2} - \frac{3}{2} = -1 = \frac{-4}{4}$$

$$= -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

Product of zeroes $=\left(\frac{1}{2}\right)\left(-\frac{3}{2}\right)=\frac{-3}{4}$

$=\frac{\text{Constan term}}{\text{Coefficient of } x^2}$ Verified

70. A teacher asked 10 of his students to write a polynomial in one variable on a paper and then to handover the paper. The following were the answers given by the students :

$2x+3$, $3x^2+7x+2$, $4x^3+3x^2+2$, $x^3+\sqrt{3x}+7$, $7x+\sqrt{7}$, $5x^3-7x+2$, $2x^2+3-\frac{5}{x}$, $5x-\frac{1}{2}$, ax^3+bx^2+cx+d, $x+\frac{1}{x}$.

b219

Answer the following question :
(i) How many of the above ten, are not polynomials?
(ii) How many of the above ten, are quadratic polynomials?

Ans : [Board 2020 OD Standard]

(i) $x^3+\sqrt{3x}+7, 2x^2+3-\frac{5}{x}$ and $x+\frac{1}{x}$ are not polynomials.

(ii) $3x^2+7x+2$ is only one quadratic polynomial.

71. Find the zeroes of the quadratic polynomial $x^2-2\sqrt{2}\,x$ and verify the relationship between the zeroes and the coefficients.

Ans : [Board Term-1 2015]

We have $p(x)x^2-2\sqrt{2}\,x=0$

$x(x-2\sqrt{2})=0$

b131

Thus zeroes are 0 and $2\sqrt{2}$.

Sum of zeroes $2\sqrt{2}=-\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

and product of zeroes $0=\frac{\text{Constan term}}{\text{Coefficient of } x^2}$

Hence verified

72. Find the zeroes of the quadratic polynomial $5x^2+8x-4$ and verify the relationship between the zeroes and the coefficients of the polynomial.

Ans : [Board Term-1 2013, Set LK-59]

We have $p(x)=5x^2+8x-4=0$

$=5x^2+10x-2x-4=0$

$=5x(x+2)-2(x+2)=0$

$=(x+2)(5x-2)$

Substituting $p(x)=0$ we get zeroes as -2 and $\frac{2}{5}$.

Verification :

Sum of zeroes $=-2+\frac{2}{5}=\frac{-8}{5}$

b132

Product of zeroes $=(-2)\times\left(\frac{2}{5}\right)=\frac{-4}{5}$

Now from polynomial we have

Sum of zeroes $-\frac{b}{a}=-\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}=\frac{-8}{5}$

Product of zeroes $\frac{c}{a}=\frac{\text{Constan term}}{\text{Coefficient of } x^2}=-\frac{4}{5}$

Hence Verified.

73. If α and β are the zeroes of a quadratic polynomial such that $\alpha+\beta=24$ and $\alpha-\beta=8$. Find the quadratic polynomial having α and β as its zeroes.

Ans : [Board Term-1 2011, Set-44]

We have $\alpha+\beta=24$ ...(1)

$\alpha-\beta=8$ ...(2)

Adding equations (1) and (2) we have

$2\alpha=32 \Rightarrow \alpha=16$

b133

Subtracting (1) from (2) we have

$2\beta=16 \Rightarrow \beta=8$

Hence, the quadratic polynomial

$$p(x)=x^2-(\alpha+\beta)x+\alpha\beta$$
$$=x^2-(16+8)x+(16)(8)$$
$$=x^2-24x+128$$

74. If α,β and γ are zeroes of the polynomial $6x^3+3x^2-5x+1$, then find the value of $\alpha^{-1}+\beta^{-1}+\gamma^{-1}$.

Ans : [KVS practice Test 2017, Board 2010]

We have $p(x)=6x^3+3x^2-5x+1$

Since α,β and γ are zeroes polynomial $p(x)$, we have

$$\alpha+\beta+\gamma=-\frac{b}{c}=-\frac{3}{6}=-\frac{1}{2}$$

$$\alpha\beta+\beta\gamma+\gamma\alpha=\frac{c}{a}=-\frac{5}{6}$$

b134

and $\alpha\beta\gamma = -\frac{d}{a} = -\frac{1}{6}$

Now $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\alpha\beta + \beta\gamma + \gamma\alpha}{\alpha\beta\gamma}$

$$= \frac{-5/6}{-1/6} = \frac{-5}{6} \times \frac{6}{-1} = 5$$

Hence $\alpha^{-1} + \beta^{-1} + \gamma^{-1} = 5$.

75. When $p(x) = x^2 + 7x + 9$ is divisible by $g(x)$, we get $(x+2)$ and -1 as the quotient and remainder respectively, find $g(x)$.

Ans : [Board Term-1 2011]

We have $p(x) = x^2 + 7x + 9$

$q(x) = x + 2$

$r(x) = -1$

Now $p(x) = g(x)q(x) + r(x)$

$$x^2 + 7x + 9 = g(x)(x+2) - 1$$

or, $g(x) = \frac{x^2 + 7x + 10}{x+2}$

$$= \frac{(x+2)(x+5)}{(x+2)} = x + 5$$

Thus $g(x) = x + 5$

76. Find the value for k for which $x^4 + 10x^3 + 25x^2 + 15x + k$ is exactly divisible by $x + 7$.

Ans : [Board Term 2010]

We have $f(x) = x^4 + 10x^3 + 25x^2 + 15x + k$

If $x + 7$ is a factor then -7 is a zero of $f(x)$ and $x = -7$ satisfy $f(x) = 0$.

Thus substituting $x = -7$ in $f(x)$ and equating to zero we have,

$$(-7)^4 + 10(-7)^3 + 25(-7)^2 + 15(-7) + k = 0$$

$$2401 - 3430 + 1225 - 105 + k = 0$$

$$3626 - 3535 + k = 0$$

$$91 + k = 0$$

$$k = -91$$

77. On dividing the polynomial $4x^4 - 5x^3 - 39x^2 -$ by the polynomial $g(x)$, the quotient is $x^2 - 3x - 5$ and the remainder is $-5x + 8$. Find the polynomial $g(x)$.

Ans : [Board Term 2009]

Dividend = (Divisor × Quotient) + Remainder

$$4x^4 - 5x^3 - 39x^3 - 46x - 2$$

$$= g(x)(x^2 - 3x - 5) + (-5x + 8)$$

$$4x^2 - 5x^3 - 39x^2 - 46x - 2 + 5x - 8$$

$$= g(x)(x^2 - 3x - 5)$$

$$4x^4 - 5x^3 - 39x^2 - 41x - 10 = g(x)(x^2 - 3x - 5)$$

$$g(x) = \frac{4x^4 - 5x^3 - 39x^2 - 41x - 10}{(x^2 - 3x - 5)}$$

Hence, $g(x) = 4x^2 + 7x + 2$

78. If the squared difference of the zeroes of the quadratic polynomial $f(x) = x^2 + px + 45$ is equal to 144, find the value of p.

Ans : [Board 2008]

We have $f(x) = x^2 + px + 45$

Let α and β be the zeroes of the given quadratic polynomial.

Sum of zeroes, $\alpha + \beta = -p$

Product of zeroes $\alpha\beta = 45$

Given, $(\alpha - \beta)^2 = 144$

$$(\alpha + \beta)^2 - 4\alpha\beta = 144$$

Substituting value of $\alpha + \beta$ and $\alpha\beta$ we get

$$(-p)^2 - 4 \times 45 = 144$$

$$p^2 - 180 = 144$$

$$p^2 = 144 + 180 = 324$$

Thus $p = \pm\sqrt{324} = \pm 18$

Hence, the value of p is ± 18.

FOUR MARKS QUESTIONS

79. Polynomial $x^4 + 7x^3 + 7x^2 + px + q$ is exactly divisible by $x^2 + 7x + 12$, then find the value of p and q.

Ans : [Board Term-1 2015]

We have $f(x) = x^4 + 7x^3 + 7x^2 + px + q$

Now $x^2 + 7x + 12 = 0$

$$x^2 + 4x + 3x + 12 = 0$$

$$x(x+4) + 3(x+4) = 0$$

$$(x+4)(x+3) = 0$$

$$x = -4, -3$$

Since $f(x) = x^4 + 7x^3 + 7x^2 + px + q$ is exactly divisible by $x^2 + 7x + 12$, then $x = -4$ and $x = -3$ must be its zeroes and these must satisfy $f(x) = 0$

So putting $x = -4$ and $x = -3$ in $f(x)$ and equating to zero we get

$$f(-4): \; (-4)^4 + 7(-4)^3 + 7(-4)^2 + p(-4) + q = 0$$

$$256 - 448 + 112 - 4p + q = 0$$

$$-4p + q - 80 = 0$$

$$4p - q = -80 \quad ...(1)$$

$$f(-3): \; (-3)^4 + 7(-3)^3 + 7(-3)^2 + p(-3) + q = 0$$

$$81 - 189 + 63 - 3p + q = 0$$

$$-3p + q - 45 = 0$$

$$3p - q = -45 \quad ...(2)$$

Subtracting equation (2) from (1) we have

$$p = -35$$

Substituting the value of p in equation (1) we have

$$4(-35) - q = -80$$

$$-140 - q = -80$$

$$-q = 140 - 80$$

or $$-q = 60$$

$$q = -60$$

Hence, $p = -35$ and $q = -60$.

80. If α and β are the zeroes of the polynomial $p(x) = 2x^2 + 5x + k$ satisfying the relation, $\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4}$, then find the value of k.

Ans : [Board Term-1 2012]

We have $$p(x) = 2x^2 + 5x + k$$

Sum of zeroes, $$\alpha + \beta = -\frac{b}{a} = -\left(\frac{5}{2}\right)$$

Product of zeroes $$\alpha\beta = \frac{c}{a} = \frac{k}{2}$$

b136

According to the question,

$$\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4}$$

$$\alpha^2 + \beta^2 + 2\alpha\beta - \alpha\beta = \frac{21}{4}$$

$$(\alpha + b)^2 - \alpha\beta = \frac{21}{4}$$

Substituting values we have

$$\left(\frac{-5}{2}\right)^2 - \frac{k}{2} = \frac{21}{4}$$

$$\frac{k}{2} = \frac{25}{4} - \frac{21}{4}$$

$$\frac{k}{2} = \frac{4}{4} = 1$$

Hence, $k = 2$

81. If α and β are the zeroes of polynomial $p(x) = 3x^2 + 2x + 1$, find the polynomial whose zeroes are $\frac{1-\alpha}{1+\alpha}$ and $\frac{1-\beta}{1+\beta}$.

Ans : [Board Term-1 2010, 2012]

We have $$p(x) = 3x^2 + 2x + 1$$

Since α and β are the zeroes of polynomial $3x^2 + 2x + 1$, we have

$$\alpha + \beta = -\frac{2}{3}$$

b137

and $$\alpha\beta = \frac{1}{3}$$

Let α_1 and β_1 be zeros of new polynomial $q(x)$.

Then for $q(x)$, sum of the zeroes,

$$\alpha_1 + \beta_1 = \frac{1-\alpha}{1+\alpha} + \frac{1-\beta}{1+\beta}$$

$$= \frac{(1 - \alpha + \beta - \alpha\beta) + (1 + \alpha - \beta - \alpha\beta)}{(1+\alpha)(1+\beta)}$$

$$= \frac{2 - 2\alpha\beta}{1 + \alpha + \beta + \alpha\beta} = \frac{2 - \frac{2}{3}}{1 - \frac{2}{3} + \frac{1}{3}}$$

$$= \frac{\frac{4}{3}}{\frac{2}{3}} = 2$$

For $q(x)$, product of the zeroes,

$$\alpha_1\beta_1 = \left[\frac{1-\alpha}{1+\alpha}\right]\left[\frac{1-\beta}{1+\beta}\right]$$

$$= \frac{(1-\alpha)(1-\beta)}{(1+\alpha)(1+\beta)}$$

$$= \frac{1 - \alpha - \beta + \alpha\beta}{1 + \alpha + \beta + \alpha\beta}$$

$$= \frac{1-(\alpha+\beta)+\alpha\beta}{1+(\alpha+\beta)+\alpha\beta}$$

$$= \frac{1+\frac{2}{3}+\frac{1}{3}}{1-\frac{2}{3}+\frac{1}{3}} = \frac{\frac{6}{3}}{\frac{2}{3}} = 3$$

Hence, Required polynomial

$$q(x) = x^2 - (\alpha_1+\beta_1)2x + \alpha_1\beta_1$$

$$= x^2 - 2x + 3$$

82. If α and β are the zeroes of the polynomial x^2+4x+3, find the polynomial whose zeroes are $1+\frac{\beta}{\alpha}$ and $1+\frac{\alpha}{\beta}$.

Ans : [Board Term-1 2013]

We have $p(x) = x^2+4x+3$

Since α and β are the zeroes of the quadratic polynomial x^2+4x+3,

b138

So, $\alpha+\beta = -4$

and $\alpha\beta = 3$

Let α_1 and β_1 be zeros of new polynomial $q(x)$.

Then for $q(x)$, sum of the zeroes,

$$\alpha_1+\beta_1 = 1+\frac{\beta}{\alpha}+1+\frac{\alpha}{\beta}$$

$$= \frac{\alpha\beta+\beta^2+\alpha\beta+\alpha^2}{\alpha\beta}$$

$$= \frac{\alpha^2+\beta^2+2\alpha\beta}{\alpha\beta}$$

$$= \frac{(\alpha+\beta)^2}{\alpha\beta} = \frac{(-4)}{3} = \frac{16}{3}$$

For $q(x)$, product of the zeroes,

$$\alpha_1\beta_1 = \left(1+\frac{\beta}{\alpha}\right)\left(1+\frac{\alpha}{\beta}\right)$$

$$= \left(\frac{\alpha+\beta}{\alpha}\right)\left(\frac{\beta+\alpha}{\beta}\right)$$

$$= \frac{(\alpha+\beta)^2}{\alpha\beta}$$

$$= \frac{(-4)^2}{3} = \frac{16}{3}$$

Hence, required polynomial

$$q(x) = x^2 - (\alpha_1+\beta_1)x + \alpha_1\beta_1$$

$$= x^2 - \left(\frac{16}{3}\right)x + \frac{16}{3}$$

$$= \left(x^2 - \frac{16}{3}x + \frac{16}{3}\right)$$

$$= \frac{1}{3}(3x^2 - 16x + 16)$$

83. If α and β are zeroes of the polynomial $p(x) = 6x^2-5x+k$ such that $\alpha-\beta = \frac{1}{6}$, Find the value of k.

Ans : [Board 2007]

We have $p(x) = 6x^2-5x+k$

b139

Since α and β are zeroes of

$$p(x) = 6x^2-5x+k,$$

Sum of zeroes, $\alpha+\beta = -\left(\frac{-5}{6}\right) = \frac{5}{6}$ ...(1)

Product of zeroes $\alpha\beta = \frac{k}{6}$ ...(2)

Given $\alpha-\beta = \frac{1}{6}$ ...(3)

Solving (1) and (3) we get $\alpha = \frac{1}{2}$ and $\beta = \frac{1}{3}$ and substituting the values of (2) we have

$$\alpha\beta = \frac{k}{6} = \frac{1}{2}\times\frac{1}{3}$$

Hence, $k = 1$.

84. If β and $\frac{1}{\beta}$ are zeroes of the polynomial $(a^2+a)x^2+61x+6a$. Find the value of β and α.

Ans :

We have $p(x) = (a^2+a)x^2+61x+6$

Since β and $\frac{1}{\beta}$ are the zeroes of polynomial, $p(x)$

Sum of zeroes, $\beta + \frac{1}{\beta} = -\frac{61}{a^2+a}$

or, $\frac{\beta^2+1}{\beta} = \frac{-61}{a^2+a}$ ...(1)

Product of zeroes $\beta\frac{1}{\beta} = \frac{6a}{a^2+a}$

or, $1 = \frac{6}{a+1}$

$a+1 = 6$

$a = 5$

Substituting this value of a in (1) we get

$$\frac{\beta^2+1}{\beta} = \frac{-61}{5^2+5} = -\frac{61}{30}$$

$$30\beta^2+30 = -61\beta$$

$$30\beta^2+61\beta+30 = 0$$

Now $\beta \frac{-61 \pm \sqrt{(-61)^2 - 4\times 30\times 30}}{2\times 30}$

$$= \frac{-61 \pm \sqrt{3721-3600}}{60}$$

$$\frac{-61 \mp 11}{60}$$

Thus $\beta = \frac{-5}{6}$ or $\frac{-6}{5}$

Hence, $\alpha = 5, \beta = \frac{-5}{6}, \frac{-6}{5}$

85. If α and β are the zeroes the polynomial $2x^2-4x+5$, find the values of

(i) $\alpha^2+\beta^2$ (ii) $\frac{1}{\alpha}+\frac{1}{\beta}$

(iii) $(\alpha-\beta)^2$ (iv) $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$

(v) $\alpha^2+\beta^2$

Ans : [Board 2007]

We have $p(x) = 2x^2-4x+5$

If α and β are then zeroes of $p(x) = 2x^2-4x+5$, then

$$\alpha+\beta = -\frac{b}{a} = \frac{-(-4)}{2} = 2$$

and $\alpha\beta = \frac{c}{a} = \frac{5}{2}$

(i) $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$$= 2^2 - 2\times\frac{5}{2}$$

$$= 4-5 = -1$$

(ii) $\frac{1}{\alpha}+\frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{2}{\frac{5}{2}} = \frac{4}{5}$

(iii) $(\alpha-\beta)^2 = (\alpha-\beta)^2 - 4\alpha\beta$

$$= 2^2 - \frac{4\times 5}{2}$$

$$4-10 = -6$$

(iv) $\frac{1}{\alpha^2}+\frac{1}{\beta^2} = \frac{\alpha^2+\beta^2}{(\alpha\beta)^2} = \frac{-1}{(\frac{5}{2})^2} = \frac{-4}{25}$

(v) $(\alpha^3+\beta^3) = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$

$$= 2^3 - 3\times\frac{5}{2}\times 2 = 8-15 = -7$$

CHAPTER 3

PAIR OF LINEAR EQUATION IN TWO VARIABLES

ONE MARK QUESTIONS

Multiple Choice Questions

1. The value of k for which the system of linear equations $x+2y=3,\ 5x+ky+7=0$ is inconsistent is

(a) $-\frac{14}{3}$ (b) $\frac{2}{5}$

(c) 5 (d) 10

c183

Ans : [Board 2020 OD Standard]

We have $x+2y-3=0$

and $5x+ky+7=0$

If system is inconsistent, then

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$$

From first two orders, we have

$$\frac{1}{5}=\frac{2}{k}\Rightarrow k=10$$

Thus (d) is correct option.

2. The value of k for which the system of equations $x+y-4=0$ and $2x+ky=3$, has no solution, is

(a) -2 (b) $\neq 2$

(c) 3 (d) 2

Ans : [Board 2020 Delhi Standard]

We have $x+y-4=0$

and $2x+ky-3=0$

c184

Here, $\frac{a_1}{a_1}=\frac{1}{2},\ \frac{b_1}{b_2}=\frac{1}{k}$ and $\frac{c_1}{c_2}=\frac{-4}{-3}=\frac{4}{3}$

Since system has no solution, we have

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$$

$$\frac{1}{2}=\frac{1}{k}\neq\frac{4}{3}$$

$$k=2 \text{ and } k\neq\frac{3}{4}$$

Thus (d) is correct option.

3. For which value(s) of p, will the lines represented by the following pair of linear equations be parallel

$$3x-y-5=0$$

$$6x-2y-p=0$$

(a) all real values except 10 (b) 10

(c) 5/2 (d) 1/2

Ans :

We have, $3x-y-5=0$

and $6x-2y-p=0$

c185

Here, $a_1=3,\ b_1=-1,\ c_1=-5$

and $a_2=6,\ b_2=-2,\ c_2=-p$

Since given lines are parallel,

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$$

$$\frac{-1}{-2}\neq\frac{-5}{-p}$$

$$p\neq 5\times 2\ \Rightarrow\ p\neq 10$$

4. The 2 digit number which becomes $\frac{5}{6}$th of itself when its digits are reversed. The difference in the digits of the number being 1, then the two digits number is

(a) 45 (b) 54

(c) 36 (d) None of these

Ans :

If the two digits are x and y, then the number is $10x+y$.

Now $\frac{5}{6}(10x+y)=10y+x$

c186

Solving, we get $44x+55y$

$$\frac{x}{y}=\frac{5}{4}$$

Also $x-y=1$. Solving them, we get $x=5$ and $y=4$. Therefore, number is 54.

Thus (b) is correct option.

5. In a number of two digits, unit's digit is twice the tens digit. If 36 be added to the number, the digits are reversed. The number is

(a) 36 (b) 63

(c) 48 (d) 84

Ans :

Let x be units digit and y be tens digit, then number will be $10y+x$

Then, $x = 2y$ ...(1)

If 36 be added to the number, the digits are reversed, i.e number will be $10x+y$.

$$10y+x+36 = 10x+y$$

$$9x-9y = 36$$

$$x-y = 4 \quad ...(2)$$

Solving (1) and (2) we have $x=8$ and $y=4$.

Thus number is 48.

Thus (c) is correct option.

6. If $3x+4y:x+2y = 9:4$, then $3x+5y:3x-y$ is equal to

(a) 4 : 1 (b) 1 : 4

(c) 7 : 1 (d) 1 : 7

Ans :

$$\frac{3x+4y}{x+2y} = \frac{9}{4}$$

Hence, $12x+16y = 9x+18y$

or $3x = 2y$

$$x = \frac{2}{3}y$$

Substituting $x = \frac{2}{3}y$ in the required expression we have

$$\frac{3x\frac{2}{3}y+5y}{3x\frac{2}{3}y-y} = \frac{7y}{y} = \frac{7}{1} = 7:1$$

Thus (c) is correct option.

7. A fraction becomes 4 when 1 is added to both the numerator and denominator and it becomes 7 when 1 is subtracted from both the numerator and denominator. The numerator of the given frac

(a) 2 (b) 3

(c) 5 (d) 15

Ans :

Let the fraction be $\frac{x}{y}$,

$$\frac{x+1}{y+1} = 4 \Rightarrow x = 4y+3 \quad ...(1)$$

and $$\frac{x-1}{y-1} = 7 \Rightarrow x = 7y-6 \quad ...(2)$$

Solving (1) and (2), we have $x = 15$, $y = 3$,

Thus (d) is correct option.

8. x and y are 2 different digits. If the sum of the two digit numbers formed by using both the digits is a perfect square, then value of $x+y$ is

(a) 10 (b) 11

(c) 12 (d) 13

Ans :

The numbers that can be formed are xy and yx. Hence, $(10x+y)+(10y+x) = 11(x+y)$. If this is a perfect square than $x+y = 11$.

9. The pair of equations $3^{x+y} = 81$, $81^{x-y} = 3$ has

(a) no solution

(b) unique solution

(c) infinitely many solutions

(d) $x = 2\frac{1}{8}, y = 1\frac{7}{8}$

Ans :

Given, $3^{x+y} = 81$

$$3^{x+y} = 3^4$$

$$x+y = 4 \quad ...(1)$$

and $81^{x-y} = 3$

$$3^{4(x-y)} = 3^1$$

$$4(x-y) = 1$$

$$x-y = \frac{1}{4} \quad ...(2)$$

Adding equation (1) and (2), we get

$$2x = 4+\frac{1}{4} = \frac{17}{4}$$

$$x = \frac{17}{8} = 2\frac{1}{8}$$

From equation (1), we get

$$y = \frac{15}{8} = 1\frac{7}{8}$$

Thus (d) is correct option.

10. The pair of linear equations $2kx+5y = 7$, $6x-5y = 11$

has a unique solution, if

(a) $k \neq -3$ (b) $k \neq \frac{2}{3}$

(c) $k \neq 5$ (d) $k \neq \frac{2}{9}$

Ans :

c192

Given the pair of linear equations are

$$2kx + 5y - 7 = 0$$

and $$6x - 5y - 11 = 0$$

On comparing with

$$a_1x + b_1y + c_1 = 0$$

and $$a_2x + b_2y + c_2 = 0$$

we get, $$a_1 = 2k,\ b_1 = 5,\ c_1 = -7$$

and $$a_2 = 6,\ b_2 = -5,\ c_2 = -11$$

For unique solution,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$\frac{2k}{6} \neq \frac{5}{-5}$$

$$\frac{k}{3} = \neq -1$$

$$k \neq -3$$

Thus (a) is correct option.

11. The pair of equations $x + 2y + 5 = 0$ and $-3x - 6y + 1 = 0$ has

(a) a unique solution

(b) exactly two solutions

(c) infinitely many solutions

(d) no solution

Ans :

c193

Given,equations are

$$x + 2y + 5 = 0$$

and $$-3x - 6y + 1 = 0$$

Here, $$a_1 = 1, b_1 = 2,\ c_1 = 5$$

and $$a_2 = -3,\ b_2 = -6,\ c_2 = 1$$

Now $$\frac{a_1}{a_2} = -\frac{1}{3},\ \frac{b_1}{b_2} = -\frac{2}{6} = -\frac{1}{3},\ \frac{c_1}{c_2} = \frac{5}{1}$$

Now, we observe that

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Hence, the pair of equations has no solution.

Thus (d) is correct option.

12. If a pair of linear equations is consistent, then the lines will be

(a) parallel

(b) always coincident

(c) intersecting or coincident

(d) always intersecting

c194

Ans :

Condition for a consistent pair of linear equations

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

[intersecting lines having unique solution]

and $$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$ [coincident or dependent]

Thus (c) is correct option.

13. The pair of equations $y = 0$ and $y = -7$ has

(a) one solution

(b) two solutions

(c) infinitely many solutions

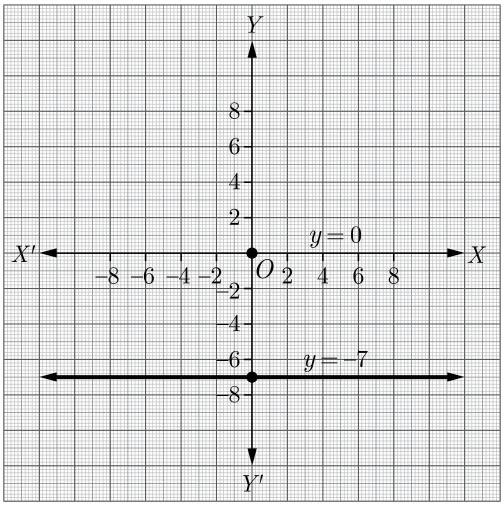
(d) no solution

c195

Ans :

The given pair of equations are

$$y = 0 \quad y = -7$$



The pair of both equations are parallel to x-axis and we know that parallel lines never intersects. So, there is no solution of these lines.

Thus (d) is correct option.

14. The pair of equations $x = a$ and $y = b$ graphically represents lines which are

(a) parallel (b) intersecting at (b, a)

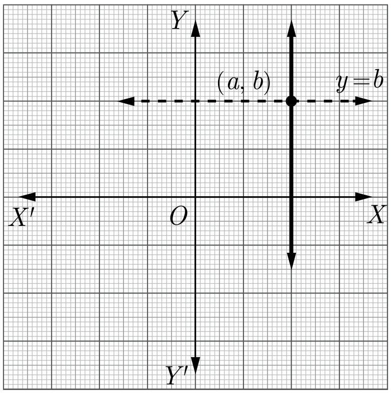
(c) coincident (d) intersecting at (a, b)

Ans :

The pair of equations

$$x = a$$

and $$y = b$$



Graphically represents lines which are intersecting at (a, b).

Thus (d) is correct option.

15. For what value of k, do the equations $3x - y + 8 = 0$ and $6x - ky = -16$ represent coincident lines ?

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$

(c) 2 (d) -2

Ans :

Given, equations,

$$3x - y + 8 = 0$$

and $$6x - ky + 16 = 0$$

Condition for coincident lines is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad ...(1)$$

Here, $a_1 = 3, b_1 = -1, c_1 = 8$

and $a_2 = 6, b_2 = -k, c_2 = 16$

From equation (1),

$$\frac{3}{6} = \frac{-1}{-k} = \frac{8}{16}$$

$$\frac{1}{k} = \frac{1}{2} \qquad \left[\text{since } \frac{3}{6} = \frac{8}{16} = \frac{1}{2}\right]$$

$$k = 2$$

Thus (c) is correct option.

16. If the lines given by $3x + 2ky = 2$ and $2x + 5y + 1 = 0$ are parallel, then the value of k is

(a) $-\frac{5}{4}$ (b) $\frac{2}{5}$

(c) $\frac{15}{4}$ (d) $\frac{3}{2}$

Ans :

We have $3x + 2ky - 2 = 0$

and $2x + 5y + 1 = 0$

Condition for parallel lines is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \quad ...(i)$$

Here, $a_1 = 3, b_1 = 2k, c_1 = -2$

and $a_2 = 2, b_2 = 5, c_2 = 1$

From equation (i), we have

$$\frac{3}{2} = \frac{2k}{5} \neq \frac{-2}{1}$$

Considering, $$\frac{3}{2} = \frac{2k}{5} \qquad \left[\frac{3}{2} \neq \frac{-2}{1} \text{ in any case}\right]$$

$$k = \frac{15}{4}$$

Thus (c) is correct option.

17. The value of c for which the pair of equations $cx - y = 2$ and $6x - 2y = 3$ will have is

(a) 3 (b) -3

(c) -12 (d) no value

Ans :

The given lines are, $cx - y = 2$

and $6x - 2y = 3$

Condition for infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad ...(i)$$

Here, $a_1 = c,\ b_1 = -1,\ c_1 = -2$

and $a_2 = 6,\ b_2 = -2,\ c_2 = -3$

From equation (i), $\frac{c}{6} = \frac{-1}{-2} = \frac{-2}{-3}$

Here, $$\frac{c}{6} = \frac{1}{2}$$

and $$\frac{c}{6} = \frac{2}{3}$$

$$c = 3$$

and $$c = 4$$

Since, c has different values.

Hence, for no value of c the pair of equations will have infinitely many solutions.

Thus (d) is correct option.

18. One equation of a pair of dependent linear equations $-5x+7y=2$ The second equation can be

(a) $10x+14y+4=0$ (b) $-10x-14y+4=0$

(c) $-10x+14y+4=0$ (d) $10x-14y=-4$

Ans :

For dependent linear equation,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

c200

Checking for option (a):

$$\frac{-5}{10} \neq \frac{7}{14}$$

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$ So, option (a) is rejected.

Checking for option (b):

$$\frac{-5}{-10} \neq \frac{7}{-14}$$

So, option (b) is also rejected.

Checking for option (c):

$$\frac{-5}{-10} = \frac{7}{14} \neq \frac{-2}{4}$$

So, option (b) is also rejected

Checking for option (d):

$$\frac{-5}{10} = \frac{7}{-14} = \frac{-2}{4}$$

Thus (d) is correct option.

19. If $x=a$ and $y=b$ is the solution of the equations $x-y=2$ and $x+y=4$, then the values of a and b are, respectively

(a) 3 and 5 (b) 5 and 3

(c) 3 and 1 (d) -1 and -3

Ans :

Since, $x = a$ and $y = b$ is the solution of the equations $x-y = 2$ and $x+y = 4$, then these values will satisfy that equation

$$a-b = 2 \quad ...(1)$$

and $$a+b = 4 \quad ...(2)$$

Adding equations (1) and (2), we get

$$2a = 6$$

c201

$$a = 3$$

Substituting $a = 3$ in equation (2),we have

$$3+b = 4 \Rightarrow b = 1$$

Thus $a = 3$ and $b = 1$.

Thus (c) is correct option.

20. Aruna has only ₹ 1 and ₹ 2 coins with her. If the total number of coins that she has is 50 and the amount of money with her is ₹ 75, then the number of ₹ 1 and ₹ 2 coins are, respectively

(a) 35 and 15 (b) 35 and 20

(c) 15 and 35 (d) 25 and 25

Ans :

Let number of ₹ 1 coins $= x$

and number of ₹ 2 coins $= y$

c202

Now, by given conditions,

$$x+y = 50 \quad ...(1)$$

Also, $$x \times 1 + y \times 2 = 75$$

$$x+2y = 75 \quad ...(2)$$

Subtracting equation (1) form equation (2), we get

$$(x+2y)-(x+y) = 75-50$$

$$y = 25$$

From equation (i), $x = 75-2x(25)$

Then, $$x = 25$$

Thus (d) is correct option.

21. The father's age is six times his son's age. Four years hence, the age of the father will be four times his son's age. The present ages (in year) of the son and the father are, respectively.

(a) 4 and 24 (b) 5 and 30

(c) 6 and 36 (d) 3 and 24

Ans :

Let the present age of father $= x$ years

and the present age of son $= y$ years

Four years hence, it has relation by given condition

$$(x+4) = 4(y+4)$$

$$x-4y = 12 \quad ...(1)$$

As the father's age is six times his son's age, so we have

$$x = 6y \quad ...(2)$$

Putting the value of x from equation (2) in equation (1), we get

$$6x-4y = 12$$

$$2y = 12$$

$$y = 6$$

From equation (1), $x = 6 \times 6$

Then, $x = 36$

Hence, present age of father is 36 year and age of son is 6 year.

Thus (c) is correct option.

22. Assertion : Pair of linear equations : $9x+3y+12=0$, $8x+6y+24=0$ have infinitely many solutions.

Reason : Pair of linear equations $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ have infinitely many solutions, if $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

From the given equations, we have

$$\frac{9}{18} = \frac{3}{6} = \frac{12}{24}$$

$$\frac{1}{2} = \frac{1}{2} = \frac{1}{2} \text{ i.e., } \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Thus (a) is correct option.

23. Assertion : $x+y-4=0$ and $2x+ky-3=0$ has no solution if $k=2$.

Reason : $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ are consistent if $\frac{a_1}{a_2} \neq \frac{k_1}{k_2}$.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

For assertion, given equation has no solution if

$$\frac{1}{2} = \frac{1}{k} \neq \frac{-4}{-3} \text{ i.e. } \frac{4}{3}$$

$$k = 2\left[\frac{1}{2} \neq \frac{4}{3} \text{holds}\right]$$

Assertion is true.

Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

Thus (b) is correct option.

Fill in the Blank Questions

24. If the lines intersect at a point, then that point gives the unique solution of the two equations. In this case, the pair of equations is ....

Ans :

consistent

25. An equation whose degree is one is known as a .... equation.

Ans :

linear

26. If the lines are parallel, then the pair of equations has no solution. In this case, the pair of equations is

Ans :

inconsistent

27. A pair of linear equations has .... solution(s) if it is represented by intersecting lines graphically.

Ans :

unique

28. Every solution of a linear equation in two variables is a point on the .... representing it.

Ans :

line

29. If a pair of linear equations has infinitely many solutions, then its graph is represented by a pair of .... lines.

Ans :

coincident

c211

30. A pair of linear equations is .... if it has no solution.

Ans :

inconsistent

c212

31. A pair of .... lines represent the pair of linear equations having no solution.

Ans :

parallel

c213

32. If a pair of linear equations has solution, either a unique or infinitely many, then it is said to be ....

Ans :

consistent

c214

33. If the equations $kx - 2y = 3$ and $3x + y = 5$ represent two intersecting lines at unique point, then the value of k is ....

Ans :

c215

For unique solution

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Here, $a_1 = k$, $b_1 = -2$, $a_2 = 3$ and $b_2 = 1$

Now $\frac{k}{3} \neq -\frac{2}{1}$

or, $k \neq -6$

Very Short Answer Questions

34. Find whether the pair of linear equations $y = 0$ and $y = -5$ has no solution, unique solution or infinitely many solutions.

Ans :

The given variable y has different values. Therefore the pair of equations $y = 0$ and $y = -5$ has no solution.

c101

35. If $am = bl$, then find whether the pair of linear equations $ax + by = c$ and $lx + my = n$ has no solution, unique solution or infinitely many solutions.

Ans :

Since, $am = bl$, we have

c102

$$\frac{a}{l} = \frac{b}{m} \neq \frac{c}{n}$$

Thus, $ax + by = c$ and $lx + my = n$ has no solution.

36. If $ad \neq bc$, then find whether the pair of linear equations $ax + by = p$ and $cx + dy = q$ has no solution, unique solution or infinitely many solutions.

Ans :

Since $ad \neq bc$ or $\frac{a}{c} \neq \frac{b}{d}$

c103

Hence, the pair of given linear equations has unique solution.

37. Two lines are given to be parallel. The equation of one of the lines is $4x + 3y = 14$, then find the equation of the second line.

Ans :

The equation of one line is $4x + 3y = 14$. We know that if two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are parallel, then

c104

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

or $\frac{4}{a_2} = \frac{3}{b_2} \neq \frac{c_1}{c_2} \Rightarrow \frac{a_2}{b_2} = \frac{4}{3} = \frac{12}{9}$

Hence, one of the possible, second parallel line is $12x + 9y = 5$.

TWO MARKS QUESTIONS

38. Find the value(s) of k so that the pair of equations $x + 2y = 5$ and $3x + ky + 15 = 0$ has a unique solution.

Ans : [Board 2019 OD]

We have $x + 2y - 5 = 0$ ...(1)

and $3x + ky + 15 = 0$ ...(2)

Comparing equation (1) with $a_1x + b_1y + c_1 = 0$, and equation (2) with $a_2x + b_2y + c_2 = 0$, we get

$$a_1 = 1,\ a_2 = 3,\ b_1 = 2,\ b_2 = k,\ c_1 = -5 \text{ and } c_2 = 15$$

Since, given equations have unique solution, So,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

c216

i.e. $\frac{1}{3} \neq \frac{2}{k}$

$$k \neq 6$$

Hence, for all values of k except 6, the given pair of equations have unique solution.

39. If $2x+y=23$ and $4x-y=19$, find the value of $(5y-2x)$ and $(\frac{y}{x}-2)$.

Ans : [Board 2020 OD Standard]

We have $\quad 2x+y = 23 \quad ...(1)$

$$4x-y = 19 \quad ...(2)$$

Adding equation (1) and (2), we have

$$6x = 42 \Rightarrow x = 7$$

Substituting the value of x in equation (1), we get

$$14+y = 23$$

$$y = 23-14 = 9$$

c217

Hence, $\quad 5y-2x = 5\times 9 - 2\times 7$

$$= 45-14 = 31$$

and $\quad \frac{y}{x}-2 = \frac{9}{7}-2 = \frac{9-14}{7} = \frac{-5}{7}$

40. Find whether the lines represented by $2x+y=3$ and $4x+2y=6$ are parallel, coincident or intersecting.

Ans : [Board Term-1 2016, MV98HN3]

Ans :

Here $a_1=2, b_1=1, c_1=-3$ and $a_1=4, b_2=2, c_2=-6$

If $\quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

then the lines are parallel.

c105

Clearly $\quad \frac{2}{4} = \frac{1}{2} = \frac{3}{6}$

Hence lines are coincident.

41. Find whether the following pair of linear equation is consistent or inconsistent:

$3x+2y=8,\ \ 6x-4y=9$

Ans : [Board Term-1 2016]

We have $\quad \frac{3}{6} \neq \frac{2}{-4}$

c106

i.e., $\quad \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence, the pair of linear equation is consistent.

42. Is the system of linear equations $2x+3y-9=0$ and $4x+6y-18=0$ consistent? Justify your answer.

Ans : [Board Term-1 2012]

For the equation $2x+3y-9=0$ we have

$$a_2 = 2, b_1 = 3 \text{ and } c_1 = -9$$

and for the equation, $4x+6y-18=0$ we have

$$a_2 = 4, b_2 = 6 \text{ and } c_2 = -18$$

c107

Here $\quad \frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$

$$\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$$

and $\quad \frac{c_1}{c_2} = \frac{-9}{-18} = \frac{1}{2}$

Thus $\quad \frac{c_1}{c_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Hence, system is consistent and dependent.

43. Given the linear equation $3x+4y=9$. Write another linear equation in these two variables such that the geometrical representation of the pair so formed is:

(1) intersecting lines

(2) coincident lines.

Ans : [Board Term-1 2016, Set-O4YP6G7]

(1) For intersecting lines $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

c108

So, one of the possible equation $3x-5y=10$

(2) For coincident lines $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

So, one of the possible equation $6x+8y=18$

44. For what value of p does the pair of linear equations given below has unique solution ?

$4x+py+8=0$ and $2x+2y+2=0$.

Ans : [Board Term-1 2012]

We have $\quad 4x+py+8 = 0$

c109

$$2x+2y+2 = 0$$

The condition of unique solution, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence, $\quad \frac{4}{2} \neq \frac{p}{2}$ or $\frac{2}{1} \neq \frac{p}{2}$

Thus $p \neq 4$. The value of p is other than 4 it may be 1,2,3, -4.....etc.

45. For what value of k, the pair of linear equations $kx-4y=3,\ 6x-12y=9$ has an infinite number of solutions ?

Ans : [Board Term-1 2012]

We have $$kx - 4y - 3 = 0$$

and $$6x - 12y - 9 = 0$$

c110

where, $$a_1 = k, b_1 = 4, c_1 = -3$$

$$a_2 = 6, b_2 = -12, c_2 - 9$$

Condition for infinite solutions:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{k}{6} = \frac{-4}{-12} = \frac{3}{9}$$

Hence, $$k = 2$$

46. For what value of k, $2x + 3y = 4$ and $(k+2)x + 6y = 3k + 2$ will have infinitely many solutions ?

Ans : [Board Term-1 2012]

We have $$2x + 3y - 4 = 0$$

and $$(k+2)x + 6y - (3k+2) = 0$$

c111

Here $a_1 = 2, b_1 = 3, c_1 = -4$

and $a_2 = k + 2, b_2 = 6, c_3 = -(3k+2)$

For infinitely many solutions

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

or, $$\frac{2}{k+2} = \frac{3}{6} = \frac{4}{3k+2}$$

From $\frac{2}{k+2} = \frac{3}{6}$ we have

$3(k+2) = 2 \times 6 \Rightarrow (k+2) = 4 \Rightarrow k = 2$

From $\frac{3}{6} = \frac{4}{3k+2}$ we have

$3(3k+2) = 4 \times 6 \Rightarrow (3k+2) = 8 \Rightarrow k = 2$

Thus $k = 2$

47. For what value of k, the system of equations $kx + 3y = 1$, $12x + ky = 2$ has no solution.

Ans : [Board Term-1 2011, NCERT]

The given equations can be written as

$kx + 3y - 1 = 0$ and $12x + ky - 2 = 0$

Here, $$a_1 = k, b_1 = 3, c_1 = -1$$

and $$a_2 = 12, b_2 = k, c_2 = -2$$

The equation for no solution if

c112

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

or, $$\frac{k}{12} = \frac{3}{k} \neq \frac{-1}{-2}$$

From $\frac{k}{12} = \frac{3}{k}$ we have $k^2 = 36 \Rightarrow k \pm 6$

From $\frac{3}{k} \neq \frac{-1}{-2}$ we have $k \neq 6$

Thus $k = -6$

48. Solve the following pair of linear equations by cross multiplication method:

$$x + 2y = 2$$

$$x - 3y = 7$$

Ans : [Board Term-1 2016]

We have $$x + 2y - 2 = 0$$

$$x - 3y - 7 = 0$$

c133

Using the formula

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

we have $$\frac{x}{-14-6} = \frac{y}{-2+7} = \frac{1}{-3-2}$$

$$\frac{x}{-20} = \frac{y}{5} = \frac{-1}{5}$$

$$\frac{x}{-20} = \frac{-1}{5} \Rightarrow x = 4$$

$$\frac{y}{5} = \frac{-1}{5} \Rightarrow y = -1$$

49. Solve the following pair of linear equations by substitution method:

$$3x + 2y - 7 = 0$$
$$4x + y - 6 = 0$$

Ans : [Board Term-1 2015]

We have $\quad 3x + 2y - 7 = 0 \quad$ ...(1)

$$4x + y - 6 = 0 \quad ...(2)$$

From equation (2), $\quad y = 6 - 4x \quad$ ...(3)

Putting this value of y in equation (1) we have

$$3x + 2(6 - 4x) - 7 = 0$$
$$3x + 12 - 8x - 7 = 0$$
$$5 - 5x = 0$$
$$5x = 5$$

c134

Thus $\quad x = 1$

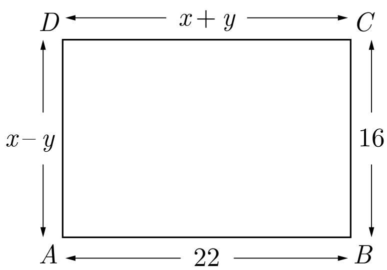
Substituting this value of x in (2), we obtain,

$$y = 6 - 4 \times 1 = 2$$

Hence, values of x and y are 1 and 2 respectively.

50. In the figure given below, $ABCD$ is a rectangle. Find the values of x and y.

Ans : [Board Term-1 2012, Set-30]



From given figure we have

$$x + y = 22 \quad ...(1)$$

and $\quad x - y = 16 \quad$ ...(2)

Adding (1) and (2), we have

c135

$$2x = 38$$
$$x = 19$$

Substituting the value of x in equation (1), we get

$$19 + y = 22$$
$$y = 22 - 19 = 3$$

Hence, $\quad x = 19$ and $y = 3$.

51. Solve : $\quad 99x + 101y = 499, \quad 101x + 99y = 501$

Ans : [Board Term-1 2012, Set-55]

We have $\quad 99x + 101y = 499 \quad$ ...(1)

$$101x + 99y = 501 \quad ...(2)$$

Adding equation (1) and (2), we have

$$200x + 200y = 1000$$
$$x + y = 5 \quad ...(3)$$

Subtracting equation (2) from equation (1), we get

$$-2x + 2y = -2$$
$$x - y = 1 \quad ...(4)$$

Adding equations (3) and (4), we have

$$2x = 6 \Rightarrow x = 3$$

Substituting the value of x in equation (3), we get

c136

$$3 + y = 5 \Rightarrow y = 2$$

52. Solve the following system of linear equations by substitution method:

$$2x - y = 2$$
$$x + 3y = 15$$

Ans : [Board Term-1 2012]

We have $\quad 2x - y = 2 \quad$ ...(1)

$$x + 3y = 15 \quad ...(2)$$

From equation (1), we get $y = 2x - 2 \quad$ ...(3)

Substituting the value of y in equation (2),

$$x + 6x - 6 = 15$$

or, $\quad 7x = 21 \Rightarrow x = 3$

c137

Substituting this value of x in (3), we get

From equation (1), we have

$$y = 2 \times 3 - 2 = 4$$

$$x = 3 \text{ and } y = 4$$

53. Find the value(s) of k for which the pair of Linear equations $kx + y = d^2$ and $x + ky = 1$ have infinitely many solutions.

Ans : [Board Term-1 2017]

We have $kx + y = k^2$

and $x + ky = 1$

$$\frac{a_1}{a_2} = \frac{k}{1}, \frac{b_1}{b_2} = \frac{1}{k}, \frac{c_1}{c_2} = \frac{k^2}{1}$$

c138

For infinitely many solution

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{k}{1} = \frac{1}{k} = \frac{k^2}{1} = k^2 = 1$$

$$k = \pm 1$$

THREE MARKS QUESTIONS

54. Solve the following system of equations.
$\frac{21}{x} + \frac{47}{y} = 110, \frac{47}{x} + \frac{21}{y} = 162, x, y \neq 0$

Ans :

We have $\frac{21}{x} + \frac{47}{y} = 110$

$$\frac{47}{x} + \frac{21}{y} = 162$$

c219

Let $\frac{1}{x} = u$ and $\frac{1}{y} = v$. then given equation become

$$21u + 47v = 110 \qquad ...(1)$$

and $47u + 21v = 162 \qquad ...(2)$

Adding equation (1) and (2) we get

$$68u + 68v = 272$$

$$u + v = 4 \qquad ...(3)$$

Subtracting equation (1) from (2) we get

$$26u - 26v = 52$$

$$u - v = 2 \qquad ...(4)$$

Adding equation (3) and (4), we get

$$2u = 6 \Rightarrow u = 3$$

Substituting $u = 3$ in equation (3), we get $v = 1$.

Thus $x = \frac{1}{u} = \frac{1}{3}$ and $y = \frac{1}{v} = \frac{1}{1} = 1$

55. A fraction becomes $\frac{1}{3}$ when 2 is subtracted from the numerator and it becomes $\frac{1}{2}$ when 1 is subtracted from the denominator- Find the fraction.

Ans : [Board 2019 Delhi]

Let the fraction be $\frac{x}{y}$. According to the first condition,

$$\frac{x-2}{y} = \frac{1}{3}$$

c221

$$3x - 6 = y$$

$$y = 3x - 6 \qquad ...(1)$$

According to the second condition,

$$\frac{x}{y-1} = \frac{1}{2}$$

$$2x = y - 1$$

$$y = 2x + 1 \qquad ...(2)$$

From equation (1) and (2), we have

$$3x - 6 = 2x + 1 \Rightarrow x = 7$$

Substitute value of x in equation (1), we get

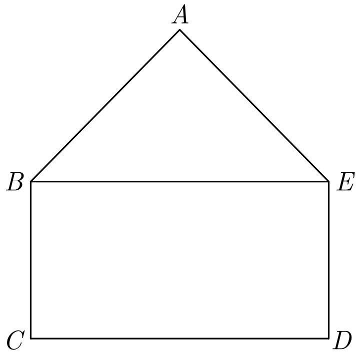
$$y = 3(7) - 6$$

$$= 21 - 6 = 15$$

Hence, fraction is $\frac{7}{15}$.

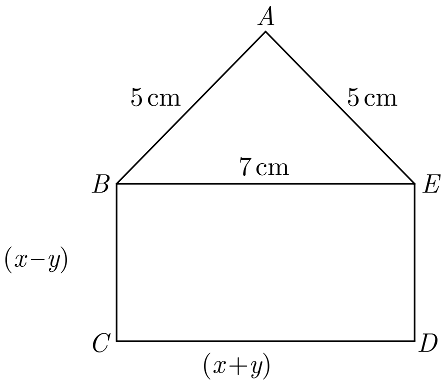
56. In the figure, $ABCDE$ is a pentagon with $BE \parallel CD$ and $BC \parallel DE$. BC is perpendicular to CD. $AB = 5$ cm, $AE = 5$ cm, $BE = 7$ cm, $BC = x - y$ and $CD = x + y$. If the perimeter of $ABCDE$ is 27 cm. Find the value of x and y, given $x, y \neq 0$.

c222



Ans : [Board 2020 SQP Standard]

We have redrawn the given figure as shown below.



We have $CD = BE$

$$x + y = 7 \quad ...(1)$$

Also, perimeter of $ABCDE$ is 27 cm, thus

$$AB + BC + CD + DE + AE = 27$$

$$5 + (x - y) + (x + y) + (x - y) + 5 = 27$$

$$3x - y = 17 \quad ...(2)$$

Adding equation (1) and (2) we have

$$4x = 24 \Rightarrow x = 6$$

Substituting $x = 6$ in equation (1) we obtain

$$y = 7 - x = 7 - 6 = 1$$

Thus $x = 6$ and $y = 1$.

57. Half the perimeter of a rectangular garden, whose length is 4 m more then its width, is 36 m. Find the dimensions of garden.

Ans : [Board Term-1 2013]

Let the length of the garden be x m and its width be y m.

Perimeter of rectangular garden

$$p = 2(x + y)$$

c113

Since half perimeter is given as 36 m,

$$(x + y) = 36 \quad ...(1)$$

Also, $x = y + 4$

or $x - y = 4 \quad ...(2)$

For $x + y = 36$

$$y = 36 - x$$

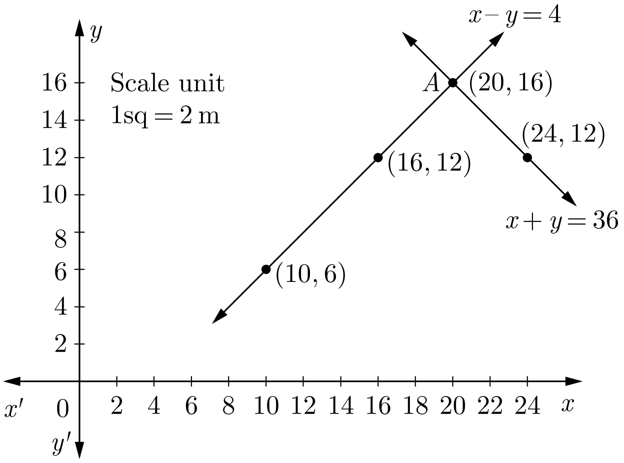
x	20	24
y	16	12

For $x - y = 4$

or, $y = x - 4$

x	10	16	20
y	6	12	16

Plotting the above points and drawing lines joining them, we get the following graph. we get two lines intersecting each other at (20, 16)



Hence, length is 20 m and width is 16 m.

58. Given the linear equation $2x + 3y - 8 = 0$, write another linear equation in two variables such that the geometrical representation of the pair so formed is :

(a) intersecting lines

(b) parallel lines

(c) coincident lines.

Ans : [Board Term-1 2014, Set-B]

Given, linear equation is $2x + 3y - 8 = 0 \quad ...(1)$

(a) For intersecting lines, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

To get its parallel line one of the possible equation may be taken as

$$5x + 2y - 9 = 0 \quad (2)$$

(b) For parallel lines, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

One of the possible line parallel to equation (1) may be taken as

c114

$$6x + 9y + 7 = 0$$

(c) For coincident lines, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

To get its coincident line, one of the possible equation may be taken as

$$4x + 6y - 16 = 0$$

59. Solve the pair of equations graphically :

$4x - y = 4$ and $3x + 2y = 14$

Ans : [Board Term-1 2014

c115

We have $4x - y = 4$

or, $y = 4x - 4$

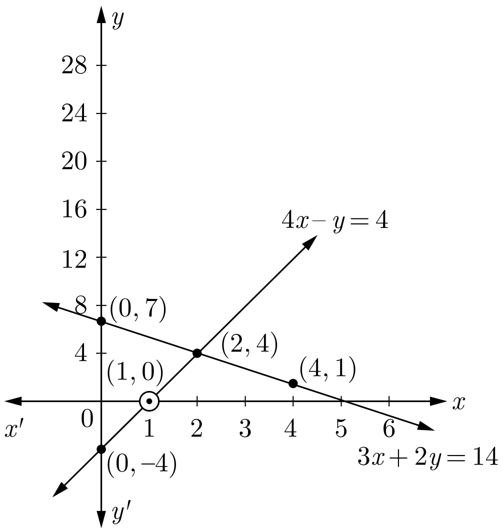
x	0	1	2
y	-4	0	4

and $3x + 2y = 14$

or, $y = \frac{14 - 3x}{2}$

x	0	2	4
y	7	4	1

Plotting the above points and drawing lines joining them, we get the following graph. We get two obtained lines intersect each other at (2, 4).



Hence, $x = 2$ and $y = 4$.

60. Determine the values of m and n so that the following system of linear equation have infinite number of solutions :

$$(2m - 1)x + 3y - 5 = 0$$

$$3x + (n - 1)y - 2 = 0$$

Ans : [Board Term-1 2013, VKH6FFC; 2011, Set-66

We have $(2m - 1)x + 3y - 5 = 0$ ...(1)

Here $a_1 = 2m - 1, b_1 = 3, c_1 = -5$

$3x + (n - 1)y - 2 = 0$ ...(2)

Here $a_2 = 3, b_2 = (n - 1), c_2 = -2$

For a pair of linear equations to have infinite number of solutions,

c116

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{2m - 1}{3} = \frac{3}{n - 1} = \frac{5}{2}$$

or $2(2m - 1) = 15$ and $5(n - 1) = 6$

Hence, $m = \frac{17}{4}, n = \frac{11}{5}$

61. Find the values of α and β for which the following pair of linear equations has infinite number of solutions : $2x + 3y = 7$; $2\alpha x + (\alpha + \beta)y = 28$.

Ans : [Board Term-1 2011]

We have $2x + 3y = 7$ and $2\alpha x + (\alpha + \beta)y = 28$.

For a pair of linear equations to be consistent and having infinite number of solutions,

c117

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{2}{2\alpha} = \frac{3}{\alpha + \beta} = \frac{7}{28}$$

$$\frac{2}{2\alpha} = \frac{7}{28}$$

$$2\alpha \times 7 = 28 \times 2 \Rightarrow \alpha = 4$$

$$\frac{3}{\alpha + \beta} = \frac{7}{28}$$

$$7(\alpha + \beta) = 28 \times 3$$

$$\alpha + \beta = 12$$

$$\beta = 12 - \alpha = 12 - 4 = 8$$

Hence $\alpha = 4$, and $\beta = 8$

62. Represent the following pair of linear equations graphically and hence comment on the condition of consistency of this pair.

$x - 5y = 6$ and $2x - 10y = 12$.

Ans : [Board Term-1 2011]

We have $x - 5y = 6$ or $x = 5y + 6$

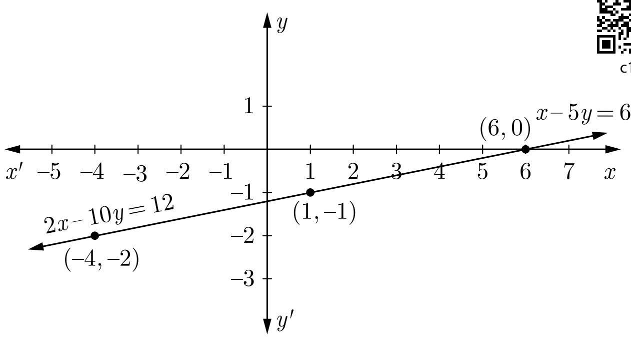
x	6	1	-4
y	0	-1	-2

and $2x - 10y = 12$ or $x = 5y + 6$

x	6	1	-4
y	0	-1	-2

Plotting the above points and drawing lines joining them, we get the following graph.

c118



Since the lines are coincident, so the system of linear equations is consistent with infinite many solutions.

63. For what value of p will the following system of equations have no solution ?

$(2p-1)x+(p-1)y = 2p+1;\ y+3x-1=0$

Ans : [Board Term-1 2011, Set-28]

We have $\quad (2p-1)x+(p-1)y-(2p+1) = 0$

Here $a_1 = 2p-1, b_1 = p-1$ and $c_1 = -(2p+1)$

Also $\quad 3x+y-1 = 0$

Here $a_2 = 3, b_2 = 1$ and $c_2 = -1$

c119

The condition for no solution is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\frac{2p-1}{3} = \frac{p-1}{1} \neq \frac{2p+1}{1}$$

From $\frac{2p-1}{3} = \frac{p-1}{1}$ we have

$$3p-3 = 2p-1$$

$$3p-2p = 3-1$$

$$p = 2$$

From $\frac{p-1}{1} \neq \frac{2p+1}{1}$ we have

$$p-1 \neq 2p+1 \text{ or } 2p-p \neq -1-1$$

$$p \neq -2$$

From $\frac{2p-1}{3} \neq \frac{2p+1}{1}$ we have

$$2p-1 \neq 6p+3$$

$$4p \neq -4$$

$$p \neq -1$$

Hence, system has no solution when $p = 2$

64. Find the value of k for which the following pair of equations has no solution :

$x+2y=3, (k-1)x+(k+1)y=(k+2)$.

Ans : [Board Term-1 2011, Set-52]

For $x+2y=3$ or $x+2y-3=0$,

c120

$$a_1 = 1, b_1 = 2, c_1 = -3$$

For $(k-1)x+(k+1)y=(k+2)$

or $(k-1)x+(k+1)y-(k-2)=0$

$$a_2 = (k-1), b_2 = (k+1), c_2 = -(k+2)$$

For no solution, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$$\frac{1}{k-1} = \frac{2}{k+1} \neq \frac{3}{k+2}$$

From $\frac{1}{k-1} = \frac{2}{k+1}$ we have

$$k+1 = 2k-2$$

$$3 = k$$

Thus $k = 3$.

65. Sum of the ages of a father and the son is 40 years. If father's age is three times that of his son, then find their respective ages.

Ans : [Board Term-1 2015]

Let age of father and son be x and y respectively.

$$x+y = 40 \quad ...(1)$$

$$x = 3y \quad ...(2)$$

Solving equations (1) and (2), we get

c139

$$x = 30 \text{ and } y = 10$$

Ages are 30 years and 10 years.

66. Solve using cross multiplication method:

$$5x+4y-4 = 0$$

$$x-12y-20 = 0$$

Ans : [Board Term-1 2015]

We have $\quad 5x+4y-4 = 0 \quad ...(1)$

$$x-12y-20 = 0 \quad ...(2)$$

By cross-multiplication method,

$$\frac{x}{b_2c_1-b_1c_2} = \frac{y}{c_1a_2-c_2a_1} = \frac{1}{b_1b_2-a_2b_1}$$

$$\frac{x}{-80-48} = \frac{y}{-4+100} = \frac{1}{-60-4}$$

$$\frac{x}{-128} = \frac{y}{96} = \frac{1}{64}$$

c140

$$\frac{x}{-128} = \frac{1}{-64} \Rightarrow x = 2$$

and $$\frac{y}{96} = \frac{1}{-64} \Rightarrow y = \frac{-3}{2}$$

Hence, $x = 2$ and $y = \frac{-3}{2}$

67. The Present age of the father is twice the sum of the ages of his 2 children. After 20 years, his age will be equal to the sum of the ages of his children. Find the age of the father.

Ans : [Board Term-1 2012, Set-39]

Let the sum of the ages of the 2 children be x and the age of the father be y years.

Now $$y = 2x$$

$$2x - y = 0 \quad ...(1)$$

and $$20 + y = x + 40$$

$$x - y = -20 \quad ...(2)$$

Subtracting (2) from (1), we get

$$x = 20$$

c166

From(1), $$y = 2x = 2 \times 20 = 40$$

Hence, the age of the father is 40 years.

68. A part of monthly hostel charge is fixed and the remaining depends on the number of days one has taken food in the mess. When Swati takes food for 20 days, she has to pay Rs. 3,000 as hostel charges whereas Mansi who takes food for 25 days Rs. 3,500 as hostel charges. Find the fixed charges and the cost of food per day.

Ans : [Board Term-1 2016, 2015]

Let fixed charge be x and per day food cost be y

$$x + 20y = 3000 \quad ...(1)$$

$$x + 25y = 3500 \quad ...(2)$$

Subtracting (1) from (2) we have

$$5y = 500 \Rightarrow y = 100$$

c141

Substituting this value of y in (1), we get

$$x + 20(100) = 3000$$

$$x = 1000$$

Thus $x = 1000$ and $y = 100$

Fixed charge and cost of food per day are Rs. 1,000 and Rs. 100.

69. Solve for x and y :

$$\frac{x}{2} + \frac{2y}{3} = -1$$

$$x - \frac{y}{3} = 3$$

Ans : [Board Term-1 2015, NCERT]

We have $$\frac{x}{2} + \frac{2y}{3} = -1$$

$$3x + 4y = -6 \quad ...(1)$$

and $$\frac{x}{1} - \frac{y}{3} = 3$$

$$3x - y = 9 \quad ...(2)$$

Subtracting equation (2) from equation (1), we have

$$5y = -15 \Rightarrow y = -1$$

Substituting $y = -3$ in eq (1), we get

c142

$$3x + 4(-3) = -6$$

$$3x - 12 = -6$$

$$3x = 12 - 6 \Rightarrow x = 2$$

Hence $x = 2$ and $y = -3$.

70. Solve the following pair of linear equations by the substitution and cross - multiplication method :

$$8x + 5y = 9$$

$$3x + 2y = 4$$

Ans : [Board Term-1 2015, SYFH4D]

We have $$8x + 5y = 9$$

or, $$8x + 5y - 9 = 0 \quad ...(1)$$

and $$3x + 2y = 4$$

or, $$3x + 2y - 4 = 0 \quad ...(2)$$

Comparing equation (1) and (2) with $ax + by + c = 0$,

$$a_1 = 8, b_1 = 5, c_1 = -9$$

and $a_2 = 3, b_2 = 2, c_2 = -4$

By cross-multiplication method,

$$\frac{x}{b_2c_1 - b_1c_2} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - b_2b_1}$$

$$\frac{x}{\{(5)(-4)-(2)(-9)\}} = \frac{y}{\{(-9)(3)-(-4)(8)\}}$$

$$= \frac{1}{\{8\times 2 - 3\times 5\}}$$

or, $\frac{x}{-2} = \frac{1}{1}$ and $\frac{y}{5} = \frac{1}{1}$

$$x = -2 \text{ and } y = 5$$

c143

We use substitution method.
From equation (2), we have

$$3x = 4 - 2y$$

or, $x = \frac{4-2y}{3}$ ...(3)

Substituting this value of y in equation (3) in (1), we get

$$8\left(\frac{4-2y}{3}\right) + 5y = 9$$

$$32 - 16y + 15y = 27$$

$$-y = 27 - 32$$

Thus $y = 5$

Substituting this value of y in equation (3)

$$x = \frac{4-2(5)}{3} = \frac{4-10}{3} = -2$$

Hence, $x = -2$ and $y = 5$.

71. 2 man and 7 boys can do a piece of work in 4 days. It is done by 4 men and 4 boys in 3 days. How long would it take for one man or one boy to do it ?

Ans : [Board Term-1 2013]

Let the man can finish the work in x days and the boy can finish work in y days.

Work done by one man in one day $= \frac{1}{x}$

And work done by one boy in one day $= \frac{1}{y}$

$$\frac{2}{x} + \frac{7}{y} = \frac{1}{4} \quad ...(1)$$

and $\frac{4}{x} + \frac{4}{y} = \frac{1}{3}$ ...(2)

Let $\frac{1}{x}$ be a and $\frac{1}{y}$ be b, then we have

$$2a + 7b = \frac{1}{4} \quad ...(3)$$

and $4a + 4b = \frac{1}{3}$ ...(4)

Multiplying equation (3) by 2 and subtract equation (4) from it

$$10b = \frac{1}{6}$$

c144

$$b = \frac{1}{60} = \frac{1}{y}$$

Thus $y = 60$ days.

Substituting $b = \frac{1}{60}$ in equation (3), we have

$$2a + \frac{7}{60} = \frac{1}{4}$$

$$2a = \frac{1}{4} - \frac{7}{60}$$

$$a = \frac{1}{15}$$

Now $\frac{1}{15} = \frac{1}{x}$

Thus $x = 15$ days.

72. In an election contested between A and B, A obtained votes equal to twice the no. of persons on the electoral roll who did not cast their votes and this later number was equal to twice his majority over B. If there were 1,8000 persons on the electoral roll. How many votes for B.

Ans : [Board Term-1 2012, Set-56]

Let x and y be the no. of votes for A and B respectively.

The no. of persons who did not vote is $18000 - x - y$.

We have $x = 2(18000 - x - y)$

$$3x + 2y = 36000 \quad ...(1)$$

and $(18000 - x - y) = 2(x - y)$

or $3x - y = 18000$ ...(2)

Subtracting equation (2) from equation (1),

$$3y = 18000$$

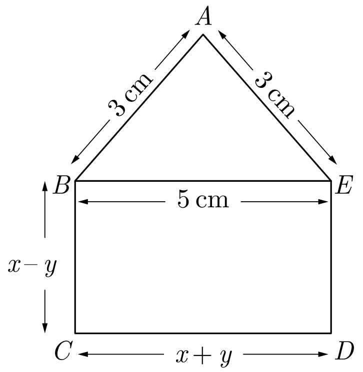
$$y = 6000$$

c179

Hence vote for B is 6000.

73. In the figure below $ABCDE$ is a pentagon with $BE || CD$ and $BC || DE$. BC is perpendicular to DC.

If the perimeter of $ABCDE$ is 21 cm, find the values of x and y.



Ans : [Board Term-1 2011]

Since $BC || DE$ and $BE || CD$ with $BC \perp DC$, $BCDE$ is a rectangle.

$$BE = CD,$$

$$x + y = 5 \qquad ...(1)$$

and $$DE = BE = x - y$$

Since perimeter of $ABCDE$ is 21,

c145

$$AB + BC + CD + DE + EA = 21$$

$$3 + x - y + x + y + x - y + 3 = 21$$

$$6 + 3x - y = 21$$

$$3x - y = 15$$

Adding equations (1) and (2), we get

$$4x = 20 \qquad ...(2)$$

$$x = 5$$

Substituting the value of x in (1), we get

$$y = 0$$

Thus $x = 5$ and $y = 0$.

74. Solve for x and y :

$$\frac{x+1}{2} + \frac{y-1}{3} = 9 \; ; \; \frac{x-1}{3} + \frac{y+1}{2} = 8.$$

Ans : [Board Term-1 2011, Set-52]

We have $$\frac{x+1}{2} + \frac{y-1}{3} = 9$$

c146

$$3(x+1) + 2(y-1) = 54$$

$$3x + 3 + 2y - 2 = 54$$

$$3x + 2y = 53 \qquad (1)$$

and $$\frac{x-1}{3} + \frac{y+1}{2} = 8$$

$$2(x-1) + 3(y+1) = 48$$

$$2x - 2 + 3y + 3 = 48$$

$$2x + 3y = 47 \qquad (2)$$

Multiplying equation (1) by 3 we have

$$9x + 6y = 159 \qquad (3)$$

Multiplying equation (2) by 2 we have

$$4x + 6y = 94 \qquad (4)$$

Subtracting equation (4) from (3) we have

$$5x = 65$$

or $$x = 13$$

Substitute the value of x in equation (2),

$$2(13) + 3y = 47$$

$$3y = 47 - 26 = 21$$

$$y = \frac{21}{3} = 7$$

Hence, $x = 13$ and $y = 7$

75. Solve for x and y :

$$\frac{6}{x-1} - \frac{3}{y-2} = 1$$

$$\frac{5}{x-1} - \frac{1}{y-2} = 2, \text{ where } x \neq 1, \; y \neq 2.$$

Ans : [Board Term-1 2011]

We have $$\frac{6}{x-1} - \frac{3}{y-2} = 1 \qquad (1)$$

$$\frac{5}{x-1} - \frac{1}{y-2} = 2, \qquad (2)$$

Let $\frac{1}{x-1} = p$ and $\frac{1}{y-2} = q$. then given equations become

$$6p - 3q = 1 \qquad ...(3)$$

and $$5p - q = 2 \qquad ...(4)$$

Multiplying equation (4) by 3 and adding in equation (3), we have

$$21p = 7$$

$$p = \frac{7}{21} = \frac{1}{3}$$

Substituting this value of p in equation (3), we have

$$6\left(\frac{1}{3}\right) - 3q = 1$$

$$2 - 3q = 1 \Rightarrow q = \frac{1}{3}$$

Now, $\frac{1}{x-1} = p = \frac{1}{3}$

or, $x - 1 = 3 \Rightarrow x = 4$

and $\frac{1}{y-2} = q = \frac{1}{3}$

or, $y - 2 = 3 \Rightarrow y = 5$

Hence $x = 4$ and, $y = 5$.

76. Seven times a two digit number is equal to four times the number obtained by reversing the order of its digits. If the difference of the digits is 3, determine the number.

Ans : [Board Term-1 2017]

Let the ten's and unit digit by y and x respectively,

So the number is $10y + x$

The number when digits are reversed becomes $10x + y$

Thus $7(10y + x) = 4(10x + y)$

$$70y + 7x = 40x + 4y$$

$$70y - 4y = 40x - 7x$$

$$2y = x \quad ...(1)$$

or $x - y = 3 \quad ...(2)$

From (1) and (2) we get

$$y = 3 \text{ and } x = 6$$

Hence the number is 36.

77. Solve the following pair of equations for x and y :

$$\frac{a^2}{x} - \frac{b^2}{y} = 0, \frac{a^2 b}{x} + \frac{b^2 a}{y} = a + b, \qquad x \neq 0; y \neq 0.$$

Ans : [Board Term-1 2011]

We have $\frac{a^2}{x} - \frac{b^2}{y} = 0$

$$\frac{a^2 b}{x} + \frac{b^2 a}{y} = a + b = a + b$$

Substituting $p = \frac{1}{x}$ and $q = \frac{1}{y}$ in the given equations,

$$a^2 p - b^2 q = 0 \quad ...(1)$$

$$a^2 bp + b^2 aq = a + b \quad ...(2)$$

Multiplying equation (1), by a

$$a^3 p - b^2 aq = 0 \quad ...(3)$$

Adding equation (2) and equation (3),

$$(a^3 + a^2 b)p = a + b$$

or, $p = \frac{(a+b)}{a^2(a+b)} = \frac{1}{a^2}$

Substituting the value of p in equation (1),

$$a^2\left(\frac{1}{a^2}\right) - b^2 q = 0 \Rightarrow q = \frac{1}{b^2}$$

Now, $p = \frac{1}{x} = \frac{1}{a^2} \Rightarrow x = a^2$

and $q = \frac{1}{y} = \frac{1}{b^2} \Rightarrow y = b^2$

Hence, $x = a^2$ and $y = b^2$

78. Solve for x and y :

$$ax + by = \frac{a+b}{2}$$

$$3x + 5y = 4$$

Ans : [Board Term-1 2011, Set-44]

We have $ax + by = \frac{a+b}{2}$

or $2ax + 2by = a + b \quad ...(1)$

and $3x + 5y = 4 \quad ...(2)$

Multiplying equation (1) by 5 we have

$$10ax + 10by = 5a + 5b \quad ...(3)$$

Multiplying equation (2) by $2b$, we have

$$6bx + 10by = 8b \quad ...(4)$$

Subtracting (4) from (3) we have

$$(10a - 6b)x = 5a - 3b$$

or $x = \frac{5a - 3b}{10a - 6b} = \frac{1}{2}$

Substitute $x = \frac{1}{2}$ in equation (2), we get

$$3 \times \frac{1}{2} + 5y = 4$$

$$5y = 4 - \frac{3}{2} = \frac{5}{2}$$

$$y = \frac{5}{2 \times 5} = \frac{1}{2}$$

Hence $x = \frac{1}{2}$ and $y = \frac{1}{2}$.

79. Solve the following pair of equations for x and y :

$4x+\frac{6}{y}=15, 6x-\frac{8}{y}=14$

and also find the value of p such that $y=px-2$.

Ans : [Board Term-1 2011, Set-60]

We have $$4x+\frac{6}{y} = 15 \quad (1)$$

$$6x-\frac{8}{y} = 14, \quad (2)$$

Let $\frac{1}{y}=z$, the given equations become

$$4x+6z = 15 \quad ...(3)$$

$$6x-8z = 14 \quad ...(4)$$

Multiply equation (3) by 4 we have

$$16x+24z = 60 \quad (5)$$

Multiply equation (4) by 3 we have

$$18x-24z = 24 \quad (6)$$

Adding equation (5) and (6) we have

$$34x = 102$$

$$x = \frac{102}{34}=3$$

c150

Substitute the value of x in equation (3),

$$4(3)+6z = 15$$

$$6z = 15-12=3$$

$$z = \frac{3}{6}=\frac{1}{2}$$

Now $$z = \frac{1}{y}=\frac{1}{2} \Rightarrow y = 2$$

Hence $x = 3$ and $y=2$.

Again $$y = px-2$$

$$2 = p(3)-2$$

$$3p = 4$$

Thus $$p = \frac{4}{3}$$

80. A chemist has one solution which is 50 % acid and a second which is 25 % acid. How much of each should be mixed to make 10 litre of 40 % acid solution.

Ans : [Board Term-1 2015, JRTSY]

Let 50 % acids in the solution be x and 25 % of other solution be y.

Total volume in the mixture

$$x+y = 10 \quad ...(1)\ 1$$

and $$\frac{50}{100}x+\frac{25}{100}y = \frac{40}{100}\times 10$$

$$2x+y = 16 \quad ...(2)\ 1$$

Subtracting equation (1) from (2) we have

$$x = 6$$

Substituting this value of x in equation (1) we get

c177

$$6+y = 10$$

$$y = 4$$

Hence, $x = 6$ and $y=4$.

81. Find whether the following pair of linear equations has a unique solutions. If yes, find the solution :

$$7x-4y = 49, 5x-6y=57.$$

Ans : [Board Term-1 2011]

We have $$7x-4y = 49 \quad (1)$$

$$5x-6y = 57 \quad (2)$$

Comparing with the equation $a_1x+b_1y=c_1$,

$$a_1 = 7, b_1=-4, c_1=49$$

$$a_2 = 5, b_2=-6, c_2=57$$

c151

Since, $$\frac{a_1}{a_2} = \frac{7}{5} \text{ and } \frac{b_1}{b_2}=\frac{4}{6}$$

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

So, system has a unique solution.

Multiply equation (1) by 5 we get

$$35x-20y = 245 \quad (3)$$

Multiply equation (2) by 7 we get

$$35x-42y = 399 \quad (4)$$

Subtracting (4) by (3) we have

$$22y = -154$$

$$y = -7$$

Putting the value of y in equation (2),

$$5x-6(-7) = 57$$

$$5x = 57-42=15$$

$$x = 3$$

Hence $x = 3$ and $y=-7$

FOUR MARKS QUESTIONS

82. Determine graphically the coordinates of the vertices of triangle, the equations of whose sides are given by $2y-x=8$, $5y-x=14$ and $y-2x=1$.

Ans : [Board 2020 Delhi Standard]

We have $2y - x = 8$

L_1 : $x = 2y - 8$

y	0	4	5
$x = 2y-8$	-8	0	2

$5y - x = 14$

L_2 : $x = 5y - 14$

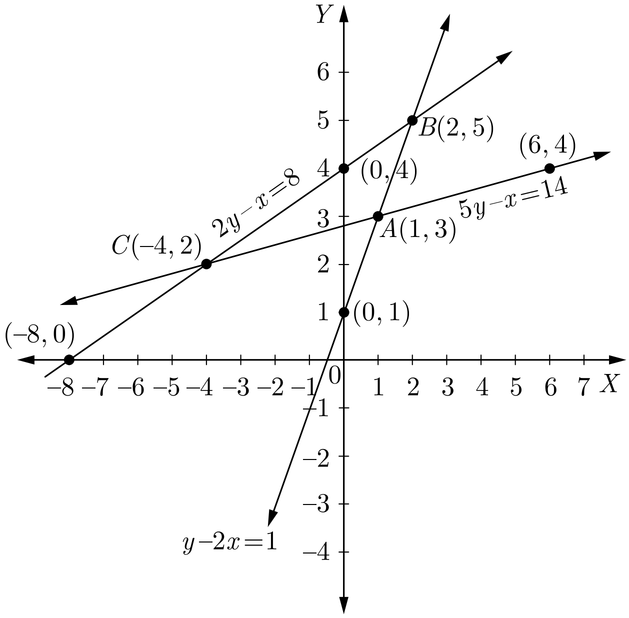
y	3	4	2
$x = 5y-14$	1	6	-4

and $y - 2x = 1$

L_3 : $y = 1 + 2x$

x	0	1	2
$y = 1+2x$	1	3	5

Plotting the above points and drawing lines joining them, we get the graphical representation:



Hence, the coordinates of the vertices of a triangle ABC are $A(1,3)$, $B(2,5)$ and $C(-4,2)$.

83. A man can row a boat downstream 20 km in 2 hours and upstream 4 km in 2 hours. Find his speed of rowing in still water. Also find the speed of the stream.

Ans : [Board 2020 Delhi Standard]

Let x be the speed of the boat in still water and y be the speed of the stream.

Relative Speed of boat in upstream will be $(x-y)$ and relative speed of boat in downstream will be $(x+y)$.

According to question, we have

$$\frac{20}{x+y} = 2$$

$$x+y = 10 \quad ...(1)$$

and $$\frac{4}{x-y} = 2$$

$$x-y = 2 \quad ...(2)$$

Adding equation (1) and (2), we have

$$2x = 12 \Rightarrow x = 6 \text{ km/hr}$$

Substituting the value of x is equation (1) we have,

$$6+y = 10 \Rightarrow y = 10-6 = 4 \text{ km/hr}$$

Thus speed of a boat in still water is 6 km/hr and speed of the stream 4 km/hr.

84. It can take 12 hours to fill a swimming pool using two pipes. If the pipe of larger diameter is used for four hours and the pipe of smaller diameter for 9 hours, only half of the pool can be filled. How long would it take for each pipe to fill the pool separately?

Ans : [Board 2020 OD Standard]

Let x be time taken to fill the pool by the larger diameter pipe and y be the time taken to fill the pool by the smaller diameter pipe.

According to question,

$$\frac{1}{x}+\frac{1}{y} = \frac{1}{12} \quad ...(1)$$

and $$\frac{4}{x}+\frac{9}{y} = \frac{1}{2} \quad ...(2)$$

Multiplying equation (1) by 9 and subtracting from equation (2), we get

$$\frac{5}{x} = \frac{9}{12}-\frac{1}{2} = \frac{1}{4}$$

$$x = 20$$

Substituting the value of x in equation (1), we have

$$\frac{1}{20}+\frac{1}{y} = \frac{1}{12}$$

$$\frac{1}{y} = \frac{1}{12}-\frac{1}{20} = \frac{5-3}{60}$$

$$\frac{1}{y} = \frac{2}{60} = \frac{1}{30} \Rightarrow y = 30$$

Hence, time taken to fill the pool by the larger and smaller diameter pipe are 20 hrs and 30 hrs respectively.

85. For what value of k, which the following pair of linear equations have infinitely many solutions:
$2x+3y=7$ and $(k+1)x+(2k-1)y=4k+1$

Ans : [Board 2019 Delhi Standard]

We have $2x+3y = 7$

and $(k+1)x+(2k-1)y = 4k+1$

c226

Here $\frac{a_1}{a_2} = \frac{2}{k+1}, \frac{b_1}{b_2} = \frac{3}{(2k-1)}$

and $\frac{c_1}{c_2} = \frac{-7}{-(4k+1)} = \frac{7}{(4k+1)}$

For infinite many solutions

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

For $\frac{a_1}{a_2} = \frac{c_1}{c_2}$ we have

$$\frac{2}{k+1} = \frac{7}{4k+1}$$

$$2(4k+1) = 7(k+1)$$

$$8k+2 = 7k+7$$

$$k = 5$$

Hence, the value of k is 5, for which the given equation have infinitely many solutions.

86. Find c if the system of equations $cx+3y+(3-c)=0; 12x+cy-c=0$ has infinitely many solutions?

Ans : [Board 2019 Delhi]

We have $cx+3y+(3-c) = 0$

$$12x+cy-c = 0$$

c227

Here, $\frac{a_1}{a_2} = \frac{c}{12}, \frac{b_1}{b_2} = \frac{3}{c}, \frac{c_1}{c_2} = \frac{3-c}{-c}$

For infinite many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

For $\frac{a_1}{a_2} = \frac{c_1}{c_2}$ we have,

$$\frac{c}{12} = \frac{3-c}{-c}$$

$$-c^2 = 36-12c$$

$$-c^2+12c-36 = 0$$

$$c^2-12c+36 = 0$$

$$c^2-6c-6c+36 = 0$$

$$c(c-6)-6(c-6) = 0$$

$$(c-6)(c-6) = 0 \Rightarrow c = 6$$

and for $\frac{b_1}{b_2} = \frac{c_1}{c_2}$,

$$\frac{3}{c} = \frac{3-c}{-c}$$

$$-3c = 3c-c^2$$

$$c^2-6c = 0$$

$$c(c-6) = 0 \Rightarrow c = 6 \text{ or } c \neq 0$$

Hence, the value of c is 6, for which the given equations have infinitely many solutions.

87. A father's age is three times the sum of the ages of his two children. After 5 years his age will be two times the sum of their ages. Find the present age of the father.

Ans : [Board 2019 Delhi]

Let x be the age of father and y be the sum of the ages of his children.

After 5 years,

Father's age $= (x+5)$ years

c228

Sum of ages of his children $= (y+10)$ years

According to the given condition,

$$x = 3y \quad \text{...(1)}$$

and $x+5 = 2(y+10)$

or, $x-2y = 15$ ...(2)

Solving equation (1) and (2), we have

$$3y - 2y = 15 \Rightarrow y = 15$$

Substituting value of y in equation (1), we get

$$x = 3 \times 15 = 45$$

Hence, father's present age is 45,

88. Two water taps together can fill a tank in $1\frac{7}{8}$ hours. The tap with longer diameter takes 2 hours less than the tap with smaller one to fill the tank separately. Find the time in which each tap can fill the tank separately.

Ans : [Board 2019 Delhi]

Let t be the time taken by the smaller diameter top. Time for larger tap diameter will be $t-2$.

Total time taken $= 1\frac{7}{8} = \frac{15}{8}h$.

Portion filled in one hour by smaller diameter tap will $\frac{1}{t}$ and by lager diameter tap will be $\frac{1}{t-2}$

According to the problem,

$$\frac{1}{t} + \frac{1}{t-2} = \frac{8}{15}$$

$$\frac{t-2+t}{t(t-2)} = \frac{8}{15}$$

$$15(2t-2) = 8t(t-2)$$

$$30t - 30 = 8t^2 - 16t$$

$$8t^2 - 46t + 30 = 0$$

$$4t^2 - 23t + 15 = 0$$

$$4t^2 - 20t - 3t + 30 = 0$$

$$(4t-3)(t-5) = 0 \Rightarrow t = \frac{3}{4} \text{ or } t = 5$$

If $t = \frac{3}{4}$ then $t - 2 = \frac{3}{4} - 2 = \frac{-5}{4}$

Since, time cannot be negative, we neglect $t = \frac{3}{4}$

Therefore, $t = 5$

and $t - 2 = 5 - 2 = 3$

Hence, time taken by larger tap is 3 hours and time taken by smaller is 5 hours

89. A boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours, it can go 40 km upstream and 55 km downstream. Determine the speed of the stream and that of the boat in still water.

Ans : [Board 2019 Delhi]

Let x be the speed of boat in still water and y be the speed of stream.

Relative speed of boat in downstream will be $x+y$ and relative speed of boat in upstream will be $x-y$.

Time taken to go 30 km upstream,

$$t_1 = \frac{30}{x-y}$$

Time taken to go 44 km downstream,

$$t_2 = \frac{40}{x+y}$$

According to the first condition we have

$$\frac{30}{x-y} + \frac{44}{x+y} = 10 \quad ...(1)$$

Similarly according to the second condition we have

$$\frac{40}{x-y} + \frac{55}{x+y} = 13 \quad ...(2)$$

Let $\frac{1}{x-y} = u$ and $\frac{1}{x+y} = \upsilon$, then we have

$$30u + 44\upsilon = 10 \quad ...(3)$$

$$40u + 55\upsilon = 13 \quad ...(4)$$

Multiplying equation (3) by 4 and equation (4) by 3 and then subtracting we have

$$11v = 1 \Rightarrow \upsilon = \frac{1}{11}$$

Multiplying equation (3) by 5 and equation (4) by 4 and then subtracting we have

$$-10u = -2 \quad ...(4)$$

$$u = \frac{1}{5}$$

Now $u = \frac{1}{x-y} = \frac{1}{5}$

$$x - y = 5 \quad (5)$$

and $\upsilon = \frac{1}{x+y} = \frac{1}{11}$

$$x + y = 11 \quad (6)$$

Adding equation (5) and (6), we get

$$2x = 16 \Rightarrow x = 8$$

Substitute value of x in equation (5), we get

$$8 - y = 5 \Rightarrow y = 3$$

Hence speed of boat in still water is 8 km/hour and and speed of stream is 3 km/hour.

90. Sumit is 3 times as old as his son. Five years later he shall be two and a half times as old as his son. How old is Sumit at present?

Ans : [Board 2019 OD]

Let x be Sumit's present age and y be his son's present age.

According to given condition,

$$x = 3y$$

After five years, ...(1)

$$\text{Sumit's age} = x+5$$

and $\text{His son's age} = y+5$

Now, again according to given condition,

c231

$$x+5 = 2\tfrac{1}{2}(y+5)$$

$$x+5 = \frac{5}{2}(y+5)$$

$$2(x+5) = 5(y+5)$$

$$2x+10 = 5y+25$$

$$2x = 5y+15$$

$$2(3y) = 5y+15 \quad \text{[from eq (1)]}$$

$$6y = 5y+15$$

$$y = 15$$

Again, from eq (1)

$$x = 3y = 3 \times 15 = 45$$

Hence, Sumit's present age is 45 years.

91. For what value of k, will the following pair of equations have infinitely many solutions:

$2x+3y = 7$ and $(k+2)x-3(1-k)y = 5k+1$

Ans : [Board 2019 OD]

We have $2x+3y = 7$ ...(1)

and $(k+2)x-3(1-k)y = 5k+1$ ...(2)

Comparing equation (1) with $a_1x+b_1y = c_1$ and equation (2) by $a_2x+b_2y = c_2$ we have

$$a_1 = 2,\ b_1 = 3,\ c_1 = 7$$

and $a_2 = (k+2),\ b_2 = -3(1-k),\ c_1 = 5k+1$

Here, $\dfrac{a_1}{a_2} = \dfrac{2}{k+2}$,

$$\frac{b_1}{b_2} = \frac{3}{-3(1-k)},\ \frac{c_1}{c_2} = \frac{7}{5k+1}$$

For a pair of linear equations to have infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

c232

So, $\dfrac{2}{k+2} = \dfrac{3}{-3(1-k)} = \dfrac{7}{5k+1}$

$$\frac{2}{k+2} = \frac{3}{-3(1-k)}$$

$$2(1-k) = -(k+2)$$

$$2-2k = -k-2 \Rightarrow k = 4$$

Hence, for $k = 4$, the pair of linear equations has infinitely many solutions.

92. The total cost of a certain length of a piece of cloth is ₹200. If the piece was 5 m longer and each metre of cloth costs ₹2 less, the cost of the piece would have remained unchanged. How long is the piece and what is its original rate per metre?

Ans : [Board 2019 OD]

Let x be the length of the cloth and y be the cost of cloth per meter.

Now $x \times y = 200$

$$y = \frac{200}{x} \quad ...(1)$$

According to given conditions,

c233

1. If the piece were 5 m longer
2. Each meter of cloth costed ₹ 2 less

i.e., $(x+5)(y-2) = 200$

$$xy-2x+5y-10 = 200$$

$$xy-2x+5y = 210$$

$$x\left(\frac{200}{x}\right)-2x+5\left(\frac{200}{x}\right) = 210$$

$$200-2x+\frac{1000}{x} = 210$$

$$\frac{1000}{x}-2x = 10$$

$$1000-2x^2 = 10x$$

$$x^2+25x-20x-500 = 0$$

$$x(x+25)-20(x+25) = 0$$

$$(x+25)(x-20) = 0$$

$$x = -25,\ 20$$

Neglecting $x = -25$ we get $x = 20$.

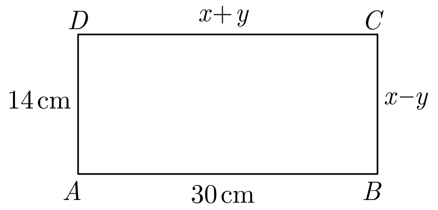
Now from equation (1), we have

$$y = \frac{200}{x} = \frac{200}{20} = 10$$

Hence, length of the piece of cloths is 20 m and rate per meter is ₹10.

93. In Figure, $ABCD$ is a rectangle. Find the values of

x and y.



Ans : [Board 2018]

Since $ABCD$ is a rectangle, we have

$$AB = CD \text{ and } BC = AD$$

Now $\quad x + y = 30 \quad$ ...(1)

$$x - y = 14 \quad ...(2)$$

Adding equation (1) and (3) we obtain,

$$2x = 44 \Rightarrow x = \frac{44}{2} = 22$$

c234

Substituting value of x in equation (1) we have

$$22 + y = 30$$

$$y = 30 - 22 = 8$$

$$x = 22\,\text{cm and } y = 8\,\text{cm}$$

94. For Uttarakhand flood victims two sections A and B of class contributed Rs. 1,500. If the contribution of X-A was Rs. 100 less than that of X-B, find graphically the amounts contributed by both the sections.

Ans : [Board Term-1 2016]

Let amount contributed by two sections X-A and X-B be Rs. x and Rs. y.

c121

$$x + y = 1{,}500 \quad ...(1)$$

$$y - x = 100 \quad ...(2)$$

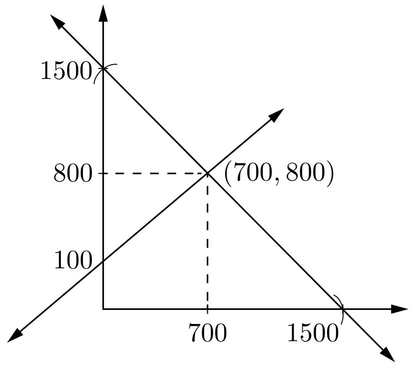
From (1) $y = 1500 - x$

x	0	700	1,500
y	1,500	800	0

From (2) $y = 100 + x$

x	0	700
y	100	800

Plotting the above points and drawing lines joining them, we get the following graph.



Clearly, the two lines intersect at point (700, 800) Hence X-A contributes 700 Rs and X-B contributes 800 Rs.

95. Determine graphically whether the following pair of linear equations :

$$3x - y = 7$$

$$2x + 5y + 1 = 0 \text{ has :}$$

a. unique solution
b. infinitely many solutions or
c. no solution.

Ans : [Board Term-1 2015]

We have $\quad 3x - y = 7$

or $\quad 3x - y - 7 = 0 \quad (1)$

Here $a_1 = 3, b_1 = 1, c_1 = -7$

$$2x + 5y + 1 = 0 \quad (2)$$

Here $a_2 = 2, b_2 = 5, c_2 = 1$

Now $\quad \frac{a_1}{a_2} = \frac{3}{2}, \frac{b_1}{b_2} = \frac{-1}{5}$

c122

Since $\frac{3}{2} \neq \frac{-1}{5}$, thus $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence, given pair of linear equations has a unique solution.

Now line (1) $\quad y = 3x - 7$

x	0	2	3
y	-7	-1	2

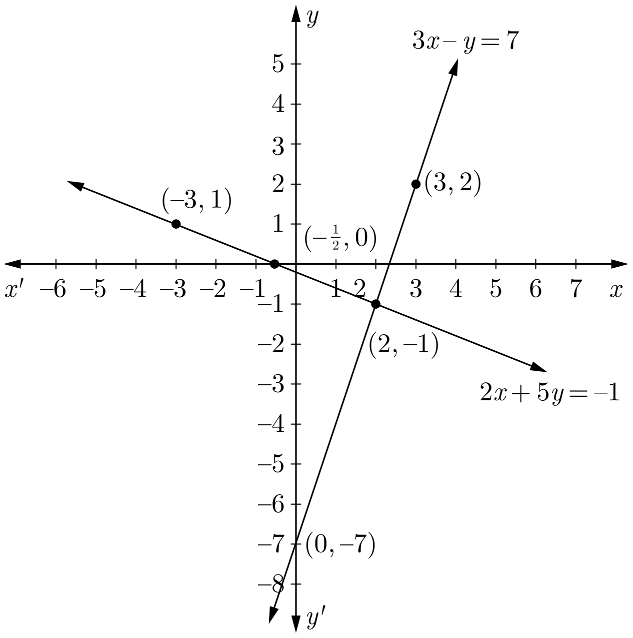
and line (2)

$$2x + 5y + 1 = 0$$

or, $$y = \frac{-1-2x}{5}$$

x	2	-3
y	-1	1

Plotting the above points and drawing lines joining them, we get the following graph.



Clearly, the two lines intersect at point $(2, -1)$.
Hence $x = 2$ and $y = -1$

96. Draw the graphs of the pair of linear equations :
$x + 2y = 5$ and $2x - 3y = -4$

Also find the points where the lines meet the x-axis.

Ans : [Board Term-1 2015]

We have $$x + 2y = 5$$

or, $$y = \frac{5-x}{2}$$

c123

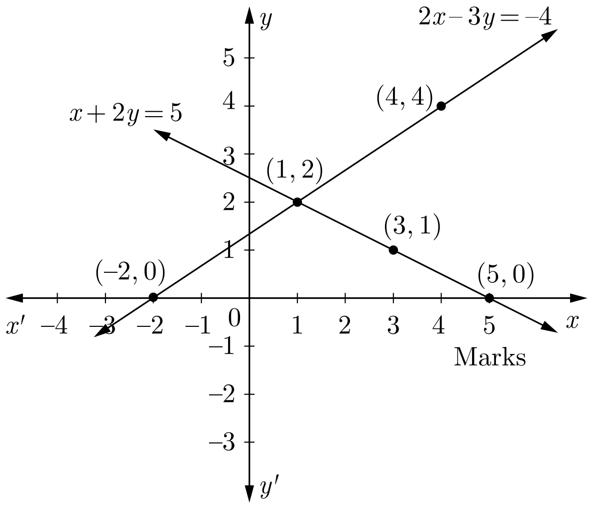
x	1	3	5
y	2	1	0

and $$2x - 3y = -4$$

or, $$y = \frac{2x+4}{3}$$

x	1	4	-2
y	2	4	0

Plotting the above points and drawing lines joining them, we get the following graph.



Clearly two lines meet x-axis at $(5, 0)$ and $(-2, 0)$ respectively.

97. Solve graphically the pair of linear equations :
$3x - 4y + 3 = 0$ and $3x + 4y - 21 = 0$
Find the co-ordinates of the vertices of the triangular region formed by these lines and x-axis. Also, calculate the area of this triangle.

Ans : [Board Term-1 2015]

We have $$3x - 4y + 3 = 0$$

or, $$y = \frac{3x+3}{4}$$

c124

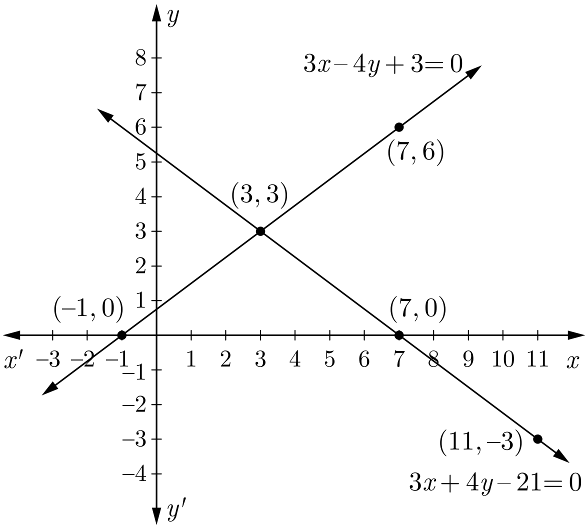
x	3	7	-1
y	3	6	0

and $$3x + 4y - 21 = 0$$

or, $$y = \frac{21-3x}{4}$$

x	3	7	11
y	3	0	-2

Plotting the above points and drawing lines joining them, we get the following graph.



Clearly, the two lines intersect at point $(3, 3)$.

(a) These lines intersect each other at point $(3, 3)$.
Hence $x = 3$ and $y = 3$

(b) The vertices of triangular region are $(3, 3)$, $(-1, 0)$ and $(7, 0)$.

(c) Area of $\Delta = \frac{1}{2} \times 8 \times 3 = 12$

Hence, Area of obtained Δ is 12 sq unit.

98. Aftab tells his daughter, '7 years ago, I was seven times as old as you were then. Also, 3 years from now, I shall be three times as old as you will be.' Represent this situation algebraically and graphically.

Ans : [Board Term-1 2015, NCERT]

Let the present age of Aftab be x years and the age of daughter be y years.

7 years ago father's(Aftab) age $= (x - 7)$years

7 years ago daughter's age $= (y - 7)$ years

According to the question,

$$(x - 7) = 7(y - 7)$$

or, $$(x - 7y) = -42 \quad (1)$$

After 3 years father's(Aftab) age $= (x + 3)$ years

After 3 years daughter's age $= (y + 3)$ years

According to the condition,

$$x + 3 = 3(y + 3)$$

or, $$x - 3y = 6 \quad (2)$$

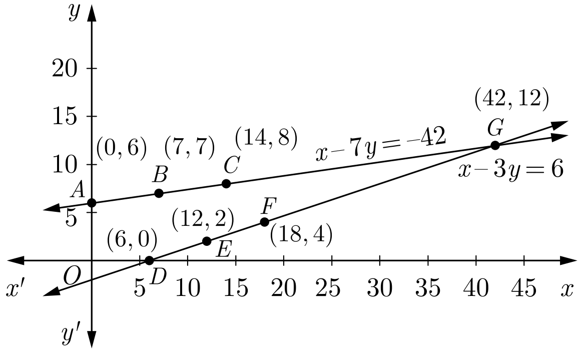
From equation(1) $x - 7y = -42$

x	0	7	14
$y = \frac{x + 42}{7}$	6	7	8

From equation (2) $x - 3y = 6$

x	6	12	18
$y = \frac{x - 6}{3}$	0	2	4

Plotting the above points and drawing lines joining them, we get the following graph.



Two lines obtained intersect each other at $(42, 12)$

Hence, father's age $= 42$ years

and daughter's age $= 12$ years

99. The cost of 2 kg of apples and 1kg of grapes on a day was found to be Rs. 160. After a month, the cost of 4kg of apples and 2kg of grapes is Rs. 300. Represent the situations algebraically and geometrically.

Ans : [Board Term-1 2013, Set DDE-E, NCERT]

Let the cost of 1 kg of apples be Rs.x and cost of 1 kg of grapes be Rs. y.

The given conditions can be represented given by the following equations :

$$2x + y = 160 \quad ...(1)$$

$$4x + 2y = 300 \quad ...(2)$$

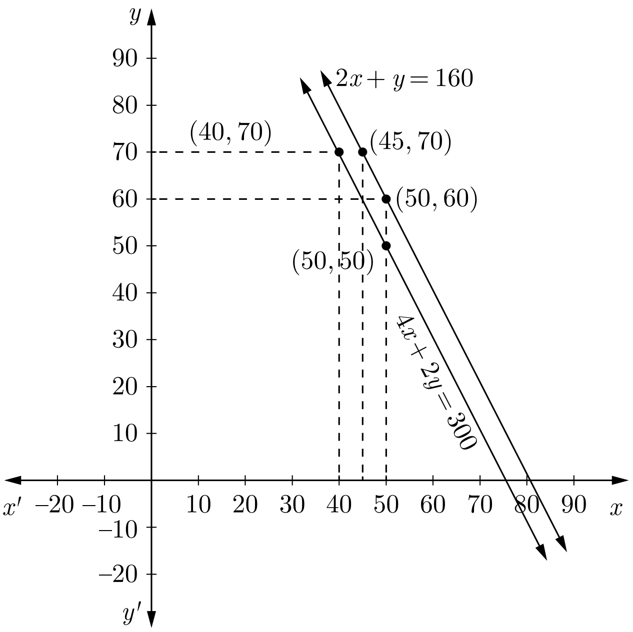
From equation (1) $y = 160 - 2x$

x	50	45
y	60	70

From equation (2) $y = 150 - 2x$

x	50	40
y	50	70

Plotting these points on graph, we get two parallel line as shown below.



100. Draw the graphs of the equations $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Determine the co-ordinates of the vertices of the triangle formed by these lines and the X-axis and shade the triangular region.

Ans : [Board Term-1 2013 NCERT]

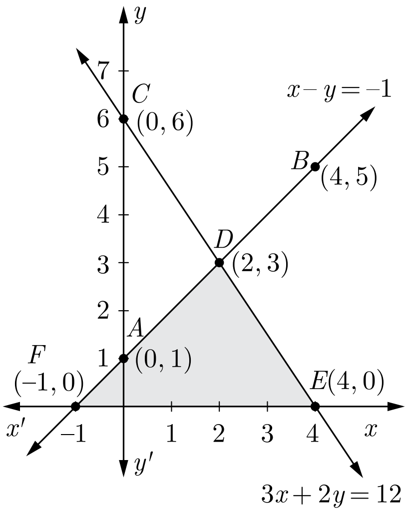
We have $\quad x - y + 1 = 0 \qquad$ ...(1)

x	0	4	2
$y = x + 1$	1	5	3

and $\quad 3x + 2y - 12 = 0 \qquad$ ...(2)

x	0	2	4
$y = \frac{12 - 3x}{2}$	6	3	0

Plotting the above points and drawing lines joining them, we get the following graph.



Clearly, the two lines intersect at point $D(2,3)$. Hence, $x = 2$ and $y = 3$ is the solution of the given pair of equations. The line CD intersects the x-axis at the point $E(4,0)$ and the line AB intersects the x-axis at the points $F(-1,0)$. Hence, the co-ordinates of the vertices of the triangle are $D(2,3)$, $E(4,0)$ and $F(-1,0)$.

101. Solve the following pair of linear equations graphically:

$2x + 3y = 12$ and $x - y = 1$

Find the area of the region bounded by the two lines representing the above equations and y-axis.

Ans : [Board Term-1 2012, Set-58]

We have $\quad 2x + 3y = 12 \Rightarrow y = \frac{12 - 2x}{3}$

x	0	6	3
y	4	0	2

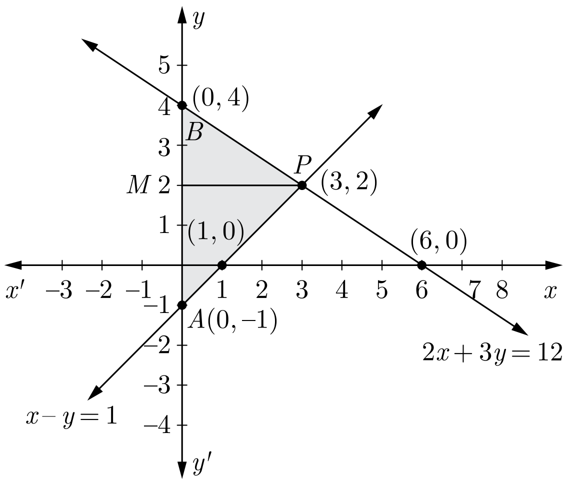
We have $\quad x - y = 1 \Rightarrow y = x - 1$

x	0	1	3
y	1	0	2

Plotting the above points and drawing lines joining them, we get the following graph.

c127

c128



Clearly, the two lines intersect at point $p(3,2)$.

Hence, $x = 3$ and $y = 2$

Area of shaded triangle region,

$$\text{Area of } \Delta PAB = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times AB \times PM$$

$$= \frac{1}{2} \times 5 \times 3$$

$$= 7.5 \text{ square unit.}$$

102. Solve the following pair of linear equations graphically:

$x + 3y = 12,\ 2x - 3y = 12$

Also shade the region bounded by the line $2x - 3y = 2$ and both the co-ordinate axes.

Ans : [Board Term-1 2013 FFC, 2012, Set-35, 48]

We have $x + 3y = 6 \Rightarrow y = \dfrac{6 - x}{3}$ ...(1)

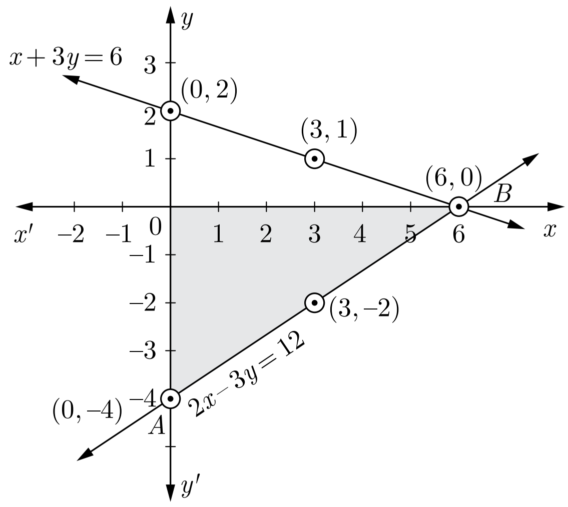
x	3	6	0
y	1	0	2

and $2x - 3y = 12 \Rightarrow y = \dfrac{2x - 12}{3}$

x	0	6	3
y	-4	0	-2

Plotting the above points and drawing lines joining them, we get the following graph.

c129



The two lines intersect each other at point $B(6,0)$.

Hence, $x = 6$ and $y = 0$

Again ΔOAB is the region bounded by the line $2x - 3y = 12$ and both the co-ordinate axes.

103. Solve the following pair of linear equations graphically:

$x - y = 1,\ 2x + y = 8$

Also find the co-ordinates of the points where the lines represented by the above equation intersect $y - axis$.

Ans : [Board Term-1 2012, Set-56]

We have $x - y = 1 \Rightarrow y = x - 1$

x	2	3	-1

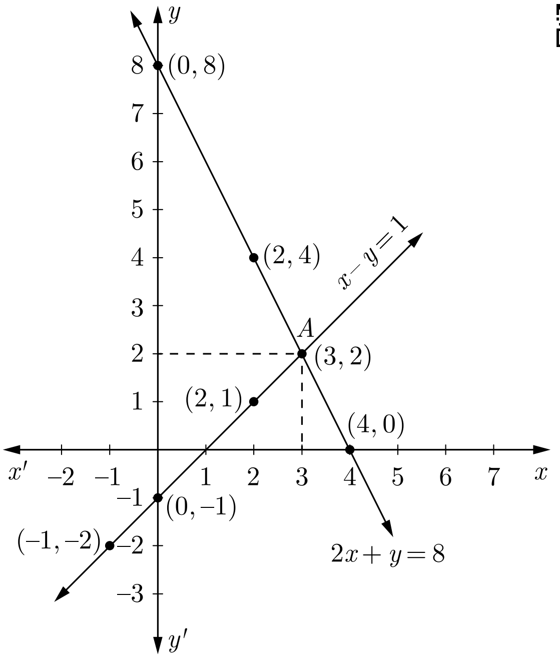
y	1	2	-2

and $\quad 2x+y = 8 \Rightarrow y = 8-2x$

x	2	4	0
y	4	0	8

Plotting the above points and drawing lines joining them, we get the following graph.

c130



The two lines intersect each other at point $A(3,2)$. Thus solution of given equations is $x=3, y=2$.

Again, $x-y=1$ intersects $y-axis$ at $(0,-1)$

and $\quad 2x+y=8\, y-axis$ at $(0,8)$.

104. Draw the graph of the following equations:

$$2x-y = 1,\ x+2y = 13$$

Find the solution of the equations from the graph and shade the triangular region formed by the lines and the y-axis.

Ans : [Board Term-1 2012 Set-52]

We have $\quad 2x-y = 1 \Rightarrow y = 2x-1$

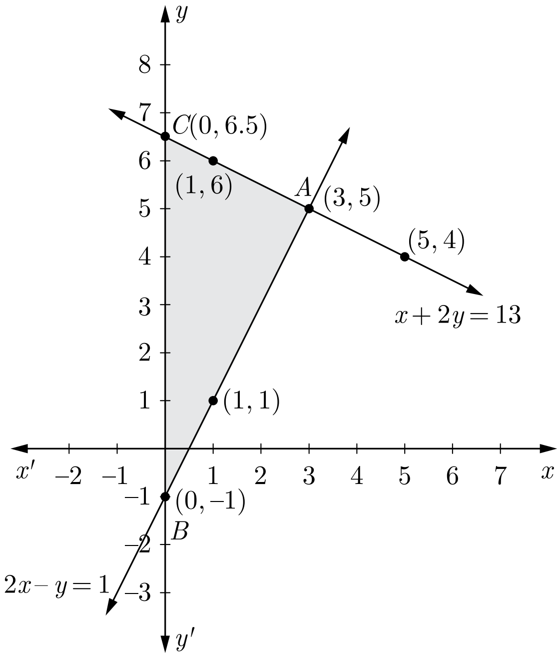
x	0	1	3
y	-1	1	5

and $\quad x+2y = 13 \Rightarrow y = \dfrac{13-x}{2}$

x	1	3	5
y	6	5	4

Plotting the above points and drawing lines joining them, we get the following graph.

c131



Clearly two obtained lines intersect at point $A(3,5)$.

Hence, $x=3$ and $y=5$

ABC is the triangular shaded region formed by the obtained lines with the y-axis.

105. Solve the following pair of equations graphically:

$$2x+3y = 12,\ x-y-1 = 0.$$

Shade the region between the two lines represented by the above equations and the X-axis.

Ans : [Board Term-1 2012, Set-48]

We have $\quad 2x+3y = 12 \Rightarrow y = \dfrac{12-2x}{3}$

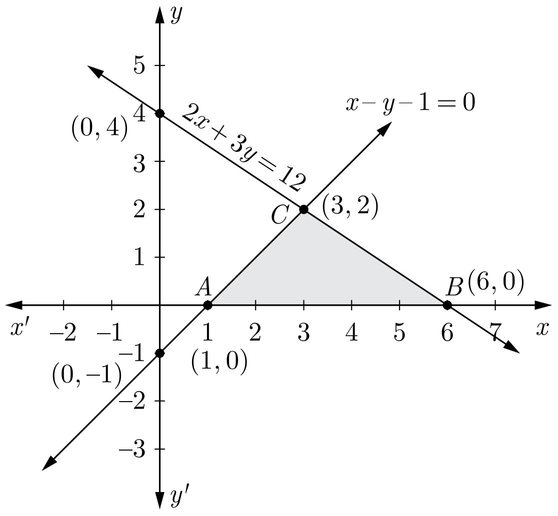
x	0	6	3
y	4	0	2

also $\quad x-y = 1 \Rightarrow y = x-1$

x	0	1	3
y	-1	0	2

Plotting the above points and drawing lines joining them, we get the following graph.

c132



The two lines intersect each other at point (3,2), Hence, $x = 3$ and $y = 2$.

ΔABC is the region between the two lines represented by the given equations and the X-axis.

106. 4 chairs and 3 tables cost Rs 2100 and 5 chairs and 2 tables cost Rs 1750. Find the cost of none chair and one table separately.

Ans : [Board Term-1 2015]

Let cost of 1 chair be Rs x and cost of 1 table be Rs y According to the question,

$$4x + 3y = 2100 \quad ...(1)$$

$$5x + 2y = 1750 \quad ...(2)$$

Multiplying equation (1) by 2 and equation (2) by 3,

$$8x + 6y = 4200 \quad ...(3)$$

$$15x + 6y = 5250 \quad ...(iv)$$

Subtracting equation (3) from (4) we have

$$7x = 1050$$

$$x = 150$$

Substituting the value of x in (1), $y = 500$

Thus cost of chair and table is Rs 150, Rs 500 respectively.

107. Solve the following pair of equations :

$$\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2 \text{ and } \frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1$$

Ans : [Board Term-1 2015]

We have $$\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2$$

$$\frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1$$

Substitute $\frac{1}{\sqrt{x}} = X$ and $\frac{1}{\sqrt{y}} = Y$

$$2X + 3Y = 2 \quad ...(1)$$

$$4X - 9Y = -1 \quad ...(2)$$

Multiplying equation (1) by 3, and adding in (2) we get

$$10X = 5 \Rightarrow X = \frac{5}{10} = \frac{1}{2}$$

Thus $$\frac{1}{\sqrt{x}} = \frac{1}{2} \Rightarrow x = 4$$

Putting the value of X in equation (1), we get

$$2 \times \frac{1}{2} + 3y = 2$$

$$3Y = 2 - 1$$

$$Y = \frac{1}{3}$$

Now $$Y = \frac{1}{3} \Rightarrow \frac{1}{\sqrt{y}} = \frac{1}{3} \Rightarrow y = 9$$

Hence $x = 4$, $y = 9$.

108. Solve for x and y :

$$2x - y + 3 = 0$$

$$3x - 5y + 1 = 0$$

Ans : [Board Term-1 2015]

We have $$2x - y + 3 = 0 \quad ...(1)$$

$$3x - 5y + 1 = 0 \quad ...(2)$$

Multiplying equation (1) by 5,and subtracting (2) from it we have

$$7x = -14$$

$$x = \frac{-14}{7} = -2$$

Substituting the value of x in equation (1) we get

$$2x - y + 3 = 0$$

$$2(-2) - y + 3 = 0$$

$$-4 - y + 3 = 0$$

$$-y - 1 = 0$$

$$y = -1$$

Hence, $x = -2$ and $y = -1$.

109. Solve $x + y = 5$ and $2x - 3y = 4$ by elimination method and the substitution method.

Ans : [Board Term-1 2015]

By Elimination Method :

We have, $x + y = 5$ ...(1)

and $2x - 3y = 4$ ...(2)

Multiplying equation (1) by 3 and adding in (2) we have

$$3(x+y) + (2x-3y) = 3 \times 5 + 4$$

or, $$3x + 3y + 2x - 3y = 15 + 4$$

c155

$$5x = 19 \Rightarrow x = \frac{19}{5}$$

Substituting $x = \frac{19}{5}$ in equation (1),

$$\frac{19}{5} + y = 5$$

$$y = 5 - \frac{19}{5} = \frac{25-19}{5} = \frac{6}{5}$$

Hence, $x = \frac{19}{5}$ and $y = \frac{6}{5}$

By Substituting Method :

We have, $x + y = 5$ ...(1)

and $2x - 3y = 4$ ...(2)

From equation (1), $y = 5 - x$ ...(3)

Substituting the value of y from equation (3) in equation (2),

$$2x - 3(5-x) = 4$$

$$2x - 15 + 3x = 4$$

$$5x = 19$$

$$x = \frac{19}{5}$$

Substituting this value of x in equation (3), we get

$$y = 5 - \frac{19}{5} = \frac{6}{5}$$

Hence $x = \frac{19}{5}$ and $y = \frac{6}{5}$

110. Solve for x and y :

$$3x + 4y = 10$$

$$2x - 2y = 2$$

Ans : [Board Term-1 2015]

By Elimination Method :

We have, $3x + 4y = 10$ ...(1)

and $2x - 2y = 2$ ...(2)

Multiplying equation (2) by 2 and adding in (1),

$$(3x+4y) + 2(2x-2y) = 10 + 2 \times 2$$

or, $$3x + 4y + 4x - 4y = 10 + 4$$

or, $$7x = 14 \Rightarrow x = 2$$

c156

Hence, $x = 2$ and $y = 1$.

By Substitution Method :

We have $3x + 4y = 10$ ...(1)

and $2x - 2y = 2$ ...(2)

From equation (2) $2y = 2x - 2$

or, $y = x - 1$ ...(3)

Substituting this value of y in equation (1),

$$3x + 4(x-1) = 10$$

$$7x = 14 \Rightarrow x = 2$$

From equation (3), $y = 2 - 1 = 1$

Hence, $x = 2$ and $y = 1$

111. Solve $3x - 5y - 4 = 0$ and $9x = 2y + 7$ by elimination method and the substitution method.

Ans : [Board Term-1 2012]

By Elimination Method :

We have, $3x - 5y = 4$ ...(1)

and $9x = 2y + 7$ ...(2)

Multiplying equation (1) by 3 and rewriting equation (2) we have

$$9x - 15y = 12 \quad ...(3)$$

$$9x - 2y = 7 \quad ...(4)$$

Subtracting equation (4) from equation (3),

$$-13y = 5$$

c157

$$y = -\frac{5}{13}$$

Substituting value of y in equation (1),

$$3x - 5\left(\frac{-5}{13}\right) = 4$$

$$3x = 4 - \frac{25}{13}$$

$$x = \frac{27}{13 \times 3} = \frac{9}{13}$$

Hence $x = \frac{9}{13}$ and $y = -\frac{5}{13}$

By Substituting Method :

We have $3x - 5y = 4$ ...(1)

and $9x = 2y + 7$ ...(2)

$$y = \frac{9x-7}{2} \quad ...(3)$$

Substituting this value of y (3) in equation (1),

$$3x - 5 \times \left(\frac{9x-7}{2}\right) = 4$$

$$6x - 45x + 35 = 8$$

$$-39x = -27$$

$$x = \frac{9}{13}$$

Substituting $x = \frac{9}{13}$ in equation (3),

$$y = \frac{9 \times \frac{9}{13} - 7}{2} = \frac{81-91}{2 \times 13}$$

$$= -\frac{10}{26} = -\frac{5}{13}$$

Hence, $x = \frac{9}{13}$ and $y = \frac{-5}{13}$

112. A train covered a certain distance at a uniform speed. If the train would have been 10 km/hr scheduled time. And, if the train were slower by 10 km/hr, it would have taken 3 hr more than the scheduled time. Find the distance covered by the train.

Ans : [Board Term-1 2012, NCERT]

Let the actual speed of the train be s and actual time taken t.

$$\text{Distance} = \text{Speed} \times \text{Time}$$

$$= st \text{ km}$$

According to the given condition, we have

$$st = (s+10)(t-2)$$

$$st = st - 2s + 10t - 20$$

$$2s - 10t + 20 = 0$$

$$s - 5t = -10 \quad (1)$$

and $st = (s-10)(t+3)$

$$st = st + 3s - 10t - 30$$

$$3s - 10t = 30 \quad ...(2)$$

Multiplying equation (1) by 3 and subtracting equation (2) from equation (1),

$$3 \times (s - 5t) - (3s - 10t) = -3 \times 10 - 30$$

$$-5t = -60 \Rightarrow t = 12$$

Substituting value of t equation (1),

$$s - 5 \times 12 = -10$$

$$s = -10 + 60 = 50$$

Hence, the distance covered by the train

$$= 50 \times 12 = 600 \text{ km.}$$

113. The ratio of incomes of two persons is 11:7 and the ratio of their expenditures is 9:5. If each of them manages to save Rs 400 per month, find their monthly incomes.

Ans : [Board Term-1 2012]

Let the incomes of two persons be $11x$ and $7x$.

Also the expenditures of two persons be $9y$ and $5y$.

$$11x - 9y = 400 \quad ...(1)$$

and $7x - 5y = 400$ ...(2)

Multiplying equation (1) by 5 and equation (2) by 9 we have

$$55x - 45y = 2000 \quad ...(3)$$

and $63x - 45y = 3600$ ...(4)

Subtracting, above equation we have

$$-8x = -1600$$

or, $x = \frac{-1,600}{-8} = 200$

Hence Their monthly incomes are $11 \times 200 =$ Rs 2200 and $7 \times 200 =$ Rs 1400.

114. A and B are two points 150 km apart on a highway. Two cars start A and B at the same time. If they move in the same direction they meet in 15 hours. But if they move in the opposite direction, they meet in 1 hours. Find their speeds.

Ans : [Board Term-1 2012]

Let the speed of the car I from A be x and speed of the car II from B be y.

Same Direction :

Distance covered by car I

$= 150 +$(distance covered by car II)

$$15x = 150 + 15y$$

$$15x - 15y = 150$$

$$x - y = 10 \quad ...(1)$$

Opposite Direction :

Distance covered by car I + distance covered by car II

$$= 150 \text{ km}$$

$$x + y = 150 \quad ...(2)$$

Adding equation (1) and (2), we have $x = 80$.

Substituting $x = 80$ in equation (1), we have $y = 70$.

Speed of the car I from $A = 80$ km/hr and speed of the car II from $B = 70$ km/hr.

115. If 2 is subtracted from the numerator and 1 is added to the denominator, a fraction becomes $\frac{1}{2}$, but when 4 is added to the numerator and 3 is subtracted from the denominator, it becomes $\frac{3}{2}$. Find the fraction.

Ans : [Board Term-1 2012]

Let the fraction be $\frac{x}{y}$ then we have

$$\frac{x-2}{y+1} = \frac{1}{2}$$

$$2x - 4 = y + 1$$

$$2x - y = 5 \quad ...(1)1$$

Also, $$\frac{x+4}{y-3} = \frac{3}{2}$$

$$2x + 8 = 3y - 9]$$

$$2x - 3y = -17 \quad ...(2)\ 1$$

Subtracting equation (2) from equation (1),

$$2y = 22 \Rightarrow y = 11$$

Substituting this value of y in equation (1) we have,

$$2x - 11 = 5$$

$$x = 8$$

Hence, Fraction $= \frac{8}{11}$

116. If a bag containing red and white balls, half the number of white balls is equal to one-third the number of red balls. Thrice the total number of balls exceeds seven times the number of white balls by 6. How many balls of each colour does the bag contain ?

Ans : [Board Term-1 2012]

Let the number of red balls be x and white balls be y.

According to the question,

$$\frac{y}{2} = \frac{1}{3}x \text{ or } 2x - 3y = 0 \quad ...(1)$$

and $$3(x+y) - 7y = 6$$

or $$3x - 4y = 6 \quad ...(2)$$

Multiplying equation (1) by 3 and equation (2) by we have

$$6x - 9y = 0 \quad ...(3)$$

$$6x - 8y = 12 \quad ...(4)$$

Subtracting equation (3) from (4) we have

$$y = 12$$

Substituting $y = 12$ in equation (1),

$$2x - 36 = 0$$

$$x = 18$$

Hence, number of red balls $= 18$

and number of white balls $= 12$

117. A two digit number is obtained by either multiplying the sum of digits by 8 and then subtracting 5 or by multiplying the difference of digits by 16 and adding 3. Find the number.

Ans : [Board Term-1 2012]

Let the digits of number be x and y, then number will $10x + y$.

According to the question, we have

$$8(x+y) - 5 = 10x + y$$

$$2x - 7y + 5 = 0 \quad ...(1)$$

also $$16(x-y) + 3 = 10x + y$$

$$6x - 17y + 3 = 0 \quad ...(2)$$

Comparing the equation with $ax + by + c = 0$ we get

$$a_1 = 2, b_1 = -1, c_1 = 5$$

$$a_2 = 6, b_2 = -17, c_2 = 3$$

Now $$\frac{x}{b_2c_1 - b_1c_2} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{c_1b_2 - a_2b_1}$$

$$\frac{x}{(-7)(3) - (-17)(5)} = \frac{y}{(5)(6) - (2)(3)}$$

$$= \frac{1}{(2)(-17) - (6)(-7)}$$

$$\frac{x}{-21+85} = \frac{y}{30-6} = \frac{1}{-34+42}$$

$$\frac{x}{64} = \frac{y}{24} = \frac{1}{8}$$

$$\frac{x}{8} = \frac{y}{3} = 1$$

Hence, $x = 8, y = 3$

So required number $= 10 \times 8 + 3 = 83$.

118. The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and the breadth is increased by 3 units. The area is increased by 67 square units if length is increased by 3 units and breadth is increased by 2 units. find the perimeter of the rectangle.

Ans : [Board Term-1 2012, Set-48]

Let length of given rectangle be x and breadth be y, then area of rectangle will be xy.

According to the first condition we have

$$(x-5)(y+3) = xy - 9$$

or, $3x - 5y = 6$ ...(1)

According to the second condition, we have

$$(x+3)(y+2) = xy - 67$$

or, $2x + 5y = 61$ ...(2)

Multiplying equation (1) by 3 and equation (2) by 5 and then adding,

$$9x - 15y = 18$$

$$10x + 15y = 305$$

$$x = \frac{323}{19} = 17$$

c164

Substituting this value of x in equation (1),

$$3(17) - 5y = 6$$

$$5y = 51 - 6$$

$$y = 9$$

Hence, perimeter $= 2(x+y) = 2(17+9) = 52$ units.

119. Solve for x and y : $2(3x-y) = 5xy, 2(x+3y) = 5xy$.

Ans : [Board Term-1 2012, Set-25]

We have $2(3x-y) = 5xy$ ...(1)

$2(x+3y) = 5xy$ ...(2)

Divide equation (1) and (2) by xy,

$$\frac{6}{y} - \frac{2}{x} = 5 \quad ...(3)$$

and $\frac{2}{y} + \frac{6}{x} = 5$ ...(4)

Let $\frac{1}{y} = a$ and $\frac{1}{x} = b$, then equations (3) and (4) become

$6a - 2b = 5$ ...(5)

$2a + 6b = 5$ ...(6)

Multiplying equation (5) by 3 and then adding with equation (6),

$$20a = 20$$

$$a = 1$$

c165

Substituting this value of a in equation (5),

$$b = \frac{1}{2}$$

Now $\frac{1}{y} = a = 1 \Rightarrow y = 1$

and $\frac{1}{x} = b = \frac{1}{2} \Rightarrow x = 2$

Hence, $x = 2, y = 1$

120. The students of a class are made to stand in rows. If 3 students are extra in a row, there would be 1 row less. If 3 students are less in a row, there would be 2 rows more. Find the number of students in the class.

Ans : [Board Term-1 2012, Set-68, NCERT]

Let the number of students in a row be x and the number of rows be y. Thus total will be xy.

Now $(x+3)(y-1) = xy$

$$xy + 3y - x - 3 = xy$$

$-x + 3y - 3 = 0$ ...(1)

and $(x-3)(y+2) = xy$

$$xy - 3y + 2x - 6 = xy$$

$2x - 3y - 6 = 0$ ...(2)

Multiply equation (1) 2 we have

$-2x + 6y - 6 = 0$ ...(3)

Adding equation (2) and (3) we have

$$3y - 12 = 0$$

$$y = 4$$

c167

Substitute $y = 4$ in equation (1)

$$-x + 12 - 3 = 0$$

$$x = 9$$

Total students $xy = 9 \times 4 = 36$

Total students in the class is 36.

121. The ages of two friends ani and Biju differ by 3 years. Ani's father Dharam is twice as old as ani and Biju is twice as old as his sister Cathy. The ages of Cathy and Dharam differ by 30 year. Find the ages of Ani and Biju.

Ans : [Board Term-1 2012, Set-64]

Let the ages of Ani and Biju be x and y, respectively.

According to the given condition,

$$x - y = \pm 3 \quad ...(1)$$

Also, age of Ani's father Dharam $= 2x$ years

And age of Biju's sister $= \frac{y}{2}$ years

c168

According to the given condition,

$$2x - \frac{y}{2} = 30$$

$$4x - y = 60 \quad ...(2)$$

Case I : When $x - y = 3 \quad ...(3)$

Subtracting equation (3) from equation (2),

$$3x = 57$$

$$x = 19 \text{ years}$$

Putting $x = 19$ in equation (3),

$$19 - y = 3$$

$$y = 16 \text{ years}$$

Case II : When $x - y = -3 \quad ...(4)$

Subtracting equation (iv) from equation (2),

$$3x = 60 + 3$$

$$3x = 63$$

$$x = 21 \text{ years}$$

Subtracting equation (4), we get

$$21 - y = -3$$

$$y = 24 \text{ years}$$

Hence, Ani's age = 19 years or 21 years Biju age = 16 years or 24 years.

122. One says, "Give me a hundred, friend! I shall then become twice as rich as you." The other replies, "If you give me ten, I shall be six times as rich as you." Tell me what is the amount of their (respective) capital.

Ans : [Board Term-1 2012, Set-54]

Let the amount of their respective capitals be x and y.

According to the given condition,

$$x + 100 = 2(y - 100)$$

$$x - 2y = -300 \quad ...(1)$$

and $$6(x - 10) = y + 10$$

$$6x - y = 70 \quad ...(2)$$

Multiplying equation (2) by 2 we have

$$12x - 2y = 140 \quad ...(3)$$

Subtracting (1) from equation (3) we have

$$11x = 440$$

$$x = 40$$

c169

Substituting $x = 40$ in equation (1),

$$40 - 2y = -300$$

or, $$2y = 340$$

$$y = 170$$

Hence, the amount of their respective capitals are 40 and 170.

123. A fraction become $\frac{9}{11}$ if 2 is added to both numerator and denominator. If 3 is added to both numerator and denominator it becomes $\frac{5}{6}$. Find the fraction.

Ans : [Board Term-1 2012, Set-60]

Let the fraction be $\frac{x}{y}$, then according to the question,

$$\frac{x+2}{y+2} = \frac{9}{11}$$

c170

$$11x + 22 = 9y + 18$$

or, $$11x - 9y + 4 = 0 \quad ...(1)$$

and $$\frac{x+3}{y+3} = \frac{5}{6}$$

or, $$6x - 5y + 3 = 0 \quad ...(2)$$

Comparing with $ax + by + c = 0$

we get $$a_1 = 11, b_1 = 9, c_1 = 4,$$

$$a_2 = 6, b_2 = -5, \text{ and } c_2 = 3$$

Now, $$\frac{x}{b_2c_1 - b_1c_2} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - b_2b_1}$$

$$\frac{x}{(-9)(3) - (-5)(4)} = \frac{y}{(4)(6) - (11)(3)}$$

$$= \frac{1}{(11)(-5) - (9)(-9)}$$

or, $$\frac{x}{-27 + 20} = \frac{y}{24 - 33} = \frac{1}{-55 + 54}$$

$$\frac{x}{-7} = \frac{y}{-9} = \frac{1}{-1}$$

Hence, $x = 7, y = 9$

Thus fraction is $\frac{7}{9}$.

124. A motor boat can travel 30 km upstream and 28 km downstream in 7 hours. It can travel 21 km upstream and return in 5 hours. Find the speed of the boat in still water and the speed of the stream.

Ans : [Board Term-1 2012]

Let the speed of the boat in still water be x km/hr and speed of the stream be y km/hr.

Speed of boat up stream $= (x - y)$ km/hr.

Speed of boat down stream $= (x + y)$ km/hr.

$$\frac{30}{x - y} + \frac{28}{x + y} = 7$$

and $$\frac{21}{x - y} + \frac{21}{x + y} = 5$$

c171

Let $\frac{1}{x - y}$ be a and $\frac{1}{x + y}$ be b, then we have

$$30a + 28b = 7 \quad ...(1)$$

$$21a + 21b = 5 \quad ...(2)$$

Multiplying equation (1) by 3 and equation (2) by 4 we have

$$90a + 84b = 21 \quad ...(3)$$

$$84a + 84b = 20 \quad ...(4)$$

Subtracting (4) from (3) we have,

$$6a = 1$$

$$a = \frac{1}{6}$$

Putting this value of a in equation (1),

$$30 \times \frac{1}{6} + 28b = 7$$

$$28b = 7 - 30 \times \frac{1}{6} = 2$$

$$b = \frac{1}{14}$$

Thus $$x + y = 14 \quad ...(5)$$

Now, $$a = \frac{1}{x - y} = \frac{1}{6}$$

or, $$x - y = 6 \quad ...(6)$$

and $$x + y = 14$$

Solving equation (5) and (6), we get

$$x = 10, y = 4$$

Hence, speed of the boat in still water = 10km/hr

and speed of the stream = 4 km/hr.

125. A boat covers 32 km upstream and 36 km downstream in 7 hours. Also, it covers 40 km upstream and 48 km downstream in 9 hours. Find the speed of the boat in still water and that of the stream.

Ans : [Board Term-1 2012, Set-48]

Let the speed of the boat be x km/hr and the speed of the stream be y km/hr.

According to the question,

c172

$$\frac{32}{x - y} + \frac{36}{x + y} = 7$$

and $$\frac{40}{x - y} + \frac{48}{x + y} = 9$$

Let $\frac{1}{x - y} = A, \frac{1}{x + y} = B$, then we have

$$32A + 36B = 7 \quad ...(1)$$

and $$40A + 48B = 9 \quad ...(2)$$

Multiplying equation (1) by 5 and (2) by 4, we have

$$160A + 180B = 35 \quad ...(3)$$

and $$160A + 192B = 36 \quad ...(4$$

Subtracting (4) from (3) we have

$$-12B = -1$$

$$B = \frac{1}{12}$$

Substituting the value of B in (2) we get

$$40A + 48\left(\frac{1}{12}\right) = 9$$

$$40A + 4 = 9$$

$$40A = 5$$

$$A = \frac{1}{8}$$

Thus $A = \frac{1}{8}$ and $B = \frac{1}{12}$

Hence $$A = \frac{1}{8} = \frac{1}{x - y}$$

$$x - y = 8 \qquad ...(5)$$

and $$B = \frac{1}{12} = \frac{1}{x + y}$$

$$x + y = 12 \qquad ...(6)$$

Adding equations (5) and (6) we have,

$$2x = 20$$

$$x = 10$$

Substituting this value of x in equation (1),

$$y = x - 8 = 10 - 8 = 2$$

Hence, the speed of the boat in still water = 10 km/hr and speed of the stream = 2 km/hr.

126. For what values of a and b does the following pair of linear equations have infinite number of solution ?

$2x + 3y = 7, a(x + y) - b(x - y) = 3a + b - 2$

Ans : [Board Term-1 2015]

We have $$2x + 3y - 7 = 0$$

Here $a_1 = 2, b_1 = 3, c_1 = -7$

and $$a(x + y) - b(x - y) = 3a + b - 2$$

$$ax + ay - bx + by = 3a + b - 2$$

$$(a - b)x + (a + b)y - (3a + b - 2) = 0$$

Here $a_2 = a - b, b_2 = a + b, c_2 = -(3a + b - 2)$

For infinite many solutions

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{2}{a - b} = \frac{3}{a + b} = \frac{-7}{(3a + b - 2)}$$

From $\frac{2}{a - b} = \frac{7}{3a + b - 2}$ we have

c173

$$2(3a + b - 2) = 7(a - b)$$

$$6a + 2b - 4 = 7a - 7b$$

$$a - 9b = -4 \qquad ...(1)$$

From $\frac{3}{a + b} = \frac{7}{3a + b - 2}$ we have

$$3(3a + b - 2) = 7(a + b)$$

$$9a + 3b - 6 = 7a + 7b$$

$$2a - 4b = 6$$

$$a - 2b = 3 \qquad ...(2)$$

Subtracting equation (1) from (2),

$$-7b = -7$$

$$b = 1$$

Substituting the value of b in equation (1),

$$a = 5$$

Hence, $a = 5, b = 1$.

127. At a certain time in a deer, the number of heads and the number of legs of deer and human visitors were counted and it was found that there were 39 heads and 132 legs.

Find the number of deer and human visitors in the park.

Ans : [Board Term-1 2015]

Let the no. of deer be x and no. of human be y.
According to the question,

$$x + y = 39 \qquad ...(1)$$

and $$4x + 2y = 132 \qquad ...(2)$$

Multiply equation (1) from by 2,

$$2x + 2y = 78 \qquad ...(3)$$

Subtract equation (3) from (2),

$$2x = 54$$

$$x = 27$$

c175

Substituting this value of x in equation (1)

$$27 + y = 39$$

$$y = 12$$

So, No. of deer = 27 and No. of human = 12

128. Find the value of p and q for which the system of equations represent coincident lines $2x + 3y = 7$,

$(p+q+1)x+(p+2q+2)y=4(p+q)+1$

Ans : [Board Term-1 2012, Set-42]

We have $$2x+3y=7$$

$$(p+q+1)x+(p+2q+2)y=4(p+q)+1$$

Comparing given equation to $ab+by+c=0$ we have

$a_1=2, b_1=3, c_1=-7$

$a_2=p+q+1, b_2=p+2q+2, c_2=-4(p+q)-1$

For coincident lines,

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

$$\frac{2}{p+q+1}=\frac{3}{p+2q+2}=\frac{7}{4(p+q)+1}$$

From $\frac{3}{p+2q+2}=\frac{7}{4(p+q)+1}$ we have

$$7p+14q+14=12p+12q+3$$

$$5p-2q-11=0 \quad ...(1)$$

From $\frac{2}{p+q+1}=\frac{7}{4(p+q)+1}$ we have

c176

$$8(p+q)+2=7p+7q+7$$

$$8p+8q+2=7p+7q+7$$

$$p+q-5=0 \quad ...(2)$$

Multiplying equation (2) by 5 we have

$$5p+5q-25=0 \quad ...(3)$$

Subtracting equation (1) from (3) we get

$$7q=14$$

$$q=2$$

Hence, $p=3$ and $q=2$.

129. The length of the sides of a triangle are $2x+\frac{y}{2}$, $\frac{5x}{3}+y+\frac{1}{2}$ and $\frac{2}{3}x+2y+\frac{5}{2}$. If the triangle is equilateral , find its perimeter.

Ans : [Board Term-1 2012]

For an equilateral Δ,

$$2x+\frac{y}{2}=\frac{5x}{3}+y+\frac{1}{2}=\frac{1}{2}x+2y+\frac{5}{2}$$

Now $$\frac{4x+y}{2}=\frac{10x+6y+3}{6}$$

$$12x+3y=10x+6y+3$$

$$2x-3y=3 \quad ...(1)$$

Again, $$2x+\frac{y}{2}=\frac{2}{3}x+2y+\frac{5}{2}$$

$$\frac{4x+y}{2}=\frac{4x+12y+15}{6}$$

$$12x+3y=4x+12y+15$$

$$8x-9y=15 \quad ...(2)$$

Multiplying equation (1) by 3 we have

$$6x-9y=9 \quad ...(1)$$

Subtracting it from (2) we get

$$2x=6 \Rightarrow x=3$$

c178

Substituting this value of x into (1), we get

$$2\times 3-3y=3$$

or, $$3y=3 \Rightarrow y=1$$

Now substituting these value of x and y

$$2x+\frac{y}{2}=2\times 3+\frac{1}{2}=6.5$$

The perimeter of equilateral triangle $=$ side $\times 3$

$$=6.5\times 3=19.5 \text{ cm}$$

Hence, the perimeter of $\Delta=19.5$ m

130. When 6 boys were admitted and 6 girls left, the percentage of boys increased from 60% to 75%. Find the original no. of boys and girls in the class.

Ans : [Board Term-1 2015]

Let the no. of boys be x and no. of girls be y.

No. of students $=x+y$

Now $$\frac{x}{x+y}=\frac{60}{100} \quad ...(1)$$

and $$\frac{x+6}{(x+6)+(y-6)}=\frac{75}{100} \quad ...(2)$$

From (1), we have

c180

$$100x=60x+60y$$

$$40x-60y=0$$

$$2x-3y=0$$

$$2x=3y \quad (3)$$

From (2) we have

$$100x+600=75x+75y$$

$$25x-75y=-600$$

$$x - 3y = -24 \quad ...(4)$$

Substituting the value of $3y$ from (3) in to (4) we have,

$$x - 2x = -24 \Rightarrow x = 24$$

$$3y = 24 \times 2$$

$$y = 16$$

Hence, no. of boys is 24 and no. of girls is 16.

131. A cyclist, after riding a certain distance, stopped for half an hour to repair his bicycle, after which he completes the whole journey of 30 km at half speed in 5 hours. If the breakdown had occurred 10 km farther off, he would have done the whole journey in 4 hours. Find where the breakdown occurred and his original speed.

Ans : [Board Term-1 2013, Set-32]

Let x be the distance of the place where breakdown occurred and y be the original speed,

c181

$$\frac{x}{y} + \frac{30 - x}{\frac{y}{2}} = 5$$

or
$$\frac{x}{y} + \frac{60 - 2x}{y} = 5$$

$$x + 60 - 2x = 5y$$

$$x + 5y = 60 \quad ...(1)$$

and
$$\frac{x + 10}{y} + \frac{30 - (x + 10)}{\frac{y}{2}} = 4$$

$$\frac{x + 10}{y} + \frac{60 - 2(x + 10)}{y} = 4$$

$$x + 10 + 60 - 2x - 20 = 4y$$

$$-x + 50 = 4y$$

$$x + 4y = 50 \quad (2)$$

Subtract equation (2) from (1), $y = 10$ km/hr.

Now from (2), $x + 40 = 50$

$$x = 10 \text{ km}$$

Break down occurred at 10 km and original speed was 10 km/hr.

132. The population of a village is 5000. If in a year, the number of males were to increase by 5% and that of a female by 3% annually, the population would grow to 5202 at the end of the year. Find the number of males and females in the village.

Ans : [Board Term-1 2012, Set-60]

Let the number of males be x and females be y

Now
$$x + y = 5,000 \quad ...(1)$$

and
$$x + \frac{5}{100}x + y + \frac{3y}{100} = 5202$$

$$\frac{5x + 3y}{100} + 5000 = 5202$$

c182

$$5x + 3y = (5202 - 5000) \times 100$$

$$5x + 3y = 20200 \quad (2)$$

Multiply (1) by 3 we have

$$3x + 3y = 15,000 \quad ...(3)$$

Subtracting (2) from (3) we have

$$2x = 5200 \Rightarrow x = 2600$$

Substituting value of x in (1) we have

$$2600 - y = 5000$$

$$y = 2400$$

Thus no. of males is 2600 and no. of females is 2400.

CHAPTER 4

QUADRATIC EQUATIONS

ONE MARK QUESTIONS

MULTIPLE CHOICE QUESTIONS

1. The sum and product of the zeroes of a quadratic polynomial are 3 and -10 respectively. The quadratic polynomial is

(a) $x^2 - 3x + 10$ (b) $x^2 + 3x - 10$

(c) $x^2 - 3x - 10$ (d) $x^2 + 3x + 10$

Ans : [Board 2020 Delhi Basic]

Sum of zeroes, $\alpha + \beta = 3$

and product of zeroes, $\alpha\beta = -10$

d243

Quadratic polynomial,

$$p(x) = x^2 - (\alpha + \beta) + \alpha\beta$$
$$= x^2 - 3x - 10$$

Thus (c) is correct option.

2. If the sum of the zeroes of the quadratic polynomial $kx^2 + 2x + 3k$ is equal to their product, then k equals

(a) $\frac{1}{3}$ (b) $-\frac{1}{3}$

(c) $\frac{2}{3}$ (d) $-\frac{2}{3}$

Ans : [Board 2020 OD Basic]

We have $p(x) = kx^2 + 2x + 3k$

Comparing it by $ax^2 + bx + c$, we get $a = k$, $b = 2$ and $c = 3k$.

Sum of zeroes, $\alpha + \beta = -\frac{b}{a} = -\frac{2}{k}$

d244

Product of zeroes, $\alpha\beta = \frac{c}{a} = \frac{3k}{k} = 3$

According to question, we have

$$\alpha + \beta = \alpha\beta$$
$$-\frac{2}{k} = 3 \Rightarrow k = -\frac{2}{3}$$

Thus (d) is correct option.

3. If α and β are the zeroes of the polynomial $x^2 + 2x + 1$, then $\frac{1}{\alpha} + \frac{1}{\beta}$ is equal to

(a) -2 (b) 2

(c) 0

Ans : [Board 2020 Delhi Basic]

Since α and β are the zeros of polynomial $x^2 + 2x + 1$,

Sum of zeroes, $\alpha + \beta = -\frac{2}{1} = -2$

d245

and product of zeroes, $\alpha\beta = \frac{1}{1} = 1$

Now, $$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = -\frac{2}{1} = -2$$

Thus (a) is correct option.

4. If α and β are the zeroes of the polynomial $2x^2 - 13x + 6$, then $\alpha + \beta$ is equal to

(a) -3 (b) 3

(c) $\frac{13}{2}$ (d) $-\frac{13}{2}$

d246

Ans : [Board 2020 Delhi Basic]

We have $p(x) = 2x^2 - 13x + 6$

Comparing it with $ax^2 + bx + c$ we get $a = 2$, $b = -13$ and $c = 6$

Sum of zeroes $\alpha + \beta = -\frac{b}{a} = -\frac{(-13)}{2} = \frac{13}{2}$

Thus (c) is correct option.

5. The roots of the quadratic equation $x^2 - 0.04 = 0$ are

(a) ± 0.2 (b) ± 0.02

(c) 0.4 (d) 2

Ans : [Board 2020 OD Standard]

We have $x^2 - 0.04 = 0$

d247

$$x^2 = 0.04$$
$$x = \pm\sqrt{0.04}$$
$$x = \pm 0.2.$$

Thus (a) is correct option.

6. If ½ is a root of the equation $x^2 + kx - \frac{5}{4} = 0$, then the value of k is

(a) 2 (b) -2

(c) $\frac{1}{4}$ (d) $\frac{1}{2}$

d248

Ans :

We have $$x^2 + kx - \frac{5}{4} = 0$$

Since, ½ is a root of the given quadratic equation, it must satisfy it.

Thus $$\left(\frac{1}{2}\right)^2 + k\left(\frac{1}{2}\right) - \frac{5}{4} = 0$$

$$\frac{1}{4} + \frac{k}{2} - \frac{5}{4} = 0$$

$$\frac{1 + 2k - 5}{4} = 0$$

$$2k - 4 = 0 \Rightarrow k = 2$$

Thus (a) is correct option.

7. Each root of $x^2 - bx + c = 0$ is decreased by 2. The resulting equation is $x^2 - 2x + 1 = 0$, then

(a) $b = 6, c = 9$ (b) $b = 3, c = 5$

(c) $b = 2, c = -1$ (d) $b = -4, c = 3$

Ans :

For $x^2 - bx + c = 0$ we have

$$\alpha + \beta = b$$

$$\alpha\beta = c$$

Now $$\alpha - 2 + \beta - 2 = \alpha + \beta - 4 = b - 4$$

$$(\alpha - 2)(\beta - 2) = \alpha\beta - 2(\alpha + \beta) + 4$$

$$= c - 2b + 4$$

d249

For $x^2 - 2x + 1 = 0$ we have

$$2 = b - 4 \Rightarrow b = 6$$

and $$1 = c - 2b + 4$$

$$= c - 2 \times 6 + 4$$

$$= c - 8$$

$$c = 1 + 8 = 9$$

Thus (a) is correct option.

8. Value(s) of k for which the quadratic equation $2x^2 - kx + k = 0$ has equal roots is/are

(a) 0 (b) 4

(c) 8 (d) 0, 8

Ans :

We have $$2x^2 - kx + k = 0$$

Comparing with $ax^2 + bx + c = 0$ we $a = 2$, $b = -k$ and $c = k$.

For equal roots, the discriminant must be zero.

Thus $$b^2 - 4ac = 0$$

$$(-k)^2 - 4(2)k = 0$$

$$k^2 - 8k = 0$$

$$k(k - 8) = 0 \Rightarrow k = 0,\ 8$$

d250

Hence, the required values of k are 0 and 8.

Thus (d) is correct option.

9. If the equation $(m^2 + n^2)x^2 - 2(mp + nq)x + p^2 + q^2 = 0$ has equal roots, then

(a) $mp = nq$ (b) $mq = np$

(c) $mn = pq$ (d) $mq = \sqrt{np}$

Ans :

For equal roots, $$b^2 = 4ac$$

$$4(mp + nq)^2 = 4(m^2 + n^2)(p^2 + q^2)$$

$$m^2q^2 + n^2p^2 - 2mnpq = 0$$

$$(mq - np)^2 = 0$$

$$mq - np = 0$$

$$mq = np$$

d251

Thus (b) is correct option.

10. The linear factors of the quadratic equation $x^2 + kx + 1 = 0$ are

(a) $k \geq 2$ (b) $k \leq 2$

(c) $k \geq -2$ (d) $2 \leq k \leq -2$

Ans :

We have, $$x^2 + kx + 1 = 0$$

Comparing with $ax^2 + bx + c = 0$ we get $a = 1$, $b = k$ and $c = 1$.

For linear factors, $$b^2 - 4ac \geq 0$$

$$k^2 - 4 \times 1 \times 1 \geq 0$$

$$(k^2 - 2^2) \geq 0$$

$$(k - 2)(k + 2) \geq 0$$

d252

$$k \geq 2 \text{ and } k \leq -2$$

Thus (d) is correct option.

11. If one root of the quadratic equation $ax^2 + bx + c = 0$ is the reciprocal of the other, then

(a) $b = c$ (b) $a = b$

(c) $ac = 1$ (d) $a = c$

d253

Ans :

If one root is α, then the other $\frac{1}{\alpha}$.

Product of roots, $\alpha \cdot \frac{1}{\alpha} = \frac{c}{a}$

$$1 = \frac{c}{a} \Rightarrow a = c$$

Thus (d) is correct option.

12. The quadratic equation $2x^2 - \sqrt{5}\,x + 1 = 0$ has

(a) two distinct real roots

(b) two equal real roots

(c) no real roots

(d) more than 2 real roots

d255

Ans :

We have $2x^2 - \sqrt{5x} + 1 = 0$

Comparing with $ax^2 + bx + c = 0$ we get $a = 2$, $b = -\sqrt{5}$ and $c = 1$,

Now $b^2 - 4ac = (-\sqrt{5})^2 - 4 \times (2) \times (1)$

$$= 5 - 8 = -3 < 0$$

Since, discriminant is negative, therefore quadratic equation $2x^2 - \sqrt{5}\,x + 1 = 0$ has no real roots i.e., imaginary roots.

Thus (c) is correct option.

13. The real roots of the equation $x^{2/3} + x^{1/3} - 2 = 0$ are

(a) 1, 8 (b) $-1, -8$

(c) $-1, 8$ (d) $1, -8$

Ans :

We have $x^{2/3} + x^{1/3} - 2 = 0$

d256

Substituting $x^{1/3} = y$ we obtain,

$$y^2 + y - 2 = 0$$

$$(y-1)(y+2) = 0 \Rightarrow y = 1 \text{ or } y = -2$$

Thus $x^{1/3} = 1 \Rightarrow x = (1)^3 = 1$

or $x^{1/3} = -2 \Rightarrow x = (-2)^3 = -8$

Hence, the real roots of the given equations are 1, -8.

Thus (d) is correct option.

14. $(x^2+1)^2 - x^2 = 0$ has

(a) four real roots (b) two real roots

(c) no real roots (d) one real root

Ans :

d257

We have $(x^2+1)^2 - x^2 = 0$

$$x^4 + 1 + 2x^2 - x^2 = 0$$

$$x^4 + x^2 + 1 = 0$$

$$(x^2)^2 + x^2 + 1 = 0$$

Let $x^2 = y$ then we have

$$y^2 + y + 1 = 0$$

Comparing with $ay^2 + by + c = 0$ we get $a = 1$, $b = 1$ and $c = 1$

Discriminant, $D = b^2 - 4ac$

$$= (1)^2 - 4(1)(1)$$

$$= 1 - 4 = -3$$

Since, $D < 0$, $y^2 + y + 1 = 0$ has no real roots.

i.e. $x^4 + x^2 + 1 = 0$ or $(x^2+1)^2 - x^2 = 0$ has no real roots.

Thus (c) is correct option.

15. The equation $2x^2 + 2(p+1)x + p = 0$, where p is real, always has roots that are

(a) Equal

(b) Equal in magnitude but opposite in sign

(c) Irrational

(d) Real

Ans :

We have $2x^2 + 2(p+1)x + p = 0$,

Comparing with $ax^2 + bx + c = 0$ we get $a = 2$, $b = 2(p+1)$ and $c = p$.

Now $b^2 - 4ac = [2(p+1)]^2 - 4(2p)$

d258

$$= 4(p+1)^2 - 8p$$

$$= 4p^2 + 8p + 4 - 8p$$

$$= 4(p^2+1)$$

For any real value of p, $4(p^2+1)$ will always be positive as p^2 cannot be negative for real p. Hence, the discriminant $b^2 - 4ac$ will always be positive.

When the discriminant is greater than 0 or is positive, then the roots of a quadratic equation are real.

Thus (d) is correct option.

16. The condition for one root of the quadratic equation

$ax^2 + bx + c = 0$ to be twice the other, is

(a) $b^2 = 4ac$ (b) $2b^2 = 9ac$

(c) $c^2 = 4a + b^2$ (d) $c^2 = 9a - b^2$

d259

Ans :

Sum of zeroes $\alpha + 2\alpha = -\frac{b}{a}$

$$3\alpha = -\frac{b}{a} \Rightarrow \alpha = -\frac{b}{3a}$$

Product of zeroes $\alpha \times 2\alpha = \frac{c}{a}$

$$2\alpha^2 = \frac{c}{a}$$

$$2\left(-\frac{b}{3a}\right)^2 = \frac{c}{a}$$

$$\frac{2b^2}{9a^2} = \frac{c}{a}$$

$$2ab^2 - 9a^2c = 0$$

$$a(2b^2 - 9ac) = 0$$

Since, $a \neq 0$, $2b^2 = 9ac$

Hence, the required condition is $2b^2 = 9ac$.

Thus (b) is correct option.

17. If $x^2 + y^2 = 25$, $xy = 12$, then x is

(a) $(3, 4)$ (b) $(3, -3)$

(c) $(3, 4, -3, -4)$ (d) $(3, -3)$

Ans :

d260

We have $x^2 + y^2 = 25$

and $xy = 12$

$$x^2 + \left(\frac{12}{x}\right)^2 = 25$$

$$x^4 + 144 - 25x^2 = 0$$

$$(x^2 - 16)(x^2 - 9) = 0$$

Hence, $x^2 = 16 \Rightarrow x = \pm 4$

and $x^2 = 9 \Rightarrow x = \pm 3$

Thus (c) is correct option.

18. The quadratic equation $2x^2 - 3\sqrt{2}x + \frac{9}{4} = 0$ has

(a) two distinct real roots

(b) two equal real roots

(c) no real roots

(d) more than 2 real roots

d261

Ans :

We have $2x^2 - 3\sqrt{2}x + \frac{9}{4} = 0$

Here $a = 2$, $b = -3\sqrt{2}$, $c = \frac{9}{4}$

Discriminant $D = b^2 - 4ac$

$$= (-3\sqrt{2})^2 - 4 \times 2 \times \frac{9}{4}$$

$$= 18 - 18 = 0$$

Thus, $2x^2 - 3\sqrt{2}x + \frac{9}{4} = 0$ has real and equal roots.

Thus (b) is correct option.

19. The quadratic equation $x^2 + x - 5 = 0$ has

(a) two distinct real roots

(b) two equal real roots

(c) no real roots

(d) more than 2 real roots

d262

Ans :

We have $x^2 + x - 5 = 0$

Here, $a = 1$, $b = 1$, $c = -5$

Now, $D = b^2 - 4ac$

$$= (1)^2 - 4 \times 1 \times (-5)$$

$$= 21 > 0$$

So $x^2 + x - 5 = 0$ has two distinct real roots.

Thus (a) is correct option.

20. The quadratic equation $x^2 + 3x + 2\sqrt{2} = 0$ has

(a) two distinct real roots

(b) two equal real roots

(c) no real roots

(d) more than 2 real roots

d263

Ans :

We have $x^2 + 3x + 2\sqrt{2} = 0$

Here, $a = 1$, $b = 3$ and $c = 2\sqrt{2}$

Now, $D = b^2 - 4ac$

$$= (3)^2 - 4(1)(2\sqrt{2})$$

$$= 9 - 8\sqrt{2} < 0$$

Hence, roots of the equation are not real.
Thus (c) is correct option.

21. The quadratic equation $5x^2 - 3x + 1 = 0$ has
(a) two distinct real roots
(b) two equal real roots
(c) no real roots
(d) more than 2 real roots

d264

Ans :

We have $5x^2 - 3x + 1 = 0$

Here $a = 5,\ b = -3,\ c = 1$

Now, $D = b^2 - 4ac = (-3)^2 - 4(5)(1)$

$= 9 - 20 < 0$

Hence, roots of the equation are not real.
Thus (c) is correct option.

22. The quadratic equation $x^2 - 4x + 3\sqrt{2} = 0$ has
(a) two distinct real roots
(b) two equal real roots
(c) no real roots
(d) more than 2 real roots

d265

Ans :

We have $x^2 - 4x + 3\sqrt{2} = 0$

Here $a = 1,\ b = -4$ and $c = 3\sqrt{2}$

Now $D = b^2 - 4ac = (-4)^2 - 4(1)(3\sqrt{2})$

$= 16 - 12\sqrt{2}$

$= 16 - 12 \times (1.41)$

$= 16 - 16.92 = -0.92$

$b^2 - 4ac < 0$

Hence, the given equation has no real roots.
Thus (c) is correct option.

23. The quadratic equation $x^2 + 4x - 3\sqrt{2} = 0$ has
(a) two distinct real roots
(b) two equal real roots
(c) no real roots
(d) more than 2 real roots

d266

Ans :

We have $x^2 + 4x - 3\sqrt{2} = 0$

Here $a = 1,\ b = 4$ and $c = -3\sqrt{2}$

Now $D = b^2 - 4ac = (4)^2 - 4(1)(-3\sqrt{2})$

$= 16 + 12\sqrt{2} > 0$

Hence, the given equation has two distinct real roots,
Thus (a) is correct option.

24. The quadratic equation $x^2 - 4x - 3\sqrt{2} = 0$ has
(a) two distinct real roots
(b) two equal real roots
(c) no real roots
(d) more than 2 real roots

d267

Ans :

We have $x^2 - 4x - 3\sqrt{2} = 0$

Here $a = 1,\ b = -4$ and $c = -3\sqrt{2}$

Now $D = b^2 - 4ac$

$= (-4)^2 - 4(1)(-3\sqrt{2})$

$= 16 + 12\sqrt{2} > 0$

Hence, the given equation has two distinct real roots.
Thus (a) is correct option.

25. The quadratic equation $3x^2 + 4\sqrt{3}x + 4$ has
(a) two distinct real roots
(b) two equal real roots
(c) no real roots
(d) more than 2 real roots

d268

Ans :

We have $3x^2 + 4\sqrt{3}x + 4 = 0$

Here, $a = 3,\ b = 4\sqrt{3}$ and $c = 4$

Now $D = b^2 - 4ac = (4\sqrt{3})^2 - 4(3)(4)$

$= 48 - 48 = 0$

Hence, the equation has real and equal roots.
Thus (b) is correct option.

26. Which of the following equations has 2 as a root?
(a) $x^2 - 4x + 5 = 0$ (b) $x^2 + 3x - 12 = 0$
(c) $2x^2 - 7x + 6 = 0$ (d) $3x^2 - 6x - 2 = 0$

Ans :

(a) Substituting, $x = 2$ in $x^2 - 4x + 5$, we get

$$(2)^2-4(2)+5 = 4-8+5 = 1 \neq 0$$

So, $x = 2$ is not a root of

$$x^2-4x+5 = 0$$

d269

(b) Substituting, $x = 2$ in $x^2+3x-12$, we get

$$(2)^2+3(2)-12 = 4+6-12 = -2 \neq 0$$

So, $x = 2$ is not a root of $x^2+3x-12 = 0$.

(c) Substituting, $x = 2$ in $2x^2-7x+6$, we get

$$2(2)^2-7(2)+6 = 2(4)-14+6$$
$$= 8-14+6$$
$$= 14-14 = 0.$$

So, $x = 2$ is a root of the equation $2x^2-7x+6 = 0$.

(d) Substituting, $x = 2$ in $3x^2-6x-2$, we get

$$3(2)^2-6(2)-2 = 12-12-2 = -2 \neq 0$$

So, $x = 2$ is not a root of

$$3x^2-6x-2 = 0.$$

Thus (c) is correct option.

27. Which of the following equations has the sum of its roots as 3 ?

(a) $2x^2-3x+6=0$ (b) $-x^2+3x-3=0$

(c) $\sqrt{2}\,x^2-\frac{3}{\sqrt{2}}x+1=0$ (d) $3x^2-3x+3=0$

Ans :

Sum of the roots,

$$\alpha+\beta = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2} = -\frac{b}{a}$$

Option a : $\alpha+\beta = -\left(\frac{-3}{2}\right) = \frac{3}{2} \neq 3$

d270

Option b : $\alpha+\beta = -\left(\frac{3}{-1}\right) = 3$

Option c : $\alpha+\beta = -\left(\frac{\frac{3}{\sqrt{2}}}{\sqrt{2}}\right) = \frac{3}{2} \neq 3$

Option d $\alpha+\beta = -\left(\frac{-3}{3}\right) = 1 \neq 3$

Thus (b) is correct option.

28. Assertion : $4x^2-12x+9=0$ has repeated roots.

Reason : The quadratic equation $ax^2+bx+c=0$ have repeated roots if discriminant $D>0$.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

Reason is false because if $D=0$, equation has repeated roots.

Assertion $4x^2-12x+9 = 0$

d271

$$D = b^2-4ac$$
$$= (-12)^2-4(4)(9)$$
$$= 144-144 = 0$$

Roots are repeated.

Assertion (A) is true but reason (R) is false.

Thus (c) is correct option.

29. Assertion : The equation $x^2+3x+1=(x-2)^2$ is a quadratic equation.

Reason : Any equation of the form $ax^2+bx+c=0$ where $a \neq 0$, is called a quadratic equation.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

We have, $x^2+3x+1 = (x-2)^2 = x^2-4x+4$

$$x^2+3x+1 = x^2-4x+4$$
$$7x-3 = 0$$

d272

It is not of the form $ax^2+bx+c=0$

(d) Assertion (A) is false but reason (R) is true.

Thus (d) is correct option.

30. Assertion : The values of x are $-\frac{a}{2}, a$ for a quadratic equation $2x^2+ax-a^2=0$.

Reason : For quadratic equation $ax^2+bx+c=0$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

We have $2x^2 + ax - a^2 = 0$

$$x = \frac{-a \pm \sqrt{a^2 + 8a^2}}{4}$$

$$= \frac{-a + 3a}{4} = \frac{2a}{4}, \frac{-4a}{4}$$

$$x = \frac{a}{2}, -a$$

Assertion (A) is false but reason (R) is true.

Thus (d) is correct option.

31. Assertion : The equation $8x^2 + 3kx + 2 = 0$ has equal roots then the value of k is $\pm\frac{8}{3}$.

Reason : The equation $ax^2 + bx + c = 0$ has equal roots if $D = b^2 - 4ac = 0$

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

We have $8x^2 + 3kx + 2 = 0$

Discriminant, $D = b^2 - 4ac$

$$= (3k)^2 - 4 \times 8 \times 2 = 9k^2 - 64$$

For equal roots, $D = 0$

$$9k^2 - 64 = 0$$

$$9k^2 = 64$$

$$k^2 = \frac{64}{9} \Rightarrow k = \pm\frac{8}{3}$$

Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Thus (a) is correct option.

32. Assertion : The roots of the quadratic equation $x^2 + 2x + 2 = 0$ are imaginary.

Reason : If discriminant $D = b^2 - 4ac < 0$ then the roots of quadratic equation $ax^2 + bx + c = 0$ are imaginary.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

We have $x^2 + 2x + 2 = 0$

Discriminant, $D = b^2 - 4ac$

$$= (2)^2 - 4 \times 1 \times 2$$

$$= 4 - 8 = -4 < 0$$

Roots are imaginary.

Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Thus (a) is correct option.

FILL IN THE BLANK QUESTIONS

33. A real number α is said to be .... of the quadratic equation $ax^2 + bx + c = 0$, if $a\alpha^2 + b\alpha + c = 0$.

Ans :

root

34. For any quadratic equation $ax^2 + bx + c = 0$, $b^2 - 4ac$, is called the .... of the equation.

Ans :

discriminant

35. If the discriminant of a quadratic equation is zero, then its roots are .... and ....

Ans :

real, equal

36. If the discriminant of a quadratic equation is greater than zero, then its roots are .... and ....

Ans :

real, distinct

37. A polynomial of degree 2 is called the .... polynomial.

Ans :

quadratic

38. A quadratic equation cannot have more than .... roots.

Ans :

two

d281

39. Let $ax^2+bx+c=0$, where a,b,c are real numbers, $a\neq 0$, be a quadratic equation, then this equation has no real roots if and only if ....

Ans :

$b^2<4ac$

d282

40. If the product ac in the quadratic equation ax^2+bx+c is negative, then the equation cannot have .... roots.

Ans :

Non-real

d283

41. The equation of the form $ax^2+bx=0$ will always have .... roots.

Ans :

real

d284

42. A quadratic equation in the variable x is of the form

$$ax^2+bx+c=0,$$

where a,b,c are real numbers and a ....

Ans :

$\neq 0$

d285

43. The roots of a quadratic equation is same as the .... of the corresponding quadratic polynomial.

Ans :

zero

d286

44. Value of the roots of the quadratic equation, $x^2-x-6=0$ are ....

Ans : [Board 2020 OD Basic]

$$x^2-x-6=0$$
$$x^2-3x+2x-6=0$$
$$x(x-3)+2(x-3)=0$$
$$(x-3)(x+2)=0 \Rightarrow x=3 \text{ and } x=-2$$

d287

45. If quadratic equation $3x^2-4x+k=0$ has equal roots, then the value of k is ....

Ans : [Board 2020 Delhi Basic]

Given, quadratic equation is $3x^2-4x+k=0$

Comparing with $ax^2+bx+c=0$, we get $a=3$, $b=-4$ and $c=k$

For equal roots, $b^2-4ac=0$

$$(-4)^2-4(3)(k)=0$$
$$16-12k=0$$
$$k=\frac{16}{12}=\frac{4}{3}$$

d288

Very Short Answer Questions

46. Find the positive root of $\sqrt{3x^2+6}=9$.

Ans : [Board Term-2, 2015]

We have $\sqrt{3x^2+6}=9$

$$3x^2+6=81$$
$$3x^2=81-6=75$$
$$x^2=\frac{75}{3}=25$$

Thus $x=\pm 5$

Hence 5 is positive root.

d101

47. If $x=-\frac{1}{2}$, is a solution of the quadratic equation $3x^2+2kx-3=0$, find the value of k.

Ans : [Board Term-2, Delhi 2015]

We have $3x^2+2kx-3=0$

Substituting $x=-\frac{1}{2}$ in given equation we get

$$3\left(-\frac{1}{2}\right)^2+2k\left(-\frac{1}{2}\right)-3=0$$
$$\frac{3}{4}-k-3=0$$
$$k=\frac{3}{4}-3$$
$$=\frac{3-12}{4}=\frac{-9}{4}$$

Hence $k=\frac{-9}{4}$

d102

48. Find the roots of the quadratic equation $\sqrt{3}\,x^2-2x-\sqrt{3}=0$

Ans : [Board Term-2, 2012, 2011]

We have $\sqrt{3}\,x^2-2x-\sqrt{3}=0$

$$\sqrt{3}\,x^2-3x+x-\sqrt{3}=0$$
$$\sqrt{3}\,x(x-\sqrt{3})+1(x-\sqrt{3})=0$$

d103

$$(x-\sqrt{3})(\sqrt{3}\,x+1) = 0$$

Thus $$x = \sqrt{3}, \frac{-1}{\sqrt{3}}$$

49. Find the value of k, for which one root of the quadratic equation $kx^2 - 14x + 8 = 0$ is six times the other.

Ans : [Board Term-2, 2016]

We have $$kx^2 - 14x + 8 = 0$$

d104

Let one root be α and other root be 6α.

Sum of roots, $$\alpha + 6\alpha = \frac{14}{k}$$

$$7\alpha = \frac{14}{k} \text{ or } \alpha = \frac{2}{k} \quad ...(1)$$

Product of roots , $$\alpha(6\alpha) = \frac{8}{k} \text{ or } 6\alpha^2 = \frac{8}{k} \quad ...(2)$$

Solving (1) and (2), we obtain

$$6\left(\frac{2}{k}\right)^2 = \frac{8}{k}$$

$$6 \times \frac{4}{k^2} = \frac{8}{k}$$

$$\frac{3}{k^2} = \frac{1}{k}$$

$$3k = k^2$$

$$3k - k^2 = 0$$

$$k[3-k] = 0$$

$$k = 0 \text{ or } k = 3$$

Since $k = 0$ is not possible, therefore $k = 3$.

50. If one root of the quadratic equation $6x^2 - x - k = 0$ is $\frac{2}{3}$, then find the value of k.

Ans : [Board Term-2 Foreign-2, 2017]

We have $$6x^2 - x - k = 0$$

Substituting $x = \frac{2}{3}$, we get

d105

$$6\left(\frac{2}{3}\right)^2 - \frac{2}{3} - k = 0$$

$$6 \times \frac{4}{9} - \frac{2}{3} - k = 0$$

$$\frac{8}{3} - \frac{2}{3} - k = 0$$

$$\frac{8-2}{3} - k = 0$$

$$2 - k = 0$$

Thus $k = 2$.

51. Find the value(s) of k if the quadratic equation $3x^2 - k\sqrt{3}\,x + 4 = 0$ has real roots.

Ans : [SQP 2017]

If discriminant $D = b^2 - 4ac$ of quadratic equation is equal to zero, or more than zero, then roots are real.

We have $$3x^2 - k\sqrt{3}\,x + 4 = 0$$

Comparing with $ax^2 + bx + c = 0 = 0$ we get

$$a = 3, b = -k\sqrt{3} \text{ and } c = 4$$

d106

For real roots $$b^2 - 4ac \geq 0$$

$$(-k\sqrt{3})^2 - 4 \times 3 \times 4 \geq 0$$

$$3k^2 - 48 \geq 0$$

$$k^2 - 16 \geq 0$$

$$(k-4)(k+4) \geq 0$$

Thus $k \leq -4$ and $k \geq 4$

TWO MARKS QUESTIONS

52. For what values of k, the roots of the equation $x^2 + 4x + k = 0$ are real?

Ans : [Board 2019 Delhi]

We have $$x^2 + 4x + k = 0.$$

Comparing the given equation with $ax^2 + bx + c = 0$ we get $a = 1$, $b = 4, c = k$.

Since, given the equation has real roots,

d308

$$D \geqslant 0$$

$$b^2 - 4ac \geqslant 0$$

$$4^2 - 4 \times 1 \times k \geqslant 0$$

$$4k \leqslant 16$$

$$k \leqslant 4$$

53. Find the value of k for which the roots of the equations $3x^2 - 10x + k = 0$ are reciprocal of each other.

Ans : [Board 2019 Delhi]

We have $$3x^2 - 10x + k = 0$$

Comparing the given equation with $ax^2 + bx + c = 0$

we get $a = 3,\ b = -10, c = k$

Let one root be α so other root is $\frac{1}{\alpha}$.

Now product of roots $\alpha \times \frac{1}{\alpha} = \frac{c}{a}$

d309

$$1 = \frac{k}{3} \Rightarrow k = 3$$

Hence, value of k is 3.

54. Find the value of k such that the polynomial $x^2 - (k+6)x + 2(2k+1)$ has sum of its zeros equal to half of their product.

Ans : [Board 2019 Delhi]

Let α and β be the roots of given quadratic equation

$$x^2 - (k+6)x + 2(2k+1) = 0$$

Now sum of roots,. $\alpha + \beta = -\frac{-(k+6)}{1} = k+6$

Product of roots, $\alpha\beta = \frac{2(2k+1)}{1} = 2(2k+1)$

According to given condition,

d310

$$\alpha + \beta = \frac{1}{2}\alpha\beta$$

$$k+6 = \frac{1}{2}[2(2k+1)]$$

$$k+6 = 2k+1 \Rightarrow k = 5$$

Hence, the value of k is 5.

55. Find the nature of roots of the quadratic equation $2x^2 - 4x + 3 = 0$.

Ans : [Board 2019 OD]

We have $2x^2 - 4x + 3 = 0$

Comparing the given equation with $ax^2 + bx + c = 0$ we get $a = 2\ b = -4,\ c = 3$

d311

Now $D = b^2 - 4ac$

$$= (-4)^2 - 4(2) \times (3)$$

$$= -8 < 0 \text{ or } (-\text{ve})$$

Hence, the given equation has no real roots.

56. Find the roots of the quadratic equation $6x^2 - x - 2 = 0$.

Ans : [Board Term-2, 2012]

We have $6x^2 - x - 2 = 0$

$$6x^2 + 3x - 4x - 2 = 0 \quad (3 \times 4 = 2 \times 6)$$

$$3x(2x+1) - 2(2x+1) = 0$$

$$(2x+1)(3x-2) = 0$$

$$3x - 2 = 0 \text{ or } 2x+1 = 0$$

$$x = \frac{2}{3} \text{or } x = -\frac{1}{2}$$

Hence roots of equation are $\frac{2}{3}$ and $-\frac{1}{2}$.

d107

57. Find the roots of the following quadratic equation :

$15x^2 - 10\sqrt{6}\,x + 10 = 0$

Ans : [Board Term-2, 2012]

We have $15x^2 - 10\sqrt{6}\,x + 10 = 0$

d108

$$3x^2 - 2\sqrt{6}\,x + 2 = 0$$

$$3x^2 - \sqrt{6}\,x - \sqrt{6}\,x + 2 = 0$$

$$\sqrt{3}\,x(\sqrt{3}\,x - \sqrt{2}) - \sqrt{2}(\sqrt{3}\,x - \sqrt{2}) = 0$$

$$(\sqrt{3}\,x - \sqrt{2})(\sqrt{3}\,x - \sqrt{2}) = 0$$

Thus $x = \frac{\sqrt{2}}{\sqrt{3}}, \frac{\sqrt{2}}{\sqrt{3}}$

58. Solve the following quadratic equation for x :

$4\sqrt{3}\,x^2 + 5x - 2\sqrt{3} = 0$

Ans : [Board Term-2, 2013, 2012]

We have $4\sqrt{3}\,x^2 + 5x - 2\sqrt{3} = 0$

d109

$$4\sqrt{3}\,x^2 + 8x - 3x - 2\sqrt{3} = 0$$

$$4x(\sqrt{3}\,x + 2) - \sqrt{3}(\sqrt{3}\,x + 2) = 0$$

$$(\sqrt{3}\,x + 2)(4x - \sqrt{3}) = 0$$

Thus $x = -\frac{2}{\sqrt{3}}, \frac{\sqrt{3}}{4}$

59. Solve for x : $x^2 - (\sqrt{3}+1)x + \sqrt{3} = 0$

Ans : [Board Term-2 Foreign 2015]

We have

d110

$$x^2 - (\sqrt{3}+1)x + \sqrt{3} = 0$$

$$x^2 - \sqrt{3}\,x - 1x + \sqrt{3} = 0$$

$$x(x - \sqrt{3}) - 1(x - \sqrt{3}) = 0$$

$$(x - \sqrt{3})(x - 1) = 0$$

Thus $x = \sqrt{3}, x = 1$

60. Find the roots of the following quadratic equation :

$(x+3)(x-1) = 3\left(x - \frac{1}{3}\right)$

Ans : [Board Term-2 2012]

$$
\begin{aligned}
\text{We have} \quad (x+3)(x-1) &= 3\left(x-\frac{1}{3}\right) \\
x^2+3x-x-3 &= 3x-1 \\
x^2-x-2 &= 0 \\
x^2-2x+x-2 &= 0 \\
x(x-2)+1(x-2) &= 0 \\
(x-2)(x+1) &= 0
\end{aligned}
$$

Thus $x = 2, -1$

61. Find the roots of the following quadratic equation :

$$\frac{2}{5}x^2 - x - \frac{3}{5} = 0$$

Ans : [Board Term-2, 2012]

$$
\begin{aligned}
\text{We have} \quad \frac{2}{5}x^2 - x - \frac{3}{5} &= 0 \\
\frac{2x^2-5x-3}{5} &= 0 \\
2x^2-5x-3 &= 0 \\
2x^2-6x+x-3 &= 0 \\
2x(x-3)+1(x-3) &= 0 \\
(2x+1)(x-3) &= 0
\end{aligned}
$$

Thus $x = -\frac{1}{2},\ 3$

62. Solve the following quadratic equation for x :

$$4x^2 - 4a^2x + (a^4 - b^4) = 0$$

Ans : [Delhi Term-2, 2015]

We have $\quad 4x^2 - 4a^2x + (a^4 - b^4) = 0$

Comparing with $Ax^2 + Bx + C = 0$ we have

$$
\begin{aligned}
A &= 4, B = -4a^2, C = (a^4 - b^4) \\
x &= \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \\
&= \frac{4a^2 \pm \sqrt{(-4a^2)^2 - 4 \times 4(a^4 - b^4)}}{2 \times 4} \\
&= \frac{4a^2 \pm \sqrt{16a^2 - 16a^4 + 16b^4}}{8} \\
&= \frac{4a^2 \pm \sqrt{16b^4}}{8}
\end{aligned}
$$

or, $\quad x = \frac{4a^2 \pm 4b^2}{8} = \frac{a^2 \pm b^2}{2}$

Thus either $x = \frac{a^2+b^2}{2}$ or $x = \frac{a^2-b^2}{2}$

63. Solve the following quadratic equation for x :

$$9x^2 - 6b^2x - (a^4 - b^4) = 0$$

Ans : [Delhi Term-2, 2015]

We have $\quad 9x^2 - 6b^2x - (a^4 - b^4) = 0$

Comparing with $Ax^2 + Bx + C = 0$ we have

$$
\begin{aligned}
A &= 9, B = -6b^2, C = -(a^4 - b^4) \\
x &= \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \\
x &= \frac{6b^2 \pm \sqrt{(-6b^2)^2 - 4 \times 9 \times \{(a^4 - b^4)\}}}{2 \times 9} \\
&= \frac{6b^2 \pm \sqrt{36b^4 + 36a^4 - 36b^4}}{18} \\
&= \frac{6b^2 \pm \sqrt{36a^4}}{18} = \frac{6b^2 \pm 6a^2}{18}
\end{aligned}
$$

Thus $x = \frac{a^2+b^2}{3}, \frac{b^2-a^2}{3}$

64. Solve the following equation for x :

$$4x^2 + 4bx - (a^2 - b^2) = 0$$

Ans : [Board Term-2, OD 2012]

$$
\begin{aligned}
\text{We have} \quad 4x^2 + 4bx + b^2 - a^2 &= 0 \\
(2x+b)^2 - a^2 &= 0 \\
(2x+b+a)(2x+b-a) &= 0
\end{aligned}
$$

Thus $\quad x = \frac{-(a+b)}{2}$ and $x = \frac{a-b}{2}$

65. Solve the following quadratic equation for x :

$$x^2 - 2ax - (4b^2 - a^2) = 0$$

Ans : [Board Term-2, 2015]

$$
\begin{aligned}
\text{We have} \quad x^2 - 2ax - (4b^2 - a^2) &= 0 \\
x^2 - 2ax + a^2 - 4b^2 &= 0 \\
(x-a)^2 - (2b)^2 &= 0
\end{aligned}
$$

$$(x-a+2b)(x-a-2b) = 0$$

Thus $x = a-2b, x = a+2b$

66. Solve the quadratic equation, $2x^2+ax-a^2=0$ for x.

Ans : [Board Term-2 Delhi 2014]

We have $\quad 2x^2+ax-a^2=0$

Comparing with $Ax^2+Bx+C=0$ we have

$$A = 2, B = a, C = -a^2$$

d117

Now $\quad x = \dfrac{-B\pm\sqrt{B^2-4AC}}{2A}$

$$= \frac{-a\pm\sqrt{a^2-4\times2\times(-a^2)}}{2\times2}$$

$$= \frac{-a\pm\sqrt{a^2+8a^2}}{4}$$

$$= \frac{-a\pm\sqrt{9a^2}}{4} = \frac{-a\pm3a}{4}$$

$$x = \frac{-a+3a}{4}, \frac{-a-3a}{4}$$

Thus $x = \frac{a}{2}, -a$

67. Find the roots of the quadratic equation $4x^2-4px+(p^2-q^2)=0$

Ans : [Board Term-2, 2014]

We have $\quad 4x^2-4px+(p^2-q^2)=0$

Comparing with $ax^2+bx+c=0$ we get

$$a = 4, b = -4p, c = (p^2-q^2)$$

The roots are given by the quadratic formula,

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$= \frac{4p\pm\sqrt{16p^2-4\times4\times(p^2-q^2)}}{2\times4}$$

$$= \frac{4p\pm\sqrt{16p^2-16p^2+16q^2}}{8}$$

$$= \frac{4p\pm4q}{8}$$

d118

Thus roots are $\frac{p+q}{2}, \frac{p-q}{2}$.

68. Solve for x (in terms of a and b) :

$$\frac{a}{x-b}+\frac{b}{x-a} = 2, x\neq a, b$$

Ans : [Board Term-2 Foreign 2016]

We have $\quad \dfrac{a(x-a)+b(x-b)}{(x-b)(x-a)} = 2$

d120

$$a(x-a)+b(x-b) = 2[x^2-(a+b)x+ab]$$

$$ax-a^2+bx-b^2 = 2x^2-2(a+b)x+2ab$$

$$2x^2-3(a+b)x+(a+b)^2 = 0$$

$$2x^2-2(a+b)x-(a-b)x+(a+b)^2 = 0$$

$$[2x-(a+b)][x-(a+b)] = 0$$

Thus $\quad x = a+b, \frac{a+b}{2}$

69. Solve for x : $\sqrt{3}x^2-2\sqrt{2}x-2\sqrt{3}=0$

Ans : [Board Term-2 Foreign 2016]

We have

d121

$\sqrt{3}x^2-3\sqrt{2}x+\sqrt{2}x-2\sqrt{3}$
$=0$

$$\sqrt{3}x[x-\sqrt{6}]+\sqrt{2}[x-\sqrt{6}] = 0$$

$$(x-\sqrt{6})(\sqrt{3}x+\sqrt{2}) = 0$$

Thus $\quad x\ \sqrt{6}, -\sqrt{\frac{2}{3}}$

70. If $x=\frac{2}{3}$ and $x=-3$ are roots of the quadratic equation $ax^2+7x+b=0$, find the values of a and b.

Ans : [Board Term-2 Delhi 2016]

We have $\quad ax^2+7x+b=0 \quad (1)$

Substituting $x=\frac{2}{3}$ in above equation we obtain

$$\frac{4}{9}a+\frac{14}{3}+b = 0$$

d122

$$4a+42+9b = 0$$

$$4a+9b = -42 \quad (2)$$

and substituting $x=-3$ in (1) we obtain

$$9a-21+b = 0$$

$$9a+b = 21 \quad (3)$$

Solving (2) and (3), we get $a = 3$ and $b = -6$

71. Solve for x : $\sqrt{6x+7} - (2x-7) = 0$

Ans : [Board Term-2 OD 2016]

We have $\sqrt{6x+7} - (2x-7) = 0$

or, $\sqrt{6x+7} = (2x-7)$

Squaring both sides we get

$$6x+7 = (2x-7)^2$$
$$6x+7 = 4x^2 - 28x + 49$$
$$4x^2 - 34x + 42 = 0$$
$$2x^2 - 17x + 21 = 0$$
$$2x^2 - 14x - 3x + 21 = 0$$
$$2x(x-7) - 3(x-7) = 0$$
$$(x-7)(2x-3) = 0$$

Thus $x = 7$ and $x = \frac{2}{3}$.

72. Find the roots of $x^2 - 4x - 8 = 0$ by the method of completing square.

Ans : [Board Term-2, 2015]

We have $x^2 - 4x - 8 = 0$

$$x^2 - 4x + 4 - 4 - 8 = 0$$
$$(x-2)^2 - 12 = 0$$
$$(x-2)^2 = 12$$
$$(x-2)^2 = (2\sqrt{3})^2$$
$$x - 2 = \pm 2\sqrt{3}$$
$$x = 2 \pm 2\sqrt{3}$$

Thus $x = 2 + 2\sqrt{3},\ 2 - 2\sqrt{3}$

73. Solve for x : $\sqrt{2x+9} + x = 13$

Ans : [Board Term-2 OD 2016]

We have $\sqrt{2x+9} + x = 13$

$$\sqrt{2x+9} = 13 - x$$

Squaring both side we have

$$2x+9 = (13-x)^2$$
$$2x+9 = 169 + x^2 - 26x$$
$$0 = x^2 + 169 - 26x - 9 - 2x$$
$$x^2 - 28x + 160 = 0$$
$$x^2 - 20x - 8x + 160 = 0$$
$$x(x-20) - 8(x-20) = 0$$
$$(x-8)(x-20) = 0$$

Thus $x = 8$ and $x = 20$.

74. Find the roots of the quadratic equation $\sqrt{2}\,x^2 + 7x + 5\sqrt{2} = 0$

Ans : [Board Term-2 OD 2017]

We have $\sqrt{2}\,x^2 + 7x + 5\sqrt{2} = 0$

$$\sqrt{2}\,x^2 + 2x + 5x + 5\sqrt{2} = 0$$
$$\sqrt{2}\,x(x+\sqrt{2}) + 5(x+\sqrt{2}) = 0$$
$$(x+\sqrt{2})(\sqrt{2}\,x+5) = 0$$

Thus $x = -\sqrt{2}$ and $= -\frac{5}{\sqrt{2}} = -\frac{5}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = -\frac{5}{2}\sqrt{2}$

75. Find the value of k for which the roots of the quadratic equation $2x^2 + kx + 8 = 0$ will have the equal roots ?

Ans : [Board Term-2 OD Compt., 2017]

We have $2x^2 + kx + 8 = 0$

Comparing with $ax^2 + bx + c = 0$ we get

$$a = 2, b = k, \text{ and } c = 8$$

For equal roots, $D = 0$

$$b^2 - 4ac = 0$$
$$k^2 - 4 \times 2 \times 8 = 0$$
$$k^2 = 64$$
$$k = \pm\sqrt{64}$$

Thus $k = \pm 8$

76. Solve for x : $\sqrt{3}\,x^2 + 10x + 7\sqrt{3} = 0$

Ans : [Board Term-II Foreign 2017 Set-2]

We have $\sqrt{3}\,x^2 + 10x + 7\sqrt{3} = 0$

$$\sqrt{3}\,x^2 + 3x + 7x + 7\sqrt{3} = 0$$
$$\sqrt{3}x(x+\sqrt{3}) + 7(x+\sqrt{3}) = 0$$
$$(x+\sqrt{3})(\sqrt{3}\,x+7) = 0$$

Thus $x = -\sqrt{3}$ and $x = -\frac{7}{\sqrt{3}}$

77. Find k so that the quadratic equation $(k+1)x^2 - 2(k+1)x + 1 = 0$ has equal roots.

Ans : [Board Term-2, 2016]

We have $(k+1)x^2 - 2(k+1)x + 1 = 0$

Comparing with $Ax^2 + Bx + C = 0$ we get

$A = (k+1), B = -2(k+1), C = 1$

If roots are equal, then $D = 0$, i.e.

$$B^2 = 4AC$$

$$4(k+1)^2 = 4(k+1)$$

$$k^2 + 2k + 1 = k + 1$$

$$k^2 + k = 0$$

$$k(k+1) = 0$$

$$k = 0, -1$$

$k = -1$ does not satisfy the equation, thus $k = 0$

d194

78. If 2 is a root of the equation $x^2 + kx + 12 = 0$ and the equation $x^2 + kx + q = 0$ has equal roots, find the value of q.

Ans : [Board Term 2 SQP 2016]

We have $x^2 + kx + 12 = 0$

If 2 is the root of above equation, it must satisfy it.

$$(2)^2 + 2k + 12 = 0$$

, $$2k + 16 = 0$$

$$k = -8$$

d195

Substituting $k = -8$ in $x^2 + kx + q = 0$ we have

$$x^2 - 8x + q = 0$$

For equal roots,

$$(-8)^2 - 4(1)q = 0$$

$$64 - 4q = 0$$

$$4q = 64 \Rightarrow q = 16$$

79. Find the values of k for which the quadratic equation $9x^2 - 3kx + k = 0$ has equal roots.

Ans : [Board Term-2 Delhi, OD 2014]

We have $9x^2 - 3kx + k = 0$

Comparing with $ax^2 + bx + c = 0$ we get

d196

$$a = 9, b = -3k, c = k$$

Since roots of the equation are equal, $b^2 - 4ac = 0$

$$(-3k)^2 - (4 \times 9 \times k) = 0$$

$$9k^2 - 36k = 0$$

$$k^2 - 4k = 0$$

$$k(k-4) = 0 \Rightarrow k = 0 \text{ or } k = 4$$

Hence, $k = 4$.

80. If the equation $kx^2 - 2kx + 6 = 0$ has equal roots, then find the value of k.

Ans : [Board Term-2, 2012]

We have $kx^2 - 2kx + 6 = 0$

Comparing with $ax^2 + bx + c = 0$ we get

$$a = k, b = -2k, c = 6$$

Since roots of the equation are equal, $b^2 - 4ac = 0$

$$(-2k)^2 - 4(k)(6) = 0$$

$$4k^2 - 24k = 0$$

$$4k(k-6) = 0$$

$$k = 0, 6$$

d197

But $k \neq 0$, as coefficient of x^2 can't be zero.

Thus $k = 6$

81. Find the positive value of k for which $x^2 - 8x + k = 0$, will have real roots.

Ans : [Board Term-2, 2014]

We have $x^2 - 8x + k = 0$

Comparing with $Ax^2 + Bx + C = 0$ we get

d214

$$A = 1, B = -8, C = k$$

Since the given equation has real roots, $B^2 - 4AC > 0$

$$(-8)^2 - 4(1)(k) \geq 0$$

$$64 - 4k \geq 0$$

$$16 - k \geq 0$$

$$16 \geq k$$

Thus $k \leq 16$

82. Find the values of p for which the quadratic equation $4x^2 + px + 3 = 0$ has equal roots.

Ans : [Board Term-2, 2014]

We have $4x^2 + px + 3 = 0$

Comparing with $ax^2 + bx + c = 0$ we get

$$a = 4, b = p, c = 3$$

d198

Since roots of the equation are equal,

$$b^2 - 4ac = 0$$
$$p^2 - 4 \times 4 \times 3 = 0$$
$$p^2 - 48 = 0$$
$$p^2 = 48$$
$$p = \pm 4\sqrt{3}$$

83. Find the nature of the roots of the quadratic equation : $13\sqrt{3}\,x^2 + 10x + \sqrt{3} = 0$

Ans : [Board Term-2, 2012]

We have $13\sqrt{3}\,x^2 + 10x + \sqrt{3} = 0$

Comparing with $ax^2 + bx + c = 0$ we get

d199

$$a = 13\sqrt{3}, b = 10, c = \sqrt{3}$$
$$b^2 - 4ac = (10)^2 - 4(13\sqrt{3})(\sqrt{3})$$
$$= 100 - 156$$
$$= -56$$

As $D < 0$, the equation has not real roots.

THREE MARKS QUESTIONS

84. Solve the following equation: $\frac{1}{x} - \frac{1}{x-2} = 3,\ x \neq 0, 2$

Ans : [Board 2020 SQP Standard]

We have $\frac{1}{x} - \frac{1}{x-2} = 3 \qquad (x \neq 0, 2)$

d291

$$\frac{x - 2 - x}{x(x-2)} = 3$$
$$\frac{-2}{x(x-2)} = 3$$
$$3x(x-2) = -2$$
$$3x^2 - 6x + 2 = 0$$

Comparing it by $ax^2 + bx + c$, we get $a = 3$, $b = -6$ and $c = 2$.

Now, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(2)}}{2(3)}$$
$$= \frac{6 \pm \sqrt{36 - 24}}{6} = \frac{6 \pm \sqrt{12}}{6}$$
$$= \frac{6 \pm 2\sqrt{3}}{6}$$
$$= \frac{3 + \sqrt{3}}{3}, \frac{3 - \sqrt{3}}{3}$$

85. Find the values of k for which the quadratic equation $x^2 + 2\sqrt{2k}\,x + 18 = 0$ has equal roots.

Ans : [Board 2020 SQP Standard]

We have $x^2 + 2\sqrt{2k}\,x + 18 = 0$

Comparing it by $ax^2 + bx + c$, we get $a = 1$, $b = 2\sqrt{2k}$ and $c = 18$.

Given that, equation $x^2 + 2\sqrt{2}\,kx + 18 = 0$ has equal roots.

$$b^2 - 4ac = 0$$
$$(2\sqrt{2}\,k)^2 - 4 \times 1 \times 18 = 0$$

d293

$$8k^2 - 72 = 0$$
$$8k^2 = 72$$
$$k^2 = \frac{72}{8} = 9$$
$$k = \pm 3$$

86. If α and β are the zeroes of the polynomial $f(x) = x^2 - 4x - 5$ then find the value of $\alpha^2 + \beta^2$

Ans : [Board 2020 Delhi Basic]

We have $p(x) = x^2 - 4x - 5$

Comparing it by $ax^2 + bx + c$, we get $a = 1$, $b = -4$ and $c = -5$

Since, given α and β are the zeroes of the polynomial,

Sum of zeroes, $\alpha + \beta = -\frac{b}{a} = \frac{-(-4)}{1} = 4$

and product of zeroes, $\alpha\beta = \frac{c}{a} = \frac{-5}{1} = -5$

Now, $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

d294

$$= (4)^2 - 2(-5)$$
$$= 16 + 10 = 26$$

87. Find the quadratic polynomial, the sum and product of whose zeroes are -3 and 2 respectively. Hence find the zeroes.

Ans : [Board 2020 OD Basic]

Sum of zeroes $\alpha+\beta = -3$ ...(1)

and product of zeroes $\alpha\beta = 2$

Thus quadratic equation is

$$x^2-(\alpha+\beta)x+\alpha\beta = 0$$

$$x^2-(-3)x+2 = 0$$

$$x^2+3x+2 = 0$$

d296

Thus quadratic equation is $x^2+3x+2=0$.

Now above equation can be written as

$$x^2+2x+x+2 = 0$$

$$x(x+2)+(x+2) = 0$$

$$(x+2)(x+1) = 0$$

Hence, zeroes are -2 and -1.

88. If α and β are the zeroes of the polynomial $f(x)=5x^2-7x+1$ then find the value of $\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)$

Ans : [Board 2020 OD Basic]

Since, α and β are the zeroes of the quadratic polynomial $f(x)=5x^2-7x+1$,

Sum of zeros, $\alpha+\beta = -\left(\frac{-7}{5}\right)=\frac{7}{5}$ ...(1)

Product of zeros, $\alpha\beta = \frac{1}{5}$ ...(2)

Now, $\frac{\alpha}{\beta}+\frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta}=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$

$$= \frac{\left(\frac{7}{5}\right)^2-2\times\frac{1}{5}}{\frac{1}{5}}$$

$$= \frac{49-2\times 5}{5} = \frac{39}{5}$$

d300

89. Find the zeroes of the quadratic polynomial $6x^2-3-7x$ and verify the relationship between the zeroes and the coefficients.

Ans : [Board 2020 Delhi Basic]

We have $p(x) = 6x^2-3-7x$

For zeroes of polynomial, $p(x)=0$,

$$6x^2-7x-3 = 0$$

$$6x^2-9x+2x-3 = 0$$

$$3x(2x-3)+1(2x-3) = 0$$

$$(2x-3)(3x+1) = 0$$

d301

Thus $2x-3 = 0$ and $3x+1 = 0$

Hence $x = \frac{3}{2}$ and $x = -\frac{1}{3}$

Therefore $\alpha=\frac{3}{2}$ and $\beta=-\frac{1}{3}$ are the zeroes of the given polynomial.

Verification :

Sum of zeroes, $\alpha+\beta = \frac{3}{2}+\left(-\frac{1}{3}\right)$

$$= \frac{3}{2}-\frac{1}{3}=\frac{7}{6}$$

$$= -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$$

and product of zeroes $\alpha\beta = \left(\frac{3}{2}\right)\left(-\frac{1}{3}\right) = -\frac{1}{2}$

$$= \frac{\text{constant term}}{\text{coefficient of } x^2}$$

90. Find the zeroes of the quadratic polynomial $x^2+7x+10$, and verify the relationship between the zeroes and the coefficients.

Ans : [Board 2020 Delhi Basic]

Let, $p(x) = x^2+7x+10$

For zeroes of polynomial $p(x) = 0$,

$$x^2+7x+10 = 0$$

d302

$$x^2+5x+2x+10 = 0$$
$$x(x+5)+2(x+5) = 0$$
$$(x+5)(x+2) = 0$$

So, $x = -2$ and $x = -5$

Therefore, $\alpha = -2$ and $\beta = -5$ are the zeroes of the given polynomial.

Verification:

Sum of zeroes, $\alpha+\beta = -2+(-5)$

$$= -7 = \frac{-7}{1}$$

$$= -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$$

and product of zeroes $\alpha\beta = (-2)(-5) = 10$

$$= \frac{10}{1}$$

$$= \frac{\text{contant term}}{\text{coefficient of } x^2}$$

91. Solve for x : $\frac{1}{x+4}-\frac{1}{x+7} = \frac{11}{30}\ x \neq -4,\ -7.$

Ans : [Board 2020 OD Standard]

We have $\frac{1}{x+4}-\frac{1}{x+7} = \frac{11}{30}$

d305

$$\frac{x+7-x-4}{(x+4)(x+7)} = \frac{11}{30}$$

$$\frac{3}{x^2+4x+7x+28} = \frac{11}{30}$$

$$\frac{3}{x^2+11x+28} = \frac{11}{30}$$

$$11x^2+121x+308 = 90$$

$$11x^2+121x+218 = 0$$

Comparing with $ax^2+bx+c = 0$, we get $a = 11$, $b = 121$ and $c = 218$ we obtain

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$= \frac{-121\pm\sqrt{14641-9592}}{22}$$

$$x = \frac{-121\pm\sqrt{5049}}{22}$$

$$= \frac{-121\pm 71.06}{22}$$

$$x = \frac{-49.94}{22}, \frac{-192.06}{22}$$

$$x = -2.27, -8.73.$$

92. Solve for x :

$\frac{x+1}{x-1}+\frac{x-2}{x+2} = 4-\frac{2x+3}{x-2}; x \neq 1, -2, 2$

Ans : [Board Term-2 OD 2016]

We have $\frac{x+1}{x-1}+\frac{x-2}{x+2} = 4-\frac{2x+3}{x-2}$

$$\frac{x^2+3x+2+x^2-3x+2}{x^2+x-2} = \frac{4x-8-2x-3}{x-2}$$

$$\frac{2x^2+4}{x^2+x-2} = \frac{2x-11}{x-2}$$

$$(2x^2+4)(x-2) = (2x-11)(x^2+x-2)$$

$$5x^2+19x-30 = 0$$

$$(5x-6)(x+5) = 0$$

$$x = -5, \frac{6}{5}$$

d131

93. Solve for x :

$\frac{2x}{x-3}+\frac{1}{2x+3}+\frac{3x+9}{(x-3)(2x+3)} = 0, x \neq 3, -\frac{3}{2}$

Ans : [Board Term-2, Delhi 2016]

d132

We have

$$\frac{2x}{x-3}+\frac{1}{2x+3}+\frac{3x+9}{(x-3)(2x+3)} = 0$$

$$2x(2x+3)+(x-3)+(3x+9) = 0$$

$$4x^2+6x+x-3+3x+9 = 0$$

$$4x^2+10x+6 = 0$$

$$2x^2+5x+3 = 0$$

$$(x+1)(2x+3) = 0$$

Thus $x = -1, x = -\frac{3}{2}$

94. Solve for x : $\frac{1}{x}+\frac{2}{2x-3} = \frac{1}{x-2}, x \neq 0, \frac{2}{3}, 2.$

Ans : [Board Term-2, Foreign 2016]

We have $\frac{1}{x}+\frac{2}{2x-3} = \frac{1}{x-2}$

$$\frac{2x-3+2x}{x(2x-3)} = \frac{1}{x-2}$$

$$\frac{4x-3}{x(2x-3)} = \frac{1}{x-2}$$

d133

$$\begin{aligned}(x-2)(4x-3) &= 2x^2-3x\\ 4x^2-11x+6 &= 2x^2-3x\\ 2x^2-8x+6 &= 0\\ x^2-4x+3 &= 0\\ (x-1)(x-3) &= 0\end{aligned}$$

Thus $x = 1,3$

95. Solve the following quadratic equation for x :

$$x^2+\left(\frac{a}{a+b}+\frac{a+b}{a}\right)x+1=0$$

Ans : [Board Term-2 OD 2016]

We have

$$\begin{aligned}x^2+\left(\frac{a}{a+b}+\frac{a+b}{a}\right)x+1 &= 0\\ x^2+\frac{a}{a+b}x+\frac{a+b}{a}x+1 &= 0\\ x\left(x+\frac{a}{a+b}\right)+\frac{a+b}{a}\left(x+\frac{a}{a+b}\right) &= 0\\ \left(x+\frac{a}{a+b}\right)\left(x+\frac{a+b}{a}\right) &= 0\end{aligned}$$

d134

Thus $x = \dfrac{-a}{a+b}, \dfrac{-(a+b)}{a}$

96. Solve for x :

$$\frac{1}{(x-1)(x-2)}+\frac{1}{(x-2)(x-3)}=\frac{2}{3}; x\neq 1,2,3$$

Ans : [Board Term-2 OD 2016]

We have

$$\begin{aligned}\frac{1}{(x-1)(x-2)}+\frac{1}{(x-2)(x-3)} &= \frac{2}{3}\\ \frac{x-3+x-1}{(x-1)(x-2)(x-3)} &= \frac{2}{3}\\ \frac{2x-4}{(x-1)(x-2)(x-3)} &= \frac{2}{3}\\ \frac{2(x-2)}{(x-1)(x-2)(x-3)} &= \frac{2}{3}\\ \frac{2}{(x-1)(x-3)} &= \frac{2}{3}\\ 3 &= (x-1)(x-3)\\ x^2-4x+3 &= 3\\ x^2-4x &= 0\\ x(x-4) &= 0\end{aligned}$$

d135

Thus $x = 0$ or $x = 4$

97. Solve for x : $\sqrt{3}\,x^2-2\sqrt{2}\,x-2\sqrt{3}=0$

Ans : [Board Term-2, OD 2015, Foreign 2014]

We have

$$\begin{aligned}\sqrt{3}\,x^2-2\sqrt{2}\,x-2\sqrt{3} &= 0\\ \sqrt{3}\,x^2-[3\sqrt{2}-\sqrt{2}]x-2\sqrt{3} &= 0\\ \sqrt{3}\,x^2-3\sqrt{2}\,x+\sqrt{2}\,x-2\sqrt{3} &= 0\\ \sqrt{3}\,x^2-\sqrt{3}\sqrt{3}\sqrt{2}\,x+\sqrt{2}\,x-\sqrt{2}\sqrt{2}\sqrt{3} &= 0\\ \sqrt{3}\,x(x-\sqrt{3}\sqrt{2})+\sqrt{2}\,(x-\sqrt{2}\sqrt{3}) &= 0\\ \sqrt{3}\,x[x-\sqrt{6}]+\sqrt{2}[x-\sqrt{6}] &= 0\\ (x-\sqrt{6})(\sqrt{3}\,x+\sqrt{2}) &= 0\end{aligned}$$

d136

Thus $x = \sqrt{6} = -\sqrt{\dfrac{2}{3}}$

98. Solve for x : $2x^2+6\sqrt{3}\,x-60=0$

Ans : [Board Term-2, OD 2015]

We have

$$\begin{aligned}2x^2+6\sqrt{3}\,x-60 &= 0\\ x^2+3\sqrt{3}\,x-30 &= 0\\ x^2+5\sqrt{3}\,x-2\sqrt{3}\,x-30 &= 0\\ x(x+5\sqrt{3})-2\sqrt{3}\,(x+5\sqrt{3}) &= 0\\ (x+5\sqrt{3})(x-2\sqrt{3}) &= 0\end{aligned}$$

d137

Thus $x = -5\sqrt{3}, 2\sqrt{3}$

99. Solve for x : $x^2+5x-(a^2+a-6)=0$

Ans : [Board Term-2 Foreign Set I 2015]

We have

$$\begin{aligned}x^3+5x-(a^2+a-6) &= 0\\ x &= \frac{-b\pm\sqrt{b^2-4ac}}{2a}\end{aligned}$$

d138

Thus

$$\begin{aligned}x &= \frac{-5\pm\sqrt{25+4(a^2+a-6)}}{2}\\ &= \frac{-5\pm\sqrt{25+4a^2+4a-24}}{2}\\ &= \frac{-5\pm\sqrt{4a^2+4a+1}}{2}\\ &= \frac{-5\pm(2a+1)}{2}\\ &= \frac{2a-4}{2}, \frac{-2a-6}{2}\end{aligned}$$

Thus $x = a-2,\ x = -(a+3)$

100. Solve for x : $x^2-(2b-1)x+(b^2-b-20)=0$

Ans : [Board Term-2 Foreign 2015]

We have $x^2-(2b-1)x+(b^2-b-20)=0$

Comparing with $Ax^2+Bx+C=0$ we have

$$A=1, B=-(2b-1), C=(b^2-b-20)$$

$$x=\frac{-B\pm\sqrt{B^2-4AC}}{2A}$$

$$x=\frac{(2b-1)\pm\sqrt{(2b-1)^2-4(b^2-b-20)}}{2}$$

$$=\frac{(2b-1)\pm\sqrt{4b^2-4b+1-4b^2+4b+80}}{2}$$

$$=\frac{(2b-1)\pm\sqrt{81}}{2}=\frac{(2b-1)\pm 9}{2}$$

$$=\frac{2b+8}{2},\frac{2b-10}{2}$$

$$=b+4,\ b-5$$

d139

Thus $x=b+4$ and $x=b-5$

101. Solve for x : $\frac{16}{x}-1=\frac{15}{x+1}$; $x\neq 0,-1$

Ans : [Board Term-2, OD 2014]

We have $$\frac{16}{x}-1=\frac{15}{x+1}$$

$$\frac{16}{x}-\frac{15}{x+1}=1$$

$$16(x+1)-15x=x(x+1)$$

$$16x+16-15x=x^2+x$$

$$x+16=x^2+x$$

$$x^2-16=0$$

$$x^2=16$$

$$x=\pm 4$$

d140

Thus $x=-4$ and $x=+4$

102. Solve the quadratic equation $(x-1)^2-5(x-1)-6=0$

Ans : [Board Term-2, 2015]

We have $$(x-1)^2-5(x-1)-6=0$$

$$x^2-2x+1-5x+5-6=0$$

$$x^2-7x+6-6=0$$

$$x^2-7x=0$$

$$x(x-7)=0$$

d141

Thus $x=0,7$

103. Solve the equation for x : $\frac{4}{x}-3=\frac{5}{2x+3}$; $x\neq 0,\frac{-3}{2}$

Ans : [Board Term-2 Delhi 2014]

We have $$\frac{4}{x}-3=\frac{5}{2x+3}$$

$$\frac{4}{x}-\frac{5}{2x+3}=3$$

$$\frac{4(2x+3)-5x}{x(2x+3)}=3$$

$$8x+12-5x=3x(2x+3)$$

$$3x+12=6x^2+9x$$

$$6x^2+6x-12=0$$

$$x^2+x-2=0$$

$$x^2+2x-x-2=0$$

$$x(x+2)-(x+2)=0$$

$$(x+2)(x-1)=0$$

d142

Thus $x=1,-2$

104. Find the roots of the equation $2x^2+x-4=0$, by the method of completing the squares.

Ans : [Board Term-2, OD 2014]

We have $$2x^2+x-4=0$$

$$x^2+\frac{x}{2}-2=0$$

$$x^2+2x\left(\frac{1}{4}\right)-2=0$$

d143

Adding and subtracting $\left(\frac{1}{4}\right)^2$, we get

$$x^2+2x\left(\frac{1}{4}\right)+\left(\frac{1}{4}\right)^2-\left(\frac{1}{4}\right)^2-2=0$$

$$\left(x+\frac{1}{4}\right)^2-\left(\frac{1}{16}+2\right)=0$$

$$\left(x+\frac{1}{4}\right)^2-\left(\frac{1+32}{16}\right)=0$$

$$\left(x+\frac{1}{4}\right)^2-\frac{33}{16}=0$$

$$\left(x+\frac{1}{4}\right)^2=\frac{33}{16}$$

$$\left(x+\frac{1}{4}\right)=\pm\frac{\sqrt{33}}{4}$$

Thus roots are $x = \frac{-1+\sqrt{33}}{4}, \frac{-1-\sqrt{33}}{4}$

105. Solve for x : $9x^2 - 6ax + (a^2 - b^2) = 0$

Ans : [Board Term-2 2012]

We have $9x^2 - 6ax + a^2 - b^2 = 0$

Comparing with $Ax^2 + Bx + C = 0$ we have

$$A = 9, B = -6a, C = (a^2 - b^2)$$

d144

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$x = \frac{6a \pm \sqrt{(-6a)^2 - 4 \times 9k(a^2 - b^2)}}{2 \times 9}$$

$$= \frac{6a \pm \sqrt{36a^2 - 36a^2 + 36b^2}}{18}$$

$$= \frac{6a \pm \sqrt{36b^2}}{18} = \frac{6a \pm 6b}{18}$$

$$= \frac{a \pm b}{3}$$

$$x = \frac{(a+b)}{3}, \frac{(a-b)}{3}$$

Thus $x = \frac{a+b}{3}, x = \frac{a-b}{3}$

106. Solve the equation $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$, $x \neq -4, 7$ for x.

Ans : [Board Term-2, 2012]

We have,

$$\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$$

d145

$$\frac{x-7-x-4}{(x+4)(x-7)} = \frac{11}{30}$$

$$\frac{-11}{(x+4)(x-7)} = \frac{11}{30}$$

$$\frac{-1}{(x+4)(x-7)} = \frac{1}{30}$$

$$(x+4)(x-7) = -30$$

$$x^2 - 3x - 28 = -30$$

$$x^2 - 3x + 2 = 0$$

$$x^2 - 2x - x + 2 = 0$$

$$(x-1)(x-2) = 0$$

Thus $x = 1, 2$.

107. Find the roots of the quadratic equation :

$$a^2b^2x^2 + b^2x - a^2x - 1 = 0$$

Ans : [Board Term-2, 2012]

We have

$$a^2b^2x^2 + b^2x - a^2x - 1 = 0$$

$$b^2x(a^2x+1) - 1(a^2x+1) = 0$$

$$(b^2x-1)(a^2x+1) = 0$$

d146

$$x = \frac{1}{b^2} \text{ or } x = -\frac{1}{a^2}$$

Hence, roots are $\frac{1}{b^2}$ and $-\frac{1}{a^2}$.

108. If $(x^2+y^2)(a^2+b^2) = (ax+by)^2$, prove that $\frac{x}{a} = \frac{y}{b}$

Ans : [Board Term-2, 2014]

We have

$$(x^2+y^2)(a^2+b^2) = (ax+by)^2$$

$$x^2a^2 + x^2b^2 + y^2a^2 + y^2b^2 = a^2x^2 + b^2y^2 + 2abxy$$

$$x^2b^2 + y^2a^2 - 2abxy = 0$$

$$(xb - ya)^2 = 0$$

$$xb = ya$$

d152

Thus $\frac{x}{a} = \frac{y}{b}$

Hence Proved.

109. Solve the following quadratic equation for x :

$$p^2x^2 + (p^2 - q^2)x - q^2 = 0$$

Ans : [Board Term-2, 2012]

We have $p^2x^2 + (p^2 - q^2)x - q^2 = 0$

Comparing with $ax^2 + bx + c = 0$ we get

$$a = p^2, b = p^2 - q^2, c = -q^2$$

d147

The roots are given by the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(p^2-q^2) - \sqrt{(p^2-q^2)^2 - 4(p^2)(-q^2)}}{2p^2}$$

$$= \frac{-(p^2-q^2) - \sqrt{p^4 + q^4 - 2p^2q^2 + 4p^2q^2}}{2p^2}$$

$$= \frac{-(p^2-q^2) - \sqrt{p^4 + q^4 + 2p^2q^2}}{2p^2}$$

$$= \frac{-(p^2-q^2)-\sqrt{(p^2+q^2)^2}}{2p^2}$$

$$= \frac{-(p^2-q^2)\pm(p^2+q^2)}{2p^2}$$

Thus $\quad x = \frac{-(p^2-q^2)+(p^2+q^2)}{2p^2} = \frac{2q^2}{2p^2} = \frac{q^2}{p^2}$

and $\quad x = \frac{-(p^2-q^2)-(p^2+q^2)}{2p^2} = \frac{-2p^2}{2p^2} = -1$

Hence, roots are $\frac{q^2}{p^2}$ and -1.

110. Solve the following quadratic equation for x :

$$9x^2 - 9(a+b)x + 2a^2 + 5ab + 2b^2 = 0$$

Ans : [Board Term-2, Foreign 2016]

We have $\quad 9x^2 - 9(a+b)x + 2a^2 + 5ab + 2b^2 = 0$

Now $\quad 2a^2 + 5ab + 2b^2 = 2a^2 + 4ab + ab + 2b^2$

$$= 2a[a+2b] + b[a+2b]$$

$$= (a+2b)(2a+b)$$

Hence the equation becomes

$$9x^2 - 9(a+b)x + (a+2b)(2a+b) = 0$$

$$9x^2 - 3[3a+3b]x + (a+2b)(2a+b) = 0$$

$$9x^2 - 3[(a+2b)+(2a+b)]x + (a+2b)(2a+b) = 0$$

$$9x^2 - 3(a+2b)x - 3(2a+b)x + (a+2b)(2a+b) = 0$$

$$3x[3x-(a+2b)] - (2a+b)[3x-(a+2b)] = 0$$

$$[3x-(a+2b)][3x-(2a+b)] = 0$$

$$3x-(2a+b) = 0$$

$$x = \frac{a+2b}{3}$$

d148

$$3x-(2a+b) = 0$$

$$x = \frac{2a+b}{3}$$

Hence, roots are $\frac{a+2b}{3}$ and $\frac{2a+b}{3}$.

111. Solve for x : $x^2 + 6x - (a^2 + 2a - 8)$

Ans : [Board Term-2, Foreign 2015]

We have $\quad x^2 + 6x - (a^2+2a-8) = 0$

Comparing with $Ax^2 + Bx + C = 0$ we get

$$A = 1, B = 6, C = (a^2+2a-8)$$

The roots are given by the quadratic formula

$$x = \frac{-B \pm \sqrt{B^2-4AC}}{2A}$$

d149

$$= \frac{-6 \pm \sqrt{36+4(a^2+2a-8)}}{2}$$

$$= \frac{-6 \pm (2a+2)}{2}$$

Thus $\quad x = \frac{-6+(2a+2)}{2} = a-2$

and $\quad x = \frac{-6-(2a+2)}{2} = -a-4$

Thus $x = a-2, -a-4$

112. If the roots of the equation $(a^2+b^2)x^2 - 2(ac+bd)x + (c^2+d^2) = 0$ are equal, prove that $\frac{a}{b} = \frac{c}{d}$.

Ans : [Board Term-2 2016]

We have $(a^2+b^2)x^2 - 2(ac+bd)x + (c^2+d^2) = 0$

Comparing with $Ax^2 + Bx + C = 0$ we get

$A = (a^2+b^2), B = -2(ac+bd), C = (c^2+d^2)$

If roots are equal, $D = B^2 - 4AC = 0$

or $\quad B^2 = 4AC$

d157

Now $\quad [-2(ac+bd)]^2 = 4(a^2+b^2)(c^2+d^2)$

$$4(a^2c^2 + 2abcd + b^2d^2) = 4(a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2)$$

$$a^2c^2 + 2abcd + b^2d^2 = a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2$$

$$2abcd = a^2d^2 + b^2c^2$$

$$0 = a^2d^2 - 2abcd + b^2c^2$$

$$0 = (ad-bc)^2$$

$$0 = ad - bc$$

Thus $\quad ad = bc$

$$\frac{a}{b} = \frac{c}{d}$$

Hence Proved

113. If 2 is a root of the quadratic equation $3x^2 + px - 8 = 0$ and the quadratic equation $4x^2 - 2px + k = 0$ has equal roots, find k.

Ans : [Board Term-2 Foreign 2014]

We have $\quad 3x^2 + px - 8 = 0$

Since 2 is a root of above equation, it must satisfy it.

Substituting $x = 2$ in $3x^2 + px - 8 = 0$ we have

$$12 + 2p - 8 = 0$$

$$p = -2$$

d200

Since $4x^2 - 2px + k = 0$ has equal roots,

or $4x^2 + 4x + k = 0$ has equal roots,

$$D = b^2 - 4ac = 0$$

$$4^2 - 4(4)(k) = 0$$

$$16 - 16k = 0$$

$$16k = 16$$

Thus $k = 1$

114. For what value of k, the roots of the quadratic equation $kx(x - 2\sqrt{5}) + 10 = 0$ are equal ?

Ans : [Board Term-2 Delhi 2014, 2013]

We have $kx(x - 2\sqrt{5}) + 10 = 0$

or, $kx^2 - 2\sqrt{5}\,kx + 10 = 0$

Comparing with $ax^2 + bx + c = 0$ we get

$$a = k, b = -2\sqrt{5}\,k \text{ and } c = 10$$

Since, roots are equal, $D = b^2 - 4ac = 0$

$$(-2\sqrt{5}\,k)^2 - 4 \times k \times 10 = 0$$

$$20k^2 - 40k = 0$$

$$20k(k-2) = 0$$

$$k(k-2) = 0$$

Since $k \neq 0$, we get $k = 2$

115. Find the nature of the roots of the following quadratic equation. If the real roots exist, find them : $3x^2 - 4\sqrt{3}\,x + 4 = 0$

Ans : [Board Term-2, 2012]

We have $3x^2 - 4\sqrt{3}\,x + 4 = 0$

Comparing with $ax^2 + bx + c = 0$ we get

$$a = 3,\ b = -4\sqrt{3},\ c = 4$$

$$b^2 - 4ac = (-4\sqrt{3})^2 - 4(3)(4)$$

$$= 48 - 48 = 0$$

Thus roots are real and equal.

Roots are $\left(-\dfrac{b}{2a}\right), \left(-\dfrac{b}{2a}\right)$ or $\dfrac{2\sqrt{3}}{3}, \dfrac{2\sqrt{3}}{3}$

116. Determine the positive value of k for which the equation $x^2 + kx + 64 = 0$ and $x^2 - 8x + k = 0$ will both have real and equal roots.

Ans : [Board Term-2, 2012, 2014]

We have $x^2 + kx + 64 = 0$

Comparing with $ax^2 + bx + c = 0$ we get

$$a = 1, b = k, c = 64$$

For real and equal roots, $b^2 - 4ac = 0$

Thus $k^2 - 4 \times 1 \times 64 = 0$

$$k^2 - 256 = 0$$

$$k = \pm 16 \quad (1)$$

Now for equation $x^2 - 8x + k = 0$ we have

$$b^2 - 4ac = 0$$

$$(-8)^2 - 4 \times 1 \times k = 0$$

$$64 = 4k$$

$$k = \frac{64}{4} = 16 \quad (2)$$

From (1) and (2), we get $k = 16$. Thus for $k = 16$, given equations have equal roots.

117. Find that non-zero value of k, for which the quadratic equation $kx^2 + 1 - 2(k-1)x + x^2 = 0$ has equal roots. Hence find the roots of the equation.

Ans : [Board Term-2 Delhi 2015]

We have $kx^2 + 1 - 2(k-1)x + x^2 = 0$

$$(k+1)x^2 - 2(k-1)x + 1 = 0$$

Comparing with $ax^2 + bx + c = 0$ we get

$$a = k+1, b = -2(k-1), c = 1$$

For real and equal roots, $b^2 - 4ac = 0$

$$4(k-1)^2 - 4(k+1) \times 1 = 0$$

$$4k^2 - 8k + 4 - 4k - 4 = 0$$

$$4k^2 - 12k = 0$$

$$4k(k-3) = 0$$

As k can't be zero, thus $k = 3$.

118. Find the value of k for which the quadratic equation $(k-2)x^2 + 2(2k-3)x + (5k-6) = 0$ has equal roots.

Ans : [Board Term-2, 2015]

We have $(k-2)x^2 + 2(2k-3)x + (5k-6) = 0$

Comparing with $ax^2 + bx + c = 0$ we get

$$a = k-2, b = 2(2k-3), c = (5k-6)$$

For real and equal roots, $b^2 - 4ac = 0$

$$\{2(2k-3)\}^2 - 4(k-2)(5k-6) = 0$$
$$4(4k^2 - 12k + 9) - 4(k-2)(5k-6) = 0$$
$$4k^2 - 12k + 9 - 5k^2 + 6k + 10k - 12 = 0$$
$$k^2 - 4k + 3 = 0$$
$$k^2 - 3k - k + 3 = 0$$
$$k(k-3) - 1(k-3) = 0$$
$$(k-3)(k-1) = 0$$

Thus $k = 1, 3$

119. If the roots of the quadratic equation $(a-b)x^2 + (b-c)x + (c-a) = 0$ are equal, prove that $2a = b + c$.

Ans : [Board Term-2 Delhi 2016]

We have $(a-b)x^2 + (b-c)x + (c-a) = 0$

Comparing with $ax^2 + bx + c = 0$ we get

$$a = (a-b,),\ b = (b-c),\ c = c-a$$

For real and equal roots, $b^2 - 4ac = 0$

$$(b-c)^2 - 4(a-b)(c-a) = 0$$
$$b^2 + c^2 - 2bc - 4(ac - a^2 - bc + ab) = 0$$
$$b^2 + c^2 - 2bc - 4ac + 4a^2 + 4bc - 4ab = 0$$
$$4a^2 + b^2 + c^2 + 2bc - 4ab - 4ac = 0$$

Using $a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a+b+c)^2$,

$$(-2a + b + c)^2 = 0$$
$$-2a + b + c = 0$$

Hence, $b + c = 2a$

120. If the quadratic equation, $(1+a^2)b^2x^2 + 2abcx + (c^2 - m^2) = 0$ in x has equal roots, prove that $c^2 = m^2(1+a^2)$

Ans : [Board Term-2, 2014]

We have $(1+a^2)b^2x^2 + 2abcx + (c^2 - m^2) = 0$

Comparing with $Ax^2 + Bx + C = 0$ we get

$$A = (1+a^2)b^2, B = 2abc, C = (c^2 - m^2)$$

If roots are equal, $B^2 - 4AC = 0$

$$(2abc)^2 - 4(1+a^2)b^2(c^2 - m^2) = 0$$
$$4a^2b^2c^2 - (4b^2 + 4a^2b^2)(c^2 - m^2) = 0$$
$$4a^2b^2c^2 - [4b^2c^2 - 4b^2m^2 + 4a^2b^2c^2 - 4a^2b^2m^2] = 0$$
$$4a^2b^2c^2 - 4b^2c^2 + 4b^2m^2 - 4a^2b^2c^2 + 4a^2b^2m^2 = 0$$
$$4b^2[a^2m^2 + m^2 - c^2] = 0$$
$$c^2 = a^2m^2 + m^2$$
$$c^2 = m^2(1+a^2)$$

121. If -3 is a root of quadratic equation $2x^2 + px - 15 = 0$, while the quadratic equation $x^2 - 4px + k = 0$ has equal roots. Find the value of k.

Ans : [Board Term-2 OD Compt. 2017]

Given -3 is a root of quadratic equation.

We have $2x^2 + px - 15 = 0$

Since 3 is a root of above equation, it must satisfy it. Substituting $x = 3$ in above equation we have

$$2(-3)^2 + p(-3) - 15 = 0$$
$$2 \times 9 - 3p - 15 = 0 \Rightarrow p = 1$$

Since $x^2 - 4px + k = 0$ has equal roots,

or $x^2 - 4x + k = 0$ has equal roots,

$$b^2 - 4ac = 0$$
$$(-4)^2 - 4k = 0$$
$$16 - 4k = 0$$
$$4k = 16 \Rightarrow k = 4$$

122. If $ad \neq bc$, then prove that the equation $(a^2 + b^2)x^2 + 2(ac + bd)x + (c^2 + d^2) = 0$ has no real roots.

Ans : [Board Term-2 OD 2017]

We have $(a^2 + b^2)x^2 + 2(ac + bd)x + (c^2 + d^2) = 0$

Comparing with $Ax^2 + Bx + C = 0$ we get

$$A = (a^2 + b^2), B = 2(ac + bd) \text{ and } C = (c^2 + d^2)$$

For no real roots, $D = B^2 - 4AC < 0$

$$D = B^2 - 4AC$$
$$= [2(ac + bd)]^2 - 4(a^2 + b^2)(c^2 + d^2)$$
$$= 4[a^2c^2 + 2abcd + b^2d^2] - 4[a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2]$$
$$= 4[a^2c^2 + 2abcd + b^2d^2 - a^2c^2 - a^2d^2 - b^2c^2 - b^2d^2]$$
$$= -4[a^2d^2 + b^2c^2 - 2abcd]$$
$$= -4(ad - bc)^2$$

Since $ad \neq bc$, therefore $D \neq 0$ and always negative. Hence the equation has no real roots.

123. Find the value of c for which the quadratic equation $4x^2 - 2(c+1)x + (c+1) = 0$ has equal roots.

Ans : [Board Term-2 Delhi 2017]

We have $4x^2 - 2(c+1)x + (x+1) = 0$

Comparing with $Ax^2 + Bx + C = 0$ we get

$$A = 4, B = 2(c+1), C = (c+1)$$

d210

If roots are equal, $B^2 - 4AC = 0$

$$[2(c+1)]^2 - 4 \times 4(c+1) = 0$$

$$4(c^2 + 2c + 1) - 4(4c + 4) = 0$$

$$4(c^2 + 2c + 1 - 4c - 4) = 0$$

$$c^2 - 2c - 3 = 0$$

$$c^2 - 3c + c - 3 = 0$$

$$c(c-3) + 1(c-3) = 0$$

$$(c-3)(c+1) = 0$$

$$c = 3, -1$$

Hence for equal roots $c = 3, -1$.

124. Show that if the roots of the following equation are equal then $ad = bc$ or $\frac{a}{b} = \frac{c}{d}$.

$$x^2(a^2 + b^2) + 2(ac + bd)x + c^2 + d^2 = 0$$

Ans : [Board Term-2 OD Compt. 2017]

We have $x^2(a^2 + b^2) + 2(ac + bd)x + c^2 + d^2 = 0$

Comparing with $Ax^2 + Bx + C = 0$ we get

$$A = a^2 + b^2, B = 2(ac + bd), C = c^2 + d^2$$

d211

If roots are equal, $B^2 - 4AC = 0$

$$[2(ac + bd)]^2 - 4(a^2 + b^2)(c^2 + d^2) = 0$$

$$4(a^2c^2 + 2abcd + b^2d^2) - 4(a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2) = 0$$

$$4(a^2c^2 + 2abcd + b^2d^2 - a^2c^2 - a^2d^2 - b^2c^2 - b^2d^2) = 0$$

$$-4(a^2d^2 + b^2c^2 - 2abcd) = 0$$

$$(ad - bc)^2 = 0$$

Thus $$ad = bc$$

$$\frac{a}{b} = \frac{c}{d}$$ Hence Proved.

125. Solve $\frac{1}{(a+b+x)} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$, $a + b \neq 0$.

Ans : [Board Term-2 SQP 2016]

We have $$\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$$

$$\frac{1}{a+b+x} - \frac{1}{x} = \frac{1}{a} + \frac{1}{b}$$

d228

$$\frac{x - (a+b+x)}{x(a+b+x)} = \frac{a+b}{ab}$$

$$\frac{x - a - b - x}{x(a+b+x)} = \frac{a+b}{ab}$$

$$\frac{-(a+b)}{x(a+b+x)} = \frac{a+b}{ab}$$

$$x(a+b+x) = -ab$$

$$x^2 + (a+b)x + ab = 0$$

$$(x+a)(x+b) = 0$$

$$x = -a \text{ or } x = -b$$

FOUR MARKS QUESTIONS

126. Solve for x : $\left(\frac{2x}{x-5}\right)^2 + \left(\frac{2x}{x-5}\right) - 24 = 0, x \neq 5$

Ans : [Board Term-2 2016]

We have $$\left(\frac{2x}{x-5}\right)^2 + 5\left(\frac{2x}{x-5}\right) - 24 = 0$$

Let $\frac{2x}{x-5} = y$ then we have

$$y^2 + 5y - 24 = 0$$

d158

$$(y+8)(y-3) = 0$$

$$y = 3, -8$$

Taking $y = 3$ we have

$$\frac{2x}{x-5} = 3$$

$$2x = 3x - 15 \Rightarrow x = 15$$

Taking $y = -8$ we have

$$\frac{2x}{x-5} = -8$$

$$2x = -8x + 40$$

$$10x = 40 \Rightarrow x = 4$$

Hence, $x = 15, 4$

127. Solve for x : $\frac{1}{x+1} + \frac{2}{x+2} = \frac{4}{x+4} \quad x \neq -1, -2, -4$

Ans : [Board Term-2 OD 2016]

We have
$$\frac{1}{x+1} + \frac{2}{x+2} = \frac{4}{x+4}$$
$$\frac{x+2+2(x+1)}{(x+1)(x+2)} = \frac{4}{x+4}$$
$$\frac{3x+4}{x^2+3x+2} = \frac{4}{x+4}$$
$$(3x+4)(x+4) = 4(x^2+3x+2)$$
$$3x^2+16x+16 = 4x^2+12x+8$$
$$x^2-4x-8 = 0$$

d159

Now
$$x = \frac{-b\sqrt{b^2+4ac}}{2a}$$
$$= \frac{-(-4) \pm \sqrt{(-4)^2-4(1)(-8)}}{2 \times 1}$$
$$= \frac{4 \pm \sqrt{16+32}}{2}$$
$$= \frac{4 \pm \sqrt{48}}{2} = \frac{4 \pm 4\sqrt{3}}{2}$$
$$= 2 \pm 2\sqrt{3}$$

Hence, $x = 2+2\sqrt{3}$ and $2-2\sqrt{3}$

128. Find x in terms of a, b and c :
$$\frac{a}{x-a} + \frac{b}{x-b} = \frac{2c}{x-c}, x \neq a, b, c$$

Ans : [Board Term-2, Delhi 2016]

d160

We have
$$\frac{a}{x-a} + \frac{b}{x-b} = \frac{2c}{x-c}$$
$$a(x-b)(x-c) + b(x-a)(x-c) = 2c(x-a)(x-b)$$
$$ax^2 - abx - acx + abc + bx^2 - bax - bcx + abc$$
$$= 2cx^2 - 2cxb - 2cxa + 2abc$$
$$ax^2 + bx^2 - 2cx^2 - abx - acx - bax - bcx + 2cbx + 2acx$$
$$= 0$$
$$x^2(a+b-2c) - 2abx + acx + bcx = 0$$
$$x^2(a+b-2c) + x(-2ab+ac+bc) = 0$$

Thus $x = -\left(\frac{ac+bc-2ab}{a+b-2c}\right)$

129. Solve for x : $\frac{3}{x+1} + \frac{4}{x-1} = \frac{29}{4x-1};\ x \neq -1, 1, \frac{1}{4}$

Ans : [Board Term-2 Delhi 2015]

We have
$$\frac{3}{x+1} + \frac{4}{x-1} = \frac{29}{4x-1}$$
$$\frac{3x-3+4x+4}{x^2-1} = \frac{29}{4x-1}$$
$$\frac{7x+1}{x^2-1} = \frac{29}{4x-1}$$
$$(7x+1)(4x-1) = 29x^2-29$$
$$28x^2-7x+4x-1 = 29x^2-29$$
$$-3x = x^2-28$$
$$x^2+3x-28 = 0$$
$$x^2+7x-4x-28 = 0$$
$$x(x+7)-4(x+7) = 0$$
$$(x+7)(x-4) = 0$$

d161

Hence, $x = 4, -7$

130. Solve for x : $\frac{x-1}{2x+1} + \frac{2x+1}{x-1} = 2$ where $x \neq -\frac{1}{2}, 1$

Ans : [Board Term-2, OD 2015]

We have
$$\frac{x-1}{2x+1} + \frac{2x+1}{x-1} = 2$$

Let $\frac{x-1}{2x+1}$ be y so $\frac{2x+1}{x-1} = \frac{1}{y}$

d179

Substituting this value we obtain
$$y + \frac{1}{y} = 2$$
$$y^2+1 = 2y$$
$$y^2-2y+1 = 0$$
$$(y-1)^2 = 0$$
$$y = 1$$

Substituting $y = \frac{x-1}{2x+1}$ we have
$$\frac{x-1}{2x+1} = 1 \text{ or } x-1 = 2x+1$$

or
$$x = -2$$

131. Find for x : $\frac{1}{x-2} + \frac{2}{x-1} = \frac{6}{x};\quad x \neq 0, 1, 2$

Ans : [Board Term-2 OD 2017]

We have $\dfrac{1}{x-2}+\dfrac{2}{x-1} = \dfrac{6}{x}$

$$\frac{x-1+2x-4}{(x-2)(x-1)} = \frac{6}{x}$$

$$3x^2-5x = 6x^2-18x+12$$

$$3x^2-13x+12 = 0$$

$$3x^2-4x-9x+12 = 0$$

$$x(3x-4)-3(3x-4) = 0$$

$$(3x-4)(x-3) = 0$$

$$x = \frac{4}{3} \text{ and } 3$$

Hence, $x = 3, \dfrac{4}{3}$

132. Solve, for x : $\sqrt{3}\,x^2+10x+7\sqrt{3} = 0$

Ans : [Board Term-2 Foreign 2017]

We have $\sqrt{3}\,x^2+10x+7\sqrt{3} = 0$

$$\sqrt{3}\,x^2+3x+7x+7\sqrt{3} = 0$$

$$(x+\sqrt{3})(\sqrt{3}\,x+7) = 0$$

$$(x+\sqrt{3})(\sqrt{3}\,x+7) = 0$$

$$x = -\sqrt{3} \text{ and } x = \frac{-7}{\sqrt{3}}$$

Hence roots $x = -\sqrt{3}$ and $x = \dfrac{-7}{\sqrt{3}}$

133. Solve for x : $\dfrac{x+3}{x-2}-\dfrac{1-x}{x} = \dfrac{17}{4}$; $x \neq 0,\ 2$

Ans : [Board Term -2 Delhi Compt. 2017]

We have $\dfrac{x+3}{x-2}-\dfrac{1-x}{x} = \dfrac{17}{4}$

$$\frac{x(x+3)-(1-x)(x-2)}{x(x-2)} = \frac{17}{4}$$

$$\frac{(x^2+3x)-(-x^2+3x-2)}{x^2-2x} = \frac{17}{4}$$

$$\frac{2x^2+2}{x^2-2x} = \frac{17}{4}$$

$$8x^2+8 = 17x^2-34x$$

$$9x^2-34x-8 = 0$$

$$9x^2-36x+2x-8 = 0$$

$$9x(x-4)+2(x-4) = 0$$

$$(x-4)(9x+2) = 0$$

$$x = 4 \text{ or } x = -\frac{2}{9}$$

Hence, $x = 4,\ -\dfrac{2}{9}$

134. Solve for x : $4x^2+4bx-(a^2-b^2) = 0$

Ans : [Board Term-2 Foreign 2017]

We have $4x^2+4bx-(a^2-b^2) = 0$

Comparing with $Ax^2+Bx+C = 0$ we get

$$A = 4, B = 4b \text{ and } C = b^2-a^2$$

$$x = \frac{-B \pm \sqrt{B^2-4AC}}{2A}$$

$$= \frac{-4b \pm \sqrt{(4b)^2-4.4(b^2-a^2)}}{2.4}$$

$$= \frac{-4b \pm \sqrt{16b^2-16b^2+16a^2}}{8}$$

$$= \frac{-4b \pm 4a}{8}$$

$$= -\frac{(a+b)}{2}, \frac{(a-b)}{2}$$

Hence the roots are $-\dfrac{(a+b)}{2}$ and $\dfrac{(a-b)}{2}$

135. Find the zeroes of the quadratic polynomial $7y^2-\frac{11}{3}y-\frac{2}{3}$ and verify the relationship between the zeroes and the coefficients.

Ans : [Board 2019 OD]

We have $7y^2-\dfrac{11}{3}y-\dfrac{2}{3} = 0$

$$21y^2-11y-2 = 0 \quad \text{...(1)}$$

$$21y^2-14y+3y-2 = 0$$

$$7y(3y-2)+(3y-2) = 0$$

$$(3y-2)(7y+1) = 0$$

$$y = \frac{2}{3}, \frac{-1}{7}$$

Hence, zeros of given polynomial are,

$$y = \frac{2}{3} \text{ and } y = \frac{-1}{7}$$

Comparing the given equation with $ax^2+bx+c=0$ we get $a=21$, $b=-11$ and $c=-2$

Now, sum of roots, $\alpha+\beta = \frac{2}{3}+\left(-\frac{1}{7}\right)$

$$= \frac{2}{3}-\frac{1}{7} = \frac{11}{21}$$

Thus $\alpha+\beta = -\frac{b}{a}$ Hence verified

and product of roots, $\alpha\beta = \frac{2}{3}\times\left(-\frac{1}{7}\right) = \frac{-2}{21}$

Thus $\alpha\beta = \frac{c}{a}$ Hence verified

136. Write all the values of p for which the quadratic equation $x^2+px+16=0$ has equal roots. Find the roots of the equation so obtained.

Ans : [Board 2019 OD]

We have $x^2+px+16 = 0$ ...(1)

If this equation has equal roots, then discriminant b^2-4ac must be zero.

i.e., $b^2-4ac = 0$ ...(2)

Comparing the given equation with $ax^2+bx+c=0$ we get $a=1$, $b=p$ and $c=16$

Substituting above in equation (2) we have

d313

$$p^2-4\times1\times16 = 0$$

$$p^2 = 64 \Rightarrow p = \pm 8$$

When $p=8$, from equation (1) we have

$$x^2+8x+16 = 0$$

$$x^2+2\times4x+4^2 = 0$$

$$(x+4)^2 = 0 \Rightarrow x = -4,\ -4$$

Hence, roots are -4 and -4.

When $p=-8$ from equation (1) we have

$$x^2-8x+16 = 0$$

$$x^2-2\times4x+4^2 = 0$$

$$(x-4)^2 = 0 \Rightarrow x = 4,\ 4$$

Hence, the required roots are either $-4,\ -4$ or $4,\ 4$

137. Solve for x : $x^2+5x-(a^2+a-6)=0$

Ans : [Board 2019 OD]

d315

We have $x^2+5x-(a^2+a-6) = 0$

$$x^2+5x-[a^2+3a-2a-6] = 0$$

$$x^2+5x-[a(a+3)-2(a+3)] = 0$$

$$x^2+5x-(a+3)(a-2) = 0$$

$$x^2+[a+3-(a-2)]x-(a+3)(a-2) = 0$$

$$x^2+(a+3)x-(a-2)x-(a+3)(a-2) = 0$$

$$x[x+(a+3)]-(a-2)[x+(a+3)] = 0$$

$$[x+(a+3)][x-(a-2)] = 0$$

Thus $x=-(a+3)$ and $x=(a-2)$

Hence, roots of given equations are $x=-(a+3)$ and $x=a-2$.

138. Find the nature of the roots of the quadratic equation $4x^2+4\sqrt{3}x+3=0$.

Ans : [Board 2019 OD]

We have $4x^2+4\sqrt{3}x+3 = 0$

Comparing the given equation with $ax^2+bx+c=0$ we get $a=4$, $b=4\sqrt{3}$ and $c=3$.

Now, $D = b^2-4ac$

d316

$$= (4\sqrt{3})^2-4\times4\times3$$

$$= 48-48 = 0$$

Since, $b^2-4ac=0$, then roots of the given equation are real and equal.

139. If $x=3$ is one root of the quadratic equation $x^2-2kx-6=0$, then find the value of k.

Ans : [Board 2018]

If $x=3$ is one root of the equation $x^2-2kx-6=0$, it must satisfy it.

Thus substituting $x=3$ in given equation we have

d317

$$9-6k-6 = 0$$

$$k = \frac{1}{2}$$

140. Find the positive values of k for which quadratic equations $x^2+kx+64=0$ and $x^2-8x+k=0$ both will have the real roots.

Ans : [Board Term-2 Foreign 2016]

(1) For $x^2+kx+64=0$ to have real roots

d213

$$k^2-256 \geq 0$$

$$k^2 \geq 256$$

$$k \geq 16 \text{ or } k < -16$$

(2) For $x^2-8x+k=0$ to have real roots

$$64-4k \geq 0$$

$$16-k \geq 0$$

$$16 \geq k$$

For (1) and (2) to hold simultaneously

$$k = 16$$

141. Find the values of k for which the equation $(3k+1)^2 + 2(k+1)x + 1$ has equal roots. Also find the roots.

Ans : [Board Term-2, 2014]

We have $\quad (3k+1)^2 + 2(k+1)x + 1$

Comparing with $Ax^2 + Bx + C = 0$ we get

$$A = (3k+1), B = 2(k+1), C = 1$$

d215

If roots are equal, $B^2 - 4AC = 0$

$$[2(k+1)]^2 - 4(3k+1)(1) = 0$$

$$4(k^2+2k+1) - (12k+4) = 0$$

$$4k^2 + 8k + 4 - 12k - 4 = 0$$

$$4k^2 - 4k = 0$$

$$4k(k-1) = 0$$

$$k = 0, 1.$$

Substituting $k = 0$, in the given equation,

$$x^2 + 2x + 1 = 0$$

$$(x+1)^2 = 0$$

$$x = -1$$

Again substituting $k = 1$, in the given equation,

$$4x^2 + 4x + 1 = 0$$

$$(2x+1)^2 = 0$$

or, $$x = -\frac{1}{2}$$

Hence, roots $= -1, -\frac{1}{2}$

142. Find the values of k for which the quadratic equations $(k+4)x^2 + (k+1)x + 1 = 0$ has equal roots. Also, find the roots.

Ans : [Board Term-2 Delhi 2014]

We have $\quad (k+4)x^2 + (k+1)x + 1 = 0$

Comparing with $Ax^2 + Bx + C = 0$ we get

$$A = (k+4), B = (k+1), C = 1$$

d216

If roots are equal, $B^2 - 4AC = 0$

$$(k+1)^2 - 4(k+4)(1) = 0$$

$$k^2 + 1 + 2k - 4k - 16 = 0$$

$$k^2 - 2k - 15 = 0$$

$$(k-5)(k+3) = 0$$

$$k = 5, -3$$

For $k = 5$, equation becomes

$$9x^2 + 6x + 1 = 0$$

$$(3x+1)^2 = 0$$

or $$x = -\frac{1}{3}$$

For $k = -3$, equation becomes

$$x^2 - 2x + 1 = 0$$

$$(x-1)^2 = 0$$

$$x = 1$$

Hence roots are 1 and $-\frac{1}{3}$.

143. If $x = -2$ is a root of the equation $3x^2 + 7x + p = 0$, find the value of k so that the roots of the equation $x^2 + k(4x + k - 1) + p = 0$ are equal.

Ans : [Board Term-2 Foreign 2015]

We have $\quad 3x^2 + 7x + p = 0$

Since $x = -2$ is the root of above equation, it must satisfy it.

Thus $$3(-2) + 7(-2) + p = 0$$

$$p = 2$$

Since roots of the equation $x^2 + 4kx + k^2 - k + 2 = 0$ are equal,

$$16k^2 - 4(k^2 - k + 2) = 0$$

d217

$$16k^2 - 4k^2 + 4k - 8 = 0$$

$$12k^2 + 4k - 8 = 0$$

$$3k^2 + k - 2 = 0$$

$$(3k-2)(k+1) = 0$$

$$k = \frac{2}{3}, -1$$

Hence, roots $= \frac{2}{3}, -1$

144. If $x = -4$ is a root of the equation $x^2 + 2x + 4p = 0$

, find the values of k for which the equation $x^2 + px(1+3k) + 7(3+2k) = 0$ has equal roots.

Ans : [Board Term-2 Foreign 2015]

We have $\quad x^2 + 2x + 4p = 0$

Since $x = -4$ is the root of above equation. It must satisfy it.

$$(-4)^2 + (2 \times -4) + 4p = 0$$

$$p = -2$$

Since equation $x^2 - 2(1+3k)x + 7(3+2k) = 0$ has equal roots.

$$4(1+3k)^2 - 28(3+2k) = 0$$

d218

$$9k^2 - 8k - 20 = 0$$

$$(9k+10)(k-2) = 0$$

$$k \ \frac{-10}{9}, 2$$

Hence, the value of k are $-\frac{10}{9}$ and 2.

145. Find the value of p for which the quadratic equation $(p+1)x^2 - 6(p+1)x + 3(p+9) = 0, \quad p \neq -1$ has equal roots. Hence find the roots of the equation.

Ans : [Board Term-2, 2015]

We have $\quad (p+1)x^2 - 6(p+1)x + 3(p+9) = 0$

Comparing with $ax^2 + bx + c = 0$ we get

$$a = p+1, b = -6(p+1), c = 3(p+9)$$

For real and equal roots, $b^2 - 4ac = 0$

$$36(p+1)^2 - 4(p+1) \times 3(p+9) = 0$$

d219

$$3(p^2 + 2p + 1) - (p+1)(p+9) = 0$$

$$3p^2 + 6p + 3 - (p^2 + 9p + p + 9) = 0$$

$$2p^2 - 4p - 6 = 0$$

$$p^2 - 2p - 3 = 0$$

$$p^2 - 3p + p - 3 = 0$$

$$p(p-3) + 1(p-3) = 0$$

$$(p-3)(p+1) = 0$$

$$p = -1, 3$$

Neglecting $p \neq -1$ we get $p = 3$

Now the equation becomes

$$4x^2 - 24x + 36 = 0$$

or $\quad x^2 - 6x + 9 = 0$

or, $\quad (x-3)(x-3) = 0$

$$x = 3, 3$$

Thus roots are 3 and 3.

146. If the equation $(1+m^2)x^2 + 2mcx + (c^2 - a^2) = 0$ has equal roots, prove that $c^2 = a^2(1+m^2)$

Ans : [Board Term-2 Delhi 2015]

We have $\quad (1+m^2)x^2 + 2mcx + (c^2 - a^2) = 0$

Comparing with $Ax^2 + Bx + C = 0$ we get

$$A = 1 + m^2, B = 2mc, C = (c^2 - a^2)$$

If roots are equal, $B^2 - 4AC = 0$

$$(2mc)^2 - 4(1+m^2)(c^2 - a^2) = 0$$

d220

$$4m^2c^2 - 4(1+m^2)(c^2 - a^2) = 0$$

$$m^2c^2 - (c^2 - a^2 + m^2c^2 - m^2a^2) = 0$$

$$m^2c^2 - c^2 + a^2 - m^2c^2 + m^2a^2 = 0$$

$$-c^2 + a^2 + m^2a^2 = 0$$

, $$c^2 = a^2(1+m^2)$$

Hence Proved.

147. If (-5) is a root of the quadratic equation $2x^2 + px + 15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots, then find the values of p and k.

Ans : [Board Term-2 Delhi 2015]

We have $\quad 2x^2 + px - 15 = 0$

Since $x = -5$ is the root of above equation. It must satisfy it.

$$2(-5)^2 + p(-5) - 15 = 0$$

d221

$$50 - 5p - 15 = 0$$

$$5p = 35 \Rightarrow p = 7$$

Now $\quad p(x^2 + x) + k = 0$ has equal roots

or $\quad 7x^2 + 7x + k = 0$

Taking $b^2 - 4ac = 0$ we have

$$7^2 - 4 \times 7 \times k = 0$$

$$7 - 4k = 0$$

$$k = \frac{7}{4}$$

Hence $p = 7$ and $k = \frac{7}{4}$.

148. If the roots of the quadratic equation

$(x-a)(x-b)+(x-b)(x-c)+(x-c)(x-a)=0$ are equal. Then show that $a=b=c$.

Ans : [Board Term-2 Delhi 2015]

We have

$$(x-a)(x-b)+(x-b)(x-c)+(x-c)(x-a)=0$$

$$x^2-ax-bx+ab+$$
$$+x^2-bx-cx+bc+$$
$$+x^2-cx-ax+ac=0$$

d222

$$3x^2-2ac-2bx-2cx+ab+bc+ca=0$$

For equal roots $B^2-4AC=0$

$$\{-2(a+b+c)\}^2-4\times 3(ab+bc+ca)=0$$
$$4(a+b+c)^2-12(ab+bc+ca)=0$$
$$a^2+b^2+c^2-3(ab+bc+ca)=0$$
$$a^2+b^2+c^2+2ab+2bc+2ac-3ab-3bc-3ac=0$$
$$a^2+b^2+c^2-ab-ac-bc=0$$
$$\tfrac{1}{2}[2a^2+2b^2+2c^2-2ab-2ac-2bc]=0$$
$$\tfrac{1}{2}\left[(a^2+b^2-2ab)+(b^2+c^2-2bc)+(c^2+a^2-2ac)\right]=0$$
$$\tfrac{1}{2}\left[(a-b)^2+(b-c)^2+(c-a)^2\right]=0$$

or, $(a-b)^2+(b-c)^2+(c-a)^2=0$

If $a\neq b\neq c$

$(a-b)^2>0,\ (b-c)^2>0,\ (c-a)^2>0$

If $(a-b)^2=0\Rightarrow a=b$

$(a-c)^2=0\Rightarrow b=c$

$(c-a)^2=0\Rightarrow c=a$

Thus $a=b=c$ Hence Proved

149. If the roots of the quadratic equation $(c^2-ab)x^2-2(a^2-bc)x+b^2-ac=0$ in x are equal then show that either $a=0$ or $a^3+b^3+c^3=3abc$

Ans : [Board Term 2Outside Delhi 2017]

We have $(c^2-ab)x^2-2(a^2-bc)x+b^2-ac=0$

Comparing with $Ax^2+Bx+C=0$ we get

$A=(c^2-ab), B=(a^2-bc), C=(b^2-ac)$

If roots are equal, $B^2-4AC=0$

$$\left[2(a^2-bc)\right]^2-4(c^2-ab)(b^2-ac)=0$$
$$4[a^4+b^2c^2-2a^2bc]-4(b^2c^2-c^3a-ab^3-a^2bc)=0$$
$$4[a^4+b^2c^2-2a^2bc-b^2c^2+c^3a+ab^3-a^2bc]=0$$
$$4[a^4+ac^3+ab^3-3a^2bc]=0$$
$$a(a^3+c^3+b^3-3abc)=0$$
$$a=0 \text{ or } a^3+b^3+c^3=3abc$$

d223

150. Solve for x : $\dfrac{1}{a+b+x}=\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{x}$

where $a+b+x\neq 0$ and $a,b,x\neq 0$

Ans : [Board Term-2 Foreign 2017]

We have $\dfrac{1}{a+b+x}-\dfrac{1}{x}=\dfrac{1}{a}+\dfrac{1}{b}$

$$\frac{-(a+b)}{x^2+(a+b)x}=\frac{b+a}{ab}$$

d224

$$x^2+(a+b)x+ab=0$$
$$(x+a)(x+b)=0$$
$$x=-a, x=-b$$

Hence $x=-a,-b$

151. Check whether the equation $5x^2-6x-2=0$ has real roots if it has, find them by the method of completing the square. Also verify that roots obtained satisfy the given equation.

Ans : [Board Term-2 SQP 2017]

We have $5x^2-6x-2=0$

Comparing with $ax^2+bx+c=0$ we get

d227

$$a=5, b=(-6) \text{ and } c=(-2)$$
$$b^2-4ac=(-6)^2-4\times 5\times -2$$
$$=36+40=76>0$$

So the equation has real and two distinct roots.

$$5x^2-6x=2$$

Dividing both the sides by 5 we get

$$\frac{x^2}{5}-\frac{6}{5}x=\frac{2}{5}$$
$$x^2-2x\left(\frac{3}{5}\right)=\frac{2}{5}$$

Adding square of the half of coefficient of x

$$x^2-2x\left(\frac{3}{5}\right)+\frac{9}{25}=\frac{2}{5}+\frac{9}{25}$$
$$\left(x-\frac{3}{5}\right)^2=\frac{19}{25}$$
$$x-\frac{3}{5}=\pm\frac{\sqrt{19}}{5}$$

$$x = \frac{3+\sqrt{19}}{5} \text{ or } \frac{3-\sqrt{19}}{5}$$

Verification :

$$5\left[\frac{3+\sqrt{19}}{5}\right]^2 - 6\left[\frac{3+\sqrt{19}}{5}\right] - 2$$

$$= \frac{9+6\sqrt{19}+19}{5} - \left(\frac{18+6\sqrt{19}}{5}\right) - 2$$

$$= \frac{28+6\sqrt{19}}{5} - \frac{18+6\sqrt{19}}{5} - 2$$

$$= \frac{28+6\sqrt{19}-18-6\sqrt{19}-10}{5}$$

$$= 0$$

Similarly

$$5\left[\frac{3-\sqrt{19}}{5}\right]^2 - 6\left[\frac{3-\sqrt{19}}{5}\right] - 2 = 0$$

Hence verified.

CHAPTER 5

ARITHMETIC PROGRESSION

ONE MARK QUESTIONS

Multiple Choice Questions

1. The n^{th} term of the AP $a, 3a, 5a, ...$ is

(a) na (b) $(2n-1)a$

(c) $(2n+1)a$ (d) $2na$

Ans : [Board 2020 OD Standard]

Given AP is $a, 3a, 5a, ...$

First term is a and $d = 3a - a = 2a$

e245

n^{th} term $a_n = a + (n-1)d$

$= a + (n-1)2a$

$= a + 2na - 2a$

$= 2na - a = (2n-1)a$

Thus (b) is correct option.

2. The common difference of the AP $\frac{1}{p}, \frac{1-p}{p}, \frac{1-2p}{p}$, ... is

(a) 1 (b) $\frac{1}{p}$

(c) -1 (d) $-\frac{1}{p}$

Ans : [Board 2020 OD Standard]

Given AP is $\frac{1}{p}, \frac{1-p}{p}, \frac{1-2p}{p}$ ...

e246

Common difference

$$d = \frac{1-p}{p} - \frac{1}{p} = \frac{1-p-1}{p} = \frac{-p}{p} = -1$$

Thus (c) is correct option.

3. The value of x for which $2x, (x+10)$ and $(3x+2)$ are the three consecutive terms of an AP, is

(a) 6 (b) -6

(c) 18 (d) -18

Ans : [Board 2020 Delhi Standard]

Since $2x$, $(x+10)$ and $(3x+2)$ are in AP we obtain,

$(x+10) - 2x = (3x+2) - (x+10)$

$-x + 10 = 2x - 8$

$-x - 2x = -8 - 10$

$-3x = -18 \Rightarrow x = 6$

e247

Thus (a) is correct option.

4. The first term of AP is p and the common difference is q, then its 10th term is

(a) $q + 9p$ (b) $p - 9q$

(c) $p + 9q$ (d) $2p + 9q$

Ans : [Board 2020 Delhi Standard]

We have $a = p$ and $d = q$

$a_{10} = a + (10-1)d$

$= p + 9q$

e248

Thus (c) is correct option.

5. In an AP, if $d = -4$, $n = 7$ and $a_n = 4$, then a is equal to

(a) 6 (b) 7

(c) 20 (d) 28

Ans : (d) 28

In an AP, $a_n = a + (n-1)d$

$4 = a + (7-1)(-4)$

$4 = a + 6(-4)$

$4 + 24 = a \Rightarrow a = 28$

e249

Thus (d) is correct option.

6. In an AP, if $a = 3.5$, $d = 0$ and $n = 101$, then a_n will be

(a) 0 (b) 3.5

(c) 103.5 (d) 104.5

Ans : (b) 3.5

As, $d = 0$ all the terms are same whatever the value of n. So, $a_n = 3.5$.

e250

Alternate Method :

In an AP, $a_n = a + (n-1)\,d$

$$a_n = 3.5 + (101-1) \times 0 = 3.5$$

Thus (b) is correct option.

7. The 11th term of an AP $-5, \frac{-5}{2}, 0, \frac{5}{2}, ...,$ is

(a) -20 (b) 20

(c) -30 (d) 30

Ans : (b) 20

Here, $a = -5,\ d = \frac{-5}{2} - (-5) = \frac{5}{2}$

e251

nth term, $a_n = a + (n-1)\,d$

$$a_{11} = -5 + (11-1) \times \left(\frac{5}{2}\right)$$

$$a_{11} = -5 + 25 = 20$$

Thus (b) is correct option.

8. In an AP, if $a = 3.5$, $d = 0$ and $n = 101$, then a_n will be

(a) 0 (b) 3.5

(c) 103.5 (d) 104.5

e252

Ans : (b) 3.5

For an AP, $a_n = a + (n-1)d$

$$= 3.5 + (101-1) \times 0$$

$$= 3.5$$

Thus (b) is correct option.

9. Which term of an AP, 21, 42, 63, 84, ... is 210?

(a) 9th (b) 10th

(c) 11th (d) 12th

e253

Ans : (b) 10th

Let nth term of given AP be 210,

First term, $a = 21$

Common difference, $d = 42 - 21 = 21$

and $a_n = 210$

In an AP, $a_n = a + (n-1)\,d$

$$210 = 21 + (n-1)21$$

$$210 = 21 + 21n - 21$$

$$210 = 21n \Rightarrow n = 10$$

Hence, the 10th term of the given AP is 210.

Thus (b) is correct option.

10. If the common difference of an AP is 5, then what is $a_{18} - a_{13}$?

(a) 5 (b) 20

(c) 25 (d) 30

e254

Ans : (c) 25

Given, the common difference of AP i.e, $d = 5$

Using, $a_n = a + (n-1)\,d$

We have, $a_{18} = a + (18-1)\,d$

and $a_{13} = a + (13-1)\,d$

Now, $a_{18} - a_{13} = a + (18-1)\,d - [a + (13-1)\,d]$

$$= a + 17 \times 5 - a - 12 \times 5$$

$$= 85 - 60 = 25$$

Thus (c) is correct option.

11. What is the common difference of an AP in which $a_{18} - a_{14} = 32$?

(a) 8 (b) -8

(c) -4 (d) 4

e255

Ans : (a) 8

We have $a_{18} - a_{14} = 32$

In an AP, $a_n = a + (n-1)\,d$

$$a + (18-1)\,d - [a + (14-1)\,d] = 32$$

$$a + 17d - a - 13d = 32$$

$$4d = 32 \Rightarrow d = 8$$

Hence, the required common difference of the given AP is 8.

Thus (a) is correct option.

12. The 4th term from the end of an AP $-11, -8, -5$, ..., 49 is

(a) 37 (b) 40

(c) 43 (d) 58

e256

Ans : (b) 40

Common difference,

$$d = -8-(-11) = -8+11 = 3$$

Last term, $l = 49$

nth term of an AP from the end is

$$a_n = l-(n-1)d$$

, $a_4 = 49-(4-1)\times 3$

$$= 49-9 = 40$$

13. If the first term of an AP is -5 and the common difference is 2, then the sum of the first 6 terms is

(a) 0 (b) 5

(c) 6 (d) 15

e258

Ans : (a) 0

We have $a = -5$ and $d = 2$

$$S_n = \frac{n}{2}\{2a+(n-1)d\}$$

$$S_6 = \frac{6}{2}[2a+(6-1)d]$$

$$= 3[2(-5)+5(2)]$$

$$= 3(-10+10) = 0$$

Thus (a) is correct option.

14. The sum of first 16 terms of the AP 10, 6, 2, ... is

(a) -320 (b) 320

(c) -352 (d) -400

Ans : (a) -320

Given, AP, is 10, 6, 2 .....

e259

We have $a = 10$ and $d = (6-10) = -4$

$$S_n = \frac{n}{2}\{2a+(n-1)d\}$$

$$S_{16} = \frac{16}{2}[2a+(16-1)d]$$

$$= 8[2\times 10+15(-4)]$$

$$= 8(20-60)$$

$$= 8(-40) = -320$$

Thus (a) is correct option.

15. In an AP, if $a=1$, $a_n = 20$ and $S_n = 399$, then n is equal to

(a) 19 (b) 21

(c) 38 (d) 42

e260

Ans : (c) 38

We have $a = 1$, $a_n = 20$ and $S_n = 399$

Now, $S_n = \frac{n}{2}(a+a_n)$

$$399 = \frac{n}{2}(1+20)$$

$$n = \frac{399\times 2}{21} = 38.$$

16. The sum of first five multiples of 3 is

(a) 45 (b) 55

(c) 65 (d) 75

e261

Ans : (a) 45

The first five multiples of 3 are 3, 6, 9, 12 and 15.

Here, first term, $a = 3$, $d = 6-3 = 3$ and $n = 5$

$$S_n = \frac{n}{2}\{2a+(n-1)\}d$$

$$S_5 = \frac{5}{2}[2a+(5-1)d]$$

$$= \frac{5}{2}[2\times 3+4\times 3]$$

$$= \frac{5}{2}(6+12) = \frac{5}{2}\times 18 = 45$$

Thus (a) is correct option.

17. If the sum of the series $2+5+8+11$ .... is 60100, then the number of terms are

(a) 100 (b) 200

(c) 150 (d) 250

e262

Ans : (b) 200

We have $a = 2$. $d = 5-2 = 3$ and $S_n = 60100$

$$\frac{n}{2}[2a+(n-1)d] = S_n$$

$$\frac{n}{2}[4+(n-1)3] = 60100$$

$$n(3n+1) = 120200$$

$$3n^2+n-120200 = 0$$

$$(n-200)(3n+601) = 0 \Rightarrow n = 200, \frac{601}{3}$$

Thus $n = 200$ because n can not be fraction.

Thus (b) is correct option.

18. If the common difference of an AP is 5, then what is $a_{18} - a_{13}$?

(a) 5 (b) 20

(c) 25 (d) 30

e263

Ans : (c) 25

Given, the common difference of AP i.e., $d = 5$

Now $a_n = a + (n-1)d$

Now, $a_{18} - a_{13} = a + (18-1)d - [a + (13-1)d]$

$= a + 17 \times 5 - a - 12 \times 5$

$= 85 - 60 = 25$

Thus (c) is correct option.

19. There are 60 terms is an AP of which the first term is 8 and the last term is 185. The 31st term is

(a) 56 (b) 94

(c) 85 (d) 98

e264

Ans : (d) 98

Let d be the common difference;

Now $a_n = a + (n-1)d$

Then 60th term, $a_{60} = 8 + (60-1)d$

$185 = 8 + 59d$

$59d = 177 \Rightarrow d = 3$

31th term $a_{31} = 8 + 30 \times 3 = 98$

Thus (d) is correct option.

20. The first and last term of an AP are a and ℓ respectively. If S is the sum of all the terms of the AP and the common difference is $\frac{\ell^2 - a^2}{k - (\ell + a)}$, then k is equal to

(a) S (b) $2S$

(c) $3S$ (d) None of these

Ans : (b) $2S$

We have, $S = \frac{n}{2}(a + \ell)$

$$\frac{2S}{a+\ell} = n \qquad (1)$$

Also, $\ell = a + (n-1)d$

$$d = \frac{\ell - a}{n-1} = \frac{\ell - a}{\frac{2S}{a+\ell} - 1}$$

e265

$$= \frac{\ell^2 - a^2}{2S - (\ell + a)}$$

Thus $k = 2S$

Thus (b) is correct option.

21. If the nth term of an AP is given by $a_n = 5n - 3$, then the sum of first 10 terms if

(a) 225 (b) 245

(c) 255 (d) 270

e266

Ans : (b) 245

We have $a_n = 5n - 3$

Substituting $n = 1$ and 10 we have

$a = 2$

$a_{10} = 47$

Thus $S_n = \frac{n}{2}(a + a_n)$

$S_{10} = \frac{10}{2}(2 + 47)$

$= 5 \times 49 = 245$

Thus (b) is correct option.

22. Two APs have the same common difference. The first term of one of these is -1 and that of the other is -8. Then the difference between their 4th terms is

(a) -1 (b) -8

(c) 7 (d) -9

e267

Ans : (c) 7

4th term of first AP,

$a_4 = -1 + (4-1)d = -1 + 3d$

and 4th term of second AP,

$a'_4 = -8 + (4-1)d = -8 + 3d$

Now, the difference between their 4th terms,

$a'_4 - a_4 = (-1 + 3d) - (-8 + 3d)$

$= -1 + 3d + 8 - 3d = 7$

Hence, the required difference is 7.

Thus (c) is correct option.

23. An AP starts with a positive fraction and every alternate term is an integer. If the sum of the first 11 terms is 33, then the fourth term is

(a) 2 (b) 3

(c) 5 (d) 6

Ans : (a) 2

We have $S_{11} = 33$

e268

$$\frac{11}{2}[2a+10d] = 33$$

$$a+5d = 3$$

i.e. $a_6 = 3 \Rightarrow a_4 = 2$

Since, alternate terms are integers and the given sum is possible, $a_4 = 2$.

Thus (a) is correct option.

24. If the sum of the first $2n$ terms of 2, 5, 8, ... is equal to the sum of the first n terms of 57, 59, 61, ..., then n is equal to

(a) 10 (b) 12

(c) 11 (d) 13

e269

Ans : (c) 11

$$\frac{2n}{2}\{2\times 2+(2n-1)3\} = \frac{n}{2}\{2\times 57+(n-1)2\}$$

$$2(6n+1) = 112+2n$$

$$10n = 110 \Rightarrow n = 11$$

Thus (c) is correct option.

25. In an AP, if $d = -4$, $n = 7$ and $a_n = 4$, then a is equal to

(a) 6 (b) 7

(c) 20 (d) 28

e270

Ans : (d) 28

In an AP, $a_n = a+(n-1)d$

$$4 = a+(7-1)(-4)$$

$$4 = a+6(-4)$$

$$4+24 = a \Rightarrow a = 28$$

Thus (d) is correct option.

26. The first four terms of an AP whose first term is -2 and the common difference is -2 are

(a) $-2, 0, 2, 4$ (b) $-2, 4, -8, 16$

(c) $-2, -4, -6, -8$ (d) $-2, -4, -8, -16$

Ans : (c) $-2, -4, -6, -8$

Let the first four terms of an AP are $a, a+d, a+2d$ and $a+3d$.

Given, that first term, $a = -2$ and common difference, $d = -2$, then we have an AP as follows

e271

$$-2, \quad -2-2, \quad -2+2(-2), \quad -2+3(-2)$$

$$= -2, -4, -6, -8$$

Thus (c) is correct option.

27. The 21^{th} term of an AP whose first two terms are -3 and 4, is

(a) 17 (b) 137

(c) 143 (d) -143

e272

Ans : (b) 137

Given, first two terms of an AP are

$$a = -3$$

and $a+d = 4$

$$-3+d = 4 \Rightarrow d = 7$$

For an AP, $a_n = a+(n-1)d$

Thus $a_{21} = a+(21-1)d$

$$= -3+(20)7$$

$$= -3+140 = 137$$

Thus (b) is correct option.

28. The number of two digit numbers which are divisible by 3 is

(a) 33 (b) 31

(c) 30 (d) 29

Ans : (c) 30

Two digit numbers which are divisible by 3 are 12, 15, 18, ..., 99;

Here $a = 12$, $d = 3$ and $a_n = 99$

e273

For an AP, $a_n = a+(n-1)d$

So, $99 = 12+(n-1)\times 3$

$$99-12 = 3n-3$$

$$99-12+3 = 3n$$

$$90 = 3n \Rightarrow n = 30$$

Thus (c) is correct option.

29. The list of numbers -10, -6, -2, 2, ... is

(a) an AP with $d = -16$ (b) an AP with $d = 4$

(c) an AP with $d = -4$ (d) not an AP

Ans : (b) an AP with $d = 4$

The given numbers are $-10, -6, -2, 2, ...$

Here, $a_1 = 10, a_2 = -6, a_3 = -2$ and $a_4 = 2, ...$

Since, $d_1 = a_2 - a_1 = -6 - (-10) = -6 + 10 = 4$

$$d_2 = a_3 - a_2 = -2 - (-6) = -2 + 6 = 4$$

$$d_3 = a_4 - a_3 = 2 - (-2) = 2 + 2 = 4$$

Since, $d_1 = d_2 = d_3 = ... = 4$

i.e., each successive term of given list has same difference. So, the given list forms an AP with common difference, $d = 4$.

Thus (b) is correct option.

e274

30. If the nth term of an AP is $4n + 1$, then the common difference is

(a) 3 (b) 4

(c) 5 (d) 6

e275

Ans : (b) 4

Given that the n^{th} term of an AP is $4n + 1$.

$$a_n = 4n + 1$$

Substituting $n = 1, 2, 3, ...$ we have

$$a_1 = 4(1) + 1 = 5$$

$$a_2 = 4(2) + 1 = 9$$

Common difference,

$$d = a_2 - a_1 = 9 - 5 = 4$$

Thus (b) is correct option.

31. If a, b, c, d, e, f are in AP, then $e - c$ is equal to

(a) $2(c - a)$ (b) $2(d - c)$

(c) $2(f - d)$ (d) $(d - c)$

e276

Ans : (b) $2(d - c)$

Let x be the common difference of the AP a, b, c, d, e, f.

For an AP, $a_n = a + (n - 1)d$

$$e = a + (5 - 1)x$$

$$e = a + 4x \quad ...(1)$$

and $c = a + (3 - 1)x$

$$c = a + 2x \quad ...(2)$$

Using equation (1) and (2), we get

$$e - c = a + 4x - a - 2x$$

$$= 2x = 2(d - c)$$

Thus (b) is correct option.

32. If 7 times the 7th term of an AP is equal to 11 times its 11th term, then its term will be

(a) 7 (b) 11

(c) 18 (d) 0

e277

Ans : (d) 0

In an AP, $a_n = a + (n - 1)d$

Now, according to the question,

$$7a_7 = 11a_{11}$$

$$7[a + (7 - 1)d] = 11[a + (11 - 1)d]$$

$$7(a + 6d) = 11(a + 10d)$$

$$7a + 42d = 11a + 110d$$

$$4a + 68d = 0$$

$$4(a + 17d) = 0$$

$$a + 17d = 0 \quad ...(1)$$

18th term of an AP,

$$a_{18} = a + (18 - 1)d = a + 17d$$

But from equation (1) this is zero.

33. The sum of 11 terms of an AP whose middle term is 30, is

(a) 320 (b) 330

(c) 340 (d) 350

Ans : (b) 330

Middle term is $\frac{11+1}{2} = 6$th term.

e279

Now $a_n = a + (n - 1)d$

$$a_6 = a + 5d$$

$$30 = a + 5d$$

$$a = 30 - 5d$$

Now $S_n = \frac{n}{2}[2a + (n - 1)d]$

$$S_{11} = \frac{11}{2}(2a + 10d)$$

Substituting value of a we have

$$S_{11} = \frac{11}{2}[2(30-5d)+10d]$$

$$= \frac{11}{2}[60-10d+10d]$$

$$= 11 \times 30$$

$$S_{11} = 330$$

Thus (b) is correct option.

34. Five distinct positive integers are in a arithmetic progression with a positive common difference. If their sum is 10020, then the smallest possible value of the last term is

(a) 2002 (b) 2004

(c) 2006 (d) 2007

e280

Ans : (c) 2006

Let the five integers be $a-2d$, $a-d$, a, $a+d$, $a+2d$.

Then, we have,

$(a-2d)+(a-d)+a+(a+d)+(a+2d) = 10020$

$$5a = 10020 \Rightarrow a = 2004$$

Now, as smallest possible value of d is 1.

Hence, the smallest possible value of $a+2d$ is 2004+2 = 2006

Thus (c) is correct option.

35. If the 2nd term of an AP is 13 and 5th term is 25, what is its 7th term?

(a) 30 (b) 33

(c) 37 (d) 38

e281

Ans : (b) 33

We have $a_2 = 13$, and $a_5 = 25$

In an AP, $a_n = a+(n-1)d$

$$a_2 = a+(2-1)d = 13$$

$$a+d = 13 \quad ...(1)$$

and $a_5 = a+(5-1)d = 25$

$$a+4d = 25 \quad ...(2)$$

Subtracting equation (1) from equation (2), we get

$$3d = 25-13 = 12 \Rightarrow d = 4$$

From equation (1), $a = 13-4 = 9$

Now, 7th term, $a_7 = a+(7-1)d$

$$= 9+6\times 4 = 33$$

Thus (b) is correct option.

36. Assertion : Common difference of the AP -5, -1, 3, 7, ... is 4.

Reason : Common difference of the AP $a, a+d, a+2d, ...$ is given by $d = a_2 - a_1$

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Common difference, $d = -1-(-5) = 4$

So, both A and R are correct and R explains A.

e282

Thus (c) is correct option.

37. Assertion : Sum of first 10 terms of the arithmetic progression -0.5, -1.0, -1.5, ... is 31.

Reason : Sum of n terms of an AP is given as $S_n = \frac{n}{2}[2a+(n-1)d]$ where a is first term and d common difference.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

Assertion, S_{10}

$$= \frac{10}{2}[2(-0.5)+(10-1)(-0.5)]$$

$$= 5[-1-4.5]$$

$$= 5(-5.5) = 27.5$$

Assertion (A) is false but reason (R) is true.

e283

Thus (d) is correct option.

38. Assertion : $a_n - a_{n-1}$ is not independent of n then the given sequence is an AP.

Reason : Common difference $d = a_n - a_{n-1}$ is constant or independent of n.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

Common difference of an AP $d = a_n - a_{n-1}$ is independent of n or constant.

So, A is correct but R is incorrect.

Thus (d) is correct option.

e284

39. **Assertion :** If n^{th} term of an AP is $7-4n$, then its common differences is -4.

Reason : Common difference of an AP is given by $d = a_{n+1} - a_n$.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

e285

Assertion, $a_n = 7-4n$

$$d = a_{n+1} - a_n$$

$$= 7 - 4(n+1) - (7-4n)$$

$$= 7 - 4n - 4 - 7 + 4n = -4$$

Both are correct. Reason is the correct explanation.

Thus (a) is correct option.

40. **Assertion :** If sum of the first n terms of an AP is given by $S_n = 3n^2 - 4n$. Then its n^{th} term is $a_n = 6n - 7$.

Reason : n^{th} term of an AP, whose sum to n terms is S_n, is given by $a_n = S_n - S_{n-1}$

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

nth term of an AP,

$$a_n = S_n - S_{n-1}$$

e286

$$= 3n^2 - 4n - 3(n-1)^2 + 4(n-1)$$

$$= 6n - 7$$

So, both A and R are correct and R explains A.

Thus (a) is correct option.

Fill in the Blank Questions

41. In an AP, the letter d is generally used to denote the ....

Ans :

common difference

e287

42. If a and d are respectively the first term and the common difference of an AP, $a + 10d$, denotes the .... term of the AP.

Ans :

eleventh

e288

43. An arithmetic progression is a list of numbers in which each term is obtained by .... a fixed number to the preceding term except the first term.

Ans :

adding

e289

44. If S_n denotes the sum of n term of an AP, then $S_{12} - S_{11}$ is the .... term of the AP.

Ans :

twelfth

e290

45. The nth term of an AP whose first term is a and common difference is d is ....

Ans :

$a + (n-1)d$

e291

46. The nth term of an AP is always a .... expression.

Ans :

linear

e292

47. The difference of corresponding terms of two AP's will be ....

Ans :

another AP

e293

48. Fill the two blanks in the sequence 2 ...., 26, ... so that the sequence forms an AP.

Ans : [Board 2020 SQP Standard]

Let a and b be the two numbers. AP will be 2, a, 26, b.

Now, $$26 - a = a - 2$$

$$2a = 28 \Rightarrow a = \frac{28}{2} = 14$$

and $$b - 26 = 26 - a$$

$$a + b = 52$$

$$14 + b = 52 \Rightarrow b = 38$$

Thus $a = 14$ and $b = 38$.

Very Short Answer Questions

49. The sum of first 20 terms of the AP 1, 4, 7, 10 .... is

Ans : [Board 2020 Delhi Standard]

Given AP is 1, 4, 7, 10 ...

Here, $a = 1$, $d = 4 - 1 = 3$ and $n = 20$

$$S_{20} = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{20}{2}[2 \times 1 + (20-1)3]$$

$$= 10(2 + 57) = 10 \times 59 = 590$$

50. Show that $(a-b)^2$, (a^2+b^2) and $(a+b)^2$ are in AP.

Ans : [Board 2020 Delhi Standard]

Given, $(a-b)^2, (a^2+b^2)$ and $(a+b)^2$.

Common difference,

$$d_1 = (a^2+b^2) - (a-b)^2$$

$$= (a^2+b^2) - (a^2+b^2-2ab)$$

$$= a^2 + b^2 - a^2 - b^2 + 2ab$$

$$= 2ab$$

and $$d_2 = (a+b)^2 - (a^2+b^2)$$

$$= a^2 + b^2 + 2ab - a^2 - b^2$$

$$= 2ab$$

Since, $d_1 = d_2$, thus, $(a-b)^2$, (a^2+b^2) and $(a+b)^2$ are in AP.

51. Find the sum of all 11 terms of an AP whose middle term is 30.

Ans : [Board 2020 OD Standard]

In an AP with 11 terms, the middle term is $\frac{11+1}{2} = 6^{th}$ term.

Now, $$a_6 = a + 5d = 30$$

Thus, $$S_{11} = \frac{11}{2}[2a + 10d]$$

$$= 11(a + 5d)$$

$$= 11 \times 30 = 330$$

52. If 4 times the 4^{th} term of an AP is equal to 18 times the 18^{th} term, then find the 22^{nd} term.

Ans : [Board 2020 Delhi Basic]

Let a be the first term and d be the common difference of the AP.

Now $$a_n = a + (n-1)d$$

As per the information given in question

$$4 \times a_4 = 18 \times a_{18}$$

$$4(a + 3d) = 18(a + 17d)$$

$$2a + 6d = 9a + 153d$$

$$7a = -147d$$

$$a = -21d$$

$$a + 21d = 0$$

$$a + (22-1)d = 0$$

$$a_{22} = 0$$

Hence, the 22^{nd} term of the AP is 0.

53. If the first three terms of an AP are b, c and $2b$, then find the ratio of b and c.

Ans : [Board 2020 SQP Standard]

Given, b, c and $2b$ are in AP.

Thus $$c - b = 2b - c$$

$$2c = 3b$$

$$\frac{2}{3} = \frac{b}{c}$$

$$\frac{b}{c} = \frac{2}{3} \Rightarrow b : c = 2 : 3$$

54. The n^{th} term of an AP is $(7 - 4n)$, then what is its common difference?

Ans : [Board 2020 Delhi Basic]

We have $a_n = 7 - 4n$

Putting $n = 1$, $a_1 = 7 - 4 = 3$

Putting $n = 2$, $a_2 = 7 - 8 = -1$

Common difference $d = a_2 - a_1$

$= -1 - 3 = -4$

55. In an AP, if the common difference $d = -4$, and the seventh term a_7 is 4, then find the first term.

Ans : [Board 2018]

We have $d = -4$

and $a_7 = 4$

Now $a_n = a + (n-1)d$

$a_7 = a + (7-1)d$

$4 = a + (7-1)(-4)$

$4 = a - 24 \Rightarrow a = 4 + 24 = 28$

First term of the AP is 28.

56. Find the sum of first 8 multiples of 3.

Ans : [Board 2018]

First 8 multiples of 3 are 3, 6, 9, 12, 15, 18, 21, 24 which are in AP where $a = 3$, $d = 3$ and $n = 8$.

Now $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_8 = \frac{8}{2}[2 \times 3 + (8-1)3]$

$= 4[6 + 21]$

$S_8 = 4 \times 27 = 108$

Thus, sum of first 8 multiples of 3 is 108.

57. Find, how many two digit natural numbers are divisible by 7.

Ans : [Board 2019 Delhi]

Two digits number which are divisible by 7 form an AP given by 14, 21, 28, ..., 98

Here, $a = 14$, $d = 21 - 14 = 7$ and $a_n = 98$

Now $a_n = a + (n-1)d$

$98 = 14 + (n-1)7$

$98 - 14 = 7n - 7$

$91 = 7n \Rightarrow n = 13$

Hence, there are 13 numbers divisible by 7.

58. Find the number of natural numbers between 102 and 998 which are divisible by 2 and 5 both.

Ans : [Board 2020 SQP Standard]

If any number is divisible by 2 and 5, it must be divisible by LCM of 2 and 5, i.e. 10.

Numbers between 102 .... 998 which are divisible by 2 and 5 are 110, 120, 130, ...990

Here $a = 110$, $d = 120 - 110 = 10$ and $a_n = 990$

$a_n = a + (n-1)d$

$990 = 110 + (n-1)10$

$880 = 10(n-1)$

$88 = n - 1$

$n = 88 + 1 = 89$

59. Is -150 a term of the AP 11, 8, 5, 2, ...?

Ans : [Board Term-2 2016]

Let the first term of an AP be a and common difference be d.

We have $a = 11$, $d = -3$, $a_n = -150$

Now $a_n = a + (n-1)d$

$-150 = 11 + (n-1)(-3)$

$-150 = 11 - 3n + 3$

$3n = 164$

or, $n = \frac{164}{3} = 54.66$

Since, 54.66 is not a whole number, -150 is not a term of the given AP

60. Which of the term of AP 5, 2, -1,...... is -49?

Ans : [Board Term-2 2012]

Let the first term of an AP be a and common difference d.

We have $a = 5$, $d = -3$

Now $a_n = a + (n-1)d$

Substituting all values we have

$$-49 = 5+(n-1)(-3)$$
$$-49 = 5-3n+3$$
$$3n = 49+5+3$$
$$n = \frac{57}{3} = 19^{th} \text{ term.}$$

61. Find the first four terms of an AP Whose first term is -2 and common difference is -2.

Ans : [Board Term-2 2012]

We have $a_1 = -2,$

$$a_2 = a_1 + d = -2+(-2) = -4$$
$$a_3 = a_2 + d = -4+(-2) = -6$$
$$a_4 = a_3 + d = -6+(-2) = -8$$

e103

Hence first four terms are $-2, -4, -6, -8$

62. Find the tenth term of the sequence $\sqrt{2}, \sqrt{8}, \sqrt{18}, \ldots$.

Ans : [Board Term-2 2016]

Let the first term of an AP be a and common difference be d.

Given AP is $\sqrt{2}, \sqrt{8}, \sqrt{18}$ or $\sqrt{2},\ 2\sqrt{2},\ 3\sqrt{2}\ldots$

where, $a = \sqrt{2}, d = \sqrt{2}, n = 10$

e104

Now $a_n = a+(n-1)d$

$$a_{10} = \sqrt{2}+(10-1)\sqrt{2}$$
$$= \sqrt{2}+9\sqrt{2}$$
$$= 10\sqrt{2}$$

Therefore tenth term of the given sequence $\sqrt{200}$.

63. Find the next term of the series $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}$ ....

Ans : [Board Term-2 2012]

Let the first term of an AP be a and common difference d.

Here, $a = \sqrt{2},\ a+d = \sqrt{8} = 2\sqrt{2}$

$$d = 2\sqrt{2}-\sqrt{2} = \sqrt{2}$$

Next term $= \sqrt{32}+\sqrt{2}$

$$= 4\sqrt{2}+\sqrt{2}$$
$$= 5\sqrt{2}$$
$$= \sqrt{50}$$

e105

64. Is series $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, \ldots$ an AP? Give rea

Ans : [Board Te...

e106

Let common difference be d then we have

$$d = a_2 - a_1 = \sqrt{6}-\sqrt{3} = \sqrt{3}(\sqrt{2}-1)$$
$$d = a_3 - a_2 = \sqrt{9}-\sqrt{6} = 3-\sqrt{6}$$
$$d = a_4 - a_3 = \sqrt{12}-\sqrt{9} = 2\sqrt{3}-3$$

As common difference are not equal, the given series is not in AP

65. What is the next term of an AP $\sqrt{7},\ \sqrt{28},\ \sqrt{63}, \ldots$?

Ans : [Board Term-2 Foreign 2014]

Let the first term of an AP be a and common difference be d.

Here, $a = \sqrt{7},\ a+d = \sqrt{28}$

$$d = \sqrt{28}-\sqrt{7} = 2\sqrt{7}-\sqrt{7}$$
$$= \sqrt{7}$$

Next term $= \sqrt{63}+\sqrt{7}$

$$= 3\sqrt{7}+\sqrt{7} = 4\sqrt{7}$$
$$= \sqrt{7\times 16}$$
$$= \sqrt{112}$$

e107

66. If the common difference of an AP is -6, find $a_{16}-a_{12}$.

Ans : [Board Term-2 2014]

Let the first term of an AP be a and common difference be d.

Now $d = -6$

$$a_{16} = a+(16-1)(-6) = a-90$$
$$a_{12} = a+(12-1)(-6) = a-66$$
$$a_{16}-a_{12} = (a-90)-(a-66) = a-90-n+66$$
$$= -24$$

e108

67. For what value of k will the consecutive terms $2k+1$, $3k+3$ and $5k-1$ form an AP?

Ans : [Board Term-2 Foreign 2016]

If x, y and z are in AP then we have

$$y-x = z-y$$

Thus if $2k+1,\ 3k+3,\ 5k-1$ are in AP then

$$(5k-1)-3k+3 = (3k+3)-(2k+1)$$
$$5k-1-3k-3\ \ 3k+3-2k-1$$
$$2k-4 = k+2$$
$$2k-k = 4+2$$
$$k = 6$$

e109

68. Find the 25^{th} term of the AP $-5, \frac{-5}{2}, \frac{5}{2}, ...$

Ans : [Board Term-2 Foreign 2015]

Let the first term of an AP be a and common difference be d.

Here, $a = -5,\ d = -\frac{5}{2} - (-5) = \frac{5}{2}$

$$a_n = a + (n-1)d$$

$$a_{25} = 5 + (25-1)\times\left(\frac{5}{2}\right)$$

$$= -5 + 60 = 55$$

e110

69. The first three terms of an AP are $3y-1$, $3y+5$ and $5y+1$ respectively then find y.

Ans : [Board Term-2 Delhi 2015]

If x, y and z are in AP then we have

$$y - x = z - y$$

Therefore if $3y-1, 3y+5$ and $5y+1$ in AP

$$(3y+5)-(3y-1) = (5y+1)-(3y+5)$$

$$3y+5-3y+1 = 5y+1-3y-5$$

$$6 = 2y-4$$

$$2y = 6+4$$

$$y = \frac{10}{2} = 5$$

e111

70. For what value of k, $k+9$, $2k-1$ and $2k+7$ are the consecutive terms of an AP

Ans : [Board Term-2 OD 2016]

If x, y and z are consecutive terms of an AP then we have

$$y - x = z - y$$

Thus if $k+9, 2k-1,$ and $2k+7$ are consecutive terms of an AP then we have

$$(2k-1)-(k+9) = (2k+7)-(2k-1)$$

$$2k-1-k-9 = 2k+7-2k+1$$

$$k-10 = 8\quad k \Rightarrow 18$$

e112

71. What is the common difference of an AP in which $a_{21} - a_7 = 84$?

Ans : [Board Term-2 2016]

Let the first term of an AP be a and common difference be d.

$$a_{21} - a_7 = 84$$

e113

$$a + (21-1)d - [a + (7-1)d] = 84$$

$$a + 20d - a - 6d = 84$$

$$14d = 84$$

$$d = 6$$

72. In the AP $2, x, 26$ find the value of x.

Ans : [Board Term-2 2012]

If x, y and z are in AP then we have

$$y - x = z - y$$

Since $2, x$ and 26 are in AP we have

e114

$$x - 2 = 26 - x$$

$$2x = 26 + 2$$

$$x = \frac{28}{2} = 14$$

73. For what value of k; $k+2$, $4k-6$, $3k-2$ are three consecutive terms of an AP.

Ans : [Board Term-2 Delhi 2014, 2012]

If x, y and z are three consecutive terms of an AP then we have

$$y - x = z - y$$

Since $k+2$, $4k-6$ and $3k-2$ are three consecutive terms of an AP, we obtain

$$\begin{aligned}(4k-6)-(k+2) &= (3k-2)-(4k-6)\\ 4k-6-k-2 &= 3k-2-4k+6\\ 3k-8 &= -k+4\\ 4k &= 4+8\\ k &= \frac{12}{4} = 3\end{aligned}$$

74. If $18, a, b, -3$ are in AP, then find $a+b$.

Ans : [Board Term-2 2012]

If $18, a, b, -3$ are in AP, then,

$$\begin{aligned}a-18 &= -3-b\\ a+b &= -3+18\\ a+b &= 15\end{aligned}$$

75. Find the common difference of the AP $\frac{1}{3q}, \frac{1-6q}{3q}, \frac{1-12q}{3q}, ...$

Ans : [Board Term-2 Delhi 2011]

Let common difference be d then we have

$$d = \frac{1-6q}{3q} - \frac{1}{3q}$$

$$= \frac{1-6q-1}{3q} = \frac{-6q}{3q} = -2$$

76. Find the first four terms of an AP whose first term is $3x+y$ and common difference is $x-y$.

Ans : [Board Term-2 2012]

Let the first term of an AP be a and common difference be d.

Now
$$\begin{aligned}a_1 &= 3x+y\\ a_2 &= a_1+d = 3x+y+x-y = 4x\\ a_3 &= a_2+d = 4x+x-y = 5x-y\\ a_4 &= a_3+d = 5x-y+x-y\\ &= 6x-2y\end{aligned}$$

So, the four terms are $3x+y$, $4x, 5x-y$ and $6x-2y$.

77. Find the 37^{th} term of the AP $\sqrt{x}, 3\sqrt{x}, 5\sqrt{x}$.

Ans : [Board Term-2 2012]

Let the nth term of an AP be a_n and common difference be d.

Here, $a_1 = \sqrt{x}$

$$\begin{aligned}a_2 &= 3\sqrt{x}\\ d &= a_2 - a_1 = 3\sqrt{x} - \sqrt{x} = 2\sqrt{x}\\ a_n &= a+(n-1)d\\ a_{37} &= \sqrt{x} + (37-1)2\sqrt{x}\\ &= \sqrt{x} + 36\times 2\sqrt{x} = 73\sqrt{x}\end{aligned}$$

78. For an AP, if $a_{25} - a_{20} = 45$, then find the value of d.

Ans : [Board Term-2 2011]

Let the first term of an AP be a and common difference be d.

$$\begin{aligned}a_{25} - a_{20} &= \{a+(25-1)d\} - \{a+(20-1)d\}\\ 45 &= a+24d-a-19d\\ 45 &= 5d\\ d\ & \frac{45}{5} = 9\end{aligned}$$

79. Find the sum of first ten multiple of 5.

Ans : [Board Term-2 Delhi, 2014]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

Here, $a = 5, n = 10, d = 5$

$$\begin{aligned}S_n &= \frac{n}{2}[2a+(n-1)d]\\ S_{10} &= \frac{10}{2}[2\times 5+(10-1)5]\\ &= 5[10+9\times 5]\\ &= 5[10+45]\\ &= 5\times 55 = 275\end{aligned}$$

Hence the sum of first ten multiple of 5 is 275.

80. Find the sum of first five multiples of 2.

Ans : [Board Term-2 2012]

Let the first term be a, common difference be d, nth term be a_n and sum of $n$the term be S_n

Here, $a = 2, d = 2, n = 5$

$$\begin{aligned}S_n &= \frac{n}{2}[2a+(n-1)d]\\ S_5 &= \frac{5}{2}[2\times 2+(5-1)2]\\ &= \frac{5}{2}[4+4\times 2] = \frac{5}{2}[4+8]\\ &= \frac{5}{2}\times 12 = 5\times 6 = 30\end{aligned}$$

81. Find the sum of first 16 terms of the AP 10, 6, 2, ...

Ans : [Board Term-2 2012]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

Here, $a = 10, d = 6 - 1 = -4, n = 16$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{16} = \frac{16}{2}[2 \times 10 + (16-1)(-4)]$$

$$= 8[20 + 15 \times (-4)]$$

$$= 8[20 - 60]$$

$$= 8 \times (-40)$$

$$= -320$$

82. What is the sum of five positive integer divisible by 6.

Ans : [Board Term-2 2012]

Let the first term be a, common difference be d, nth term be a_n and sum of $n$the term be S_n

Here, $a = 6, d = 6, n = 5$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_5 = \frac{5}{2}[2 \times 6 + (5-1)(6)]$$

$$= \frac{5}{2}[12 + 4 \times 6]$$

$$= \frac{5}{2}[12 + 24] = \frac{5}{2}[36]$$

$$= 5 \times 18 = 90$$

83. If the sum of n terms of an AP is $2n^2 + 5n$, then find the 4^{th} term.

Ans : [Board Term-2 2012]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

Now, $S_n = 2n^2 + 5n$

n^{th} term of AP,

$$a_n = S_n - S_{n-1}$$

$$a_n = (2n^2 + 5n) - [2(n-1)^2 + 5(n-1)]$$

$$= 2n^2 + 5n - [2n^2 - 4n + 2 + 5n - 5]$$

$$= 2n^2 + 5n - 2n^2 - n + 3$$

$$= 4n + 3$$

Thus 4^{th} term $a_4 = 4 \times 4 + 3 = 19$

84. If the sum of first k terms of an AP is $3k^2 - k$ and its common difference is 6. What is the first term?

Ans : [Board Term-2 2012]

Let the first term be a, common difference be d, nth term be a_n. Let the sum of k terms of AP is S_k.

We have $S_k = 3k^2 - k$

Now k^{th} term of AP,

$$a_k = S_k - S_{k-1}$$

$$a_k = (3k^2 - k) - [3(k-1)^2 - (k-1)]$$

$$= 3k^2 - k - [3k^2 - 6k + 3 - k + 1]$$

$$= 3k^2 - k - 3k^2 + 7k - 4$$

$$= 6k - 4$$

First term $a = 6 \times 1 - 4 = 2$

85. Which term of the AP $8, 14, 20, 26, ...$ will be 72 more than its 41^{st} term.

Ans : [Board Term-2 OD 2017]

Let the first term be a, common difference be d and nth term be a_n.

We have $a = 8, d = 6$.

Since n^{th} term is 72 more than 41^{st} term. we get

$$a_n = a_{41} + 72$$

$$8 + (n-1)6 = 8 + 40 \times 6 + 72$$

$$6n - 6 = 240 + 72$$

$$6n = 312 + 6 = 318$$

$$n = 53$$

86. If the n^{th} term of an AP $-1, 4, 9, 14, ...$ is 129. Find the value of n.

Ans : [Board Term-2 OD Compt. 2017]

Let the first term be a, common difference be d and nth term be a_n.

We have $a = -1$ and $d = 4 - (-1) = 5$

$$-1 + (n-1) \times 5 = a_n$$

$$-1 + 5n - 5 = 129$$

$$5n = 135$$

$$n = 27$$

Hence 27^{th} term is 129.

87. Write the n^{th} term of the AP $\frac{1}{m}, \frac{1+m}{m}, \frac{1+2m}{m}, ...$

Ans : [Board Term-2 OD Compt. 2017]

Let the first term be a, common difference be d and nth term be a_n.

We have $$a = \frac{1}{m}$$

$$d = \frac{1+m}{m} - \frac{1}{m} = 1$$

$$a_n = \frac{1}{m} + (n-1)1$$

Hence , $$a_n = \frac{1}{m} + n - 1$$

88. What is the common difference of an AP which $a_{21} - a_7 = 84$.

Ans : [Board Term-2 OD 2017]

Let the first term be a, common difference be d and nth term be a_n.

We have $$a_{21} - a_7 = 84$$

$$a + 20d - a - 6d = 84$$

$$14d = 84$$

$$d = \frac{84}{14} = 6$$

Hence common difference is 6.

89. Which term of the progression $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}....$ is the first negative.

Ans : [Board Term-2 OD 2017]

Let the first term be a, common difference be d and nth term be a_n.

We have $a = 20$ and $d = -\frac{3}{4}$

Let the n^{th} term be first negative term, then

$$a + (n-1)d < 0$$

$$20 + (n-1)\left(-\frac{3}{4}\right) < 0$$

$$20 - \frac{3}{4}n + \frac{3}{4} < 0$$

$$3n > 83$$

$$n > \frac{83}{3} = 27\frac{2}{3}$$

Hence 28^{th} term is first negative.

TWO MARKS QUESTIONS

90. If the sum of first m terms of an AP is the same as the sum of its first n terms, show that the sum of its first $(m+n)$ terms is zero.

Ans : [Board 2020 SQP Standard]

Let a be the first term and d be the common difference of the given AP. Then,

$$S_m = S_n$$

$$\frac{m}{2}\{2a + (m-1)d\} = \frac{n}{2}\{2a + (n-1)d\}$$

$$2a(m-n) + \{m(m-1) - n(n-1)d\} = 0$$

$$2a(m-n) + [(m^2 - n^2) - (m-n)d] = 0$$

$$(m-n)[2a + (m+n-1)d] = 0$$

$$2a + (m+n-1)d = 0$$

Now, $$S_{m+n} = \frac{m+n}{2}\{2a + (m+n-1)d\}$$

$$= \frac{m+n}{2} \times 0 = 0$$

91. If $3k-2$, $4k-6$ and $k+2$ are three consecutive terms of AP, then find the value of k.

Ans : [Board 2020 OD Basic]

To be term of an AP the difference between two consecutive terms must be the same.

If $3k-2$, $4k-6$ and $k+2$ are terms of an AP, then

$$4k - 6 - (3k-2) = k + 2 - (4k-6)$$

$$4k - 6 - 3k + 2 = k + 2 - 4k + 6$$

$$k - 4 = 8 - 3k$$

$$4k = 12 \Rightarrow k = 3$$

Hence, the value of k is 3.

92. How many terms of AP 3, 5, 7, 9, ... must be taken to get the sum 120?

Ans : [Board 2020 OD Basic]

Given AP : 3, 5, 7, 9, ...

We have $a = 3$, $d = 2$ and $S_n = 120$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$120 = \frac{n}{2}[2 \times 3 + (n-1)2]$$

$$120 = n(3 + n - 1)$$

$$120 = n(n+2)$$

$$n^2 + 2n - 120 = 0$$

$$n^2 + 12n - 10n - 120 = 0$$

$$(n+12)(n-10) = 0 \Rightarrow n = 10 \text{ or } n = -12$$

Neglecting $n = -12$ because n can't be negative we get $n = 10$. Hence, 10 terms must be taken to get the sum 120.

93. How many two digits numbers are divisible by 3?

Ans : [Board 2019 Delhi]

Numbers divisible by 3 are 3, 6, 9, 12, 15, ..., 96 and 99. Lowest two digit number divisible by 3 is 12 and highest two digit number divisible by 3 is 99.

Hence, the sequence start with 12, ends with 99 and common difference is 3.

So, the AP is 12, 15, 18, ..., 96, 99.

Here, $a = 12,\ d = 3 \text{ and } a_n = 99$

$$a_n = a + (n-1)d$$

$$99 = 12 + (n-1)3$$

$$99 - 12 = 3(n-1)$$

$$n - 1 = \frac{87}{3} = 29 \Rightarrow n = 30$$

e315

Therefore, there are 30, two digit numbers divisible by 3.

94. Which term of the AP 3, 15, 27, 39, ... will be 120 more than its 21st term?

Ans : [Board 2019 Delhi]

Given AP is 3, 15, 27, 39......

Here, first term, $a = 3$ and common difference, $d = 12$

Now, 21^{st} term of AP is

$$a_n = a + (n-1)d$$

$$a_{21} = 3 + (21-1) \times 12$$

$$= 3 + 20 \times 12 = 243$$

e316

Therefore, 21^{st} term is 243.

Now we need to calculate term which is 120 more than 21^{st} term i.e it should be $243 + 120 = 363$

Therefore, $a_n = a + (n-1)d$

$$363 = 3 + (n-1)12$$

$$360 = 12(n-1)$$

$$n - 1 = 30 \Rightarrow n = 31$$

So, 31^{st} term is 120 more than 21^{st} term.

95. If S_n the sum of first n terms of an AP is given by $S_n = 3n^2 - 4n$, find the n^{th} term.

Ans : [Board 2019 Delhi]

We have $S_n = 3n^2 - 4n$

Substituting $n = 1$, we get

$$S_1 = 3 \times 1^2 - 4 \times 1 = -1$$

So, sum of first term of AP is -1, but sum of first term is the first term itself,

Thus first term $a_1 = -1$

e317

Now substituting $n = 2$ we have

$$S_2 = 3 \times 2^2 - 4 \times 2 = 4$$

Sum of first two terms is 4.

$$a_1 + a_2 = 4$$

$$-1 + a_2 = 4 \Rightarrow a_2 = 5$$

Hence, common difference,

$$d = a_2 - a_1 = 5 - (-1) = 6$$

Now n^{th} term, $a_n = a_1 + (n-1)d$

, $a_n = -1 + (n-1)6$

$$a_n = 6n - 7$$

Therefore, n^{th} term is $6n - 7$.

96. Find the 21^{st} term of the AP $-4\frac{1}{2}, -3, -1\frac{1}{2}, ...$

Ans : [Board 2019 OD]

Given AP is $-4\frac{1}{2}, -3, -1\frac{1}{2}, \dots$ or $-\frac{9}{2}, -3, -\frac{3}{2}, \dots$

First term, $a = \frac{-9}{2}$

Common difference,

$$d = -3 - \left(-\frac{9}{2}\right) = -3 + \frac{9}{2}$$

$$= \frac{-6+9}{2} = \frac{3}{2}$$

e323

Now $a_n = a + (n-1)d$

$$a_{21} = \left(-\frac{9}{2}\right) + (21-1)\left(\frac{3}{2}\right)$$

$$= -\frac{9}{2} + 20 \times \frac{3}{2} = -\frac{9}{2} + 30$$

$$= \frac{-9+30}{2} = \frac{51}{2} = 25\frac{1}{2}$$

Hence, 21^{st} term of given AP is $25\frac{1}{2}$.

97. If the sum of first n terms of an AP is n^2, then find its 10th term.

Ans : [Board 2019 Delhi]

We have $S_n = n^2$ ...(1)

Substituting $n = 1$ in equation (1), we have

$$S_1 = 1$$

Hence, sum of first term of AP is 1, but sum of first term is first term itself.

So, first term, $a = 1$ ...(2)

Substituting $n = 2$ in equation (1), we have

$$S_2 = (2)^2 = 4$$

Sum of first 2 terms is 4.

Now $a + a_2 = 4$ ...(3)

From equation (2) and (3) we have

$$a_2 = 3$$

e306

Now, common difference,

$$d = a_2 - a = 3 - 1 = 2$$

Now, 10^{th} term of AP,

$$a_{10} = a + (10 - 1)d$$

$$= 1 + 9 \times 2 = 19$$

Hence, the 10^{th} term of AP is 19.

98. Is 184 a term of the sequence $3, 7, 11, ...?$

Ans : [Board Term-2 2012]

Let the first term of an AP be a, common difference be d and number of terms be n.

Let $a_n = 184$

Here, $a = 3, d = 7 - 3 = 11 - 7 = 4$

e121

Now $a_n = a + (n-1)d,$

$$184 = 3 + (n-1)4$$

$$\frac{181}{4} = n - 1$$

$$45.25 = n - 1$$

$$46.25 = n$$

Since 46.25 is not an whole number, thus 184 is not a term of given AP

99. Find, 100 is a term of the AP $25, 28, 31, ...$ or not.

Ans : [Board Term-2 2012]

Let the first term of an AP be a, common difference be d and number of terms be n.

Let $a_n = 100$

Here $a = 25, d = 28 - 25 = 31 - 28 = 3$

e122

Now $a_n = a + (n-1)d,$

$$100 = 25 + (n-1) \times 3$$

$$100 - 25 = 75 = (n-1) \times 3$$

$$25 = n - 1$$

$$n = 26$$

Since 26 is an whole number, thus 100 is a term of given AP.

100. Find the 7^{th} term from the end of AP $7, 10, 13, ... 184$.

Ans : [Board Term-2 2012]

Let us write AP in reverse order i.e., $184, ... 13, 10, 7$

Let the first term of an AP be a and common difference be d.

Now $d = 7 - 10 = -3$

$$a = 184, \ n = 7$$

e123

7^{th} term from the original end,

$$a_7 = a + 6d$$

$$a_7 = 184 + 6(-3)$$

$$= 184 - 18 = 166.$$

Hence, 166 is the 7^{th} term from the end.

101. Which term of an AP 150, 147, 144, ... is its first negative term?

Ans : [KVS 2014]

Let the first term of an AP be a, common difference be d and nth term be a_n.

For first negative term $a_n < 0$

$$a + (n-1)d < 0$$

$$150 + (n-1)(-3) < 0$$

e124

$$150 - 3n + 3 < 0$$

$$-3n < -153$$

$$n > 51$$

Therefore, the first negative term is 52^{nd} term.

102. In a certain AP 32^{th} term is twice the 12^{th} term. Prove that 70^{th} term is twice the 31^{st} term.

Ans : [Board Term-2 2015, 2012]

Let the first term of an AP be a, common difference be d and nth term be a_n.

Now we have
$$a_{32} = 2a_{12}$$
$$a + 31d = 2(a + 11d)$$
$$a + 31d = 2a + 22d$$
$$a = 9d$$
$$a_{70} = a + 69d$$
$$= 9d + 69d = 78d$$
$$a_{31} = a + 30d$$
$$= 9d + 30d = 39d$$
$$a_{70} = 2a_{31}$$
Hence Proved.

103. The 8^{th} term of an AP is zero. Prove that its 38^{th} term is triple of its 18^{th} term.

Ans : [Board Term-2 2012]

Let the first term of an AP be a, common difference be d and nth term be a_n.

We have, $a_8 = 0$ or, $a + 7d = 0$ or, $a = -7d$

Now
$$a_{38} = a + 37d$$
$$a_{38} = -7d + 37d = 30d$$
$$a_{18} = a + 17d$$
$$= -7d + 17d = 10d$$
$$a_{38} = 30d = 3\times 10d = 3\times a_{18}$$
$$a_{38} = 3a_{18}$$
Hence Proved

104. If five times the fifth term of an AP is equal to eight times its eighth term, show that its 13^{th} term is zero.

Ans : [Board Term-2 2012]

Let the first term of an AP be a, common difference be d and nth term be a_n.

Now
$$5a_5 = 8a_8$$
$$5(a + 4d) = 8(a + 7d)$$
$$5a + 20d = 8a + 56d$$
$$3a + 36d = 0$$
$$3(a + 12d) = 0$$
$$a + 12d = 0$$
$$a_{13} = 0$$
Hence Proved

105. The fifth term of an AP is 20 and the sum of its seventh and eleventh terms is 64. Find the common difference.

Ans : [Board Term-2 Foreign 2015]

Let the first term be a and common difference be d.
$$a + 4d = 20 \quad ...(1)$$
$$a + 6d + a + 10d = 64$$
$$a + 8d = 32 \quad ...(2)$$
Solving equations (1) and (2), we have
$$d = 3$$

106. The ninth term of an AP is -32 and the sum of its eleventh and thirteenth term is -94. Find the common difference of the AP

Ans : [Board Term-2 Foreign 2015]

Let the first term be a and common difference be d.

Now
$$a + 8d = a_9$$
$$a + 8d = -32 \quad ...(1)$$
and
$$a_{11} + a_{13} = -94$$
$$a + 10d + a + 12d = -94$$
$$a + 11d = -47 \quad ...(2)$$
Solving equation (1) and (2), we have
$$d = -5$$

107. The seventeenth term of an AP exceeds its 10^{th} term by 7. Find the common difference.

Ans : [Board Term-2 2015, 2014]

Let the first term be a and common difference be d.

Now
$$a_{17} = a_{10} + 7$$
$$a + 16d = a + 9d + 7$$
$$16d - 9d = 7$$
$$7d = 7$$
$$d = 1$$
Thus common difference is 1.

108. The fourth term of an AP is 11. The sum of the fifth and seventh terms of the AP is 34. Find the c difference.

Ans : [Foreign

Let the first term be a and common difference be d.

Now $a_4 = 11$

$$a + 3d = 11 \quad ...(1)$$

and $a_5 + a_7 = 34$

$$a + 4d + a + 6d = 34$$

$$2a + 10d = 34$$

$$a + 5d = 17 \quad ...(2)$$

Solving equations (1) and (2) we have

$$d = 3$$

109. Find the middle term of the AP 213, 205, 197, ... 37.

Ans : [Board Term-2 Delhi 2015]

Let the first term of an AP be a, common difference be d and number of terms be m.

Here, $a = 213, d = 205 - 213 = -8, a_m = 37$

$$a_m = a + (m-1)d$$

$$37 = 213 + (m-1)(-8)$$

$$37 - 213 = -8(m-1)$$

$$m - 1 = \frac{-176}{-8} = 22$$

$$m = 22 + 1 = 23$$

The middle term will be $= \frac{23+1}{2} = 12^{th}$

$$a_{12} = a + (12-1)d$$

$$= 213 + (12-1)(-8)$$

$$= 213 - 88 = 125$$

Middle term will be 125.

110. Find the middle term of the AP 6, 13, 20, ... 216.

Ans : [Board Term-2 Delhi 2015]

Let the first term of an AP be a, common difference be d and number of terms be m.

Here, $a = 6, a_m = 216, d = 13 - 6 = 7$

$$a_m = a + (m-1)d$$

$$216 = 6 + (m-1)(7)$$

$$216 - 6 = 7(m-1)$$

$$m - 1 = \frac{210}{7} = 30$$

$$m = 30 + 1 = 31$$

The middle term will be $= \frac{31+1}{2} = 16^{th}$

$$a_{16} = a + (16-1)d$$

$$= 6 + (16-1)(7)$$

$$= 6 + 15 \times 7$$

$$= 6 + 105 = 111$$

Middle term will be 111.

111. If the 2^{nd} term of an AP is 8 and the 5^{th} term is 17, find its 19^{th} term.

Ans : [Board Term-2 2016]

Let the first term be a and common difference be d.

Now $a_2 = a + d$

$$8 = a + d \quad (1)$$

and $a_5 = a + 4d$

$$17 = a + 4d \quad (2)$$

Solving (1) and (2), we have

$$a = 5, d = 3,$$

$$a_{19} = a + 18d$$

$$= 5 + 54 = 59$$

112. If the number $x + 3, 2x + 1$ and $x - 7$ are in AP find the value of x.

Ans : [Board Term-2 2012]

If x, y and z are three consecutive terms of an AP then we have

$$y - x = z - y$$

$$(2x+1) - (x+3) = (x-7) - (2x+1)$$

$$2x + 1 - x - 3 = x - 7 - 2x - 1$$

$$x - 2 = -x - 8$$

$$2x = -6$$

$$x = -3$$

113. Find the values of a, b and c, such that the numbers a, 10, b, c, 31 are in AP

Ans : [Board Term-2 2012]

Let the first term be a and common difference be d.

Since $a, 10, b, c, 31$ are in AP, then

$$a + d = 10 \quad (1)$$

$$a + 4d = a_5$$

$$a + 4d = 31 \quad (2)$$

Solving (1) and (2) we have

$$d = 7 \text{ and } a = 3$$

e136

Now $a = 3, b = 3 + 14 = 17, c = 3 + 21 = 24$

Thus $a = 3, b = 17, c = 24$.

114. For AP show that $a_p + a_{p+2q} = 2a_{p+q}$.

Ans : [Board Term-2 2012]

Let the first term be a and the common difference be d. Let a_n be the nth term.

e137

$$a_p = a + (p-1)d$$

$$a_{p+2q} = a + (p + 2q - 1)d$$

$$a_p + a_{p+2q} = a + (p-1)d + a + (p + 2q - 1)d$$

$$= a + pd - d + a + pd + 2qd - d$$

$$= 2a + 2pd + 2qd - 2d$$

or $a_p + a_{p+2q} = 2[a + (p + q - 1)d]$ ...(1)

But $2a_{p+q} = 2[a + (p + q - 1)d]$ ...(2)

From (1) and (2), we get $a_p + a_{p+2q} = 2a_{p+q}$

115. The sum of first terms of an AP is given by $S_n = 2n^2 + 8n$. Find the sixteenth term of the AP.

Ans : [Board SQP 2017]

Let the first term be a, common difference be d and nth term be a_n.

Now $S_n = 2n^2 + 3n$

$$S_1 = 2 \times 1^2 + 3 \times 1 = 2 + 3 = 5$$

e138

Since $S_1 = a_1$,

$$a_1 = 5$$

$$S_2 = 2 \times 2^2 + 3 \times 2 = 8 + 6 = 14$$

$$a_1 + a_2 = 14$$

$$a_2 = 14 - a_1 = 14 - 5 = 9$$

$$d = a_2 - a_1 = 9 - 5 = 4$$

$$a_{16} = a + (16 - 1)d$$

$$= 5 + 15 \times 4 = 65$$

116. The 4^{th} term of an AP is zero. Prove that the 25^{th} term of the AP is three times its 11^{th} term.

Ans : [Board Term-2 OD 2016]

Let the first term be a, common difference be d and nth term be a_n.

e139

We have, $a_4 = 0$

$$a + 3d = 0 \quad [a + (n-1)d = a_n]$$

$$3d = -a$$

$$-3d = a \quad ...(1)$$

Now, $a_{25} = a + 24d = -3d + 24d = 21d$ ...(2)

$$a_{11} = a + 10d = -3d + 10d = 7d \quad ...(3)$$

From equation (2) and (3) we have

$$a_{25} = 3a_{11}$$

Hence Proved.

117. How many terms of the AP $65, 60, 55, ...$ be taken so that their sum is zero?

Ans : [Board Term-2 Delhi 2015]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We have $a = 65, d = -5, S_n = 0$

Now $S_n = \frac{n}{2}[2a + (n-1)d]$

e162

Let sum of n term be zero, then we have

$$\frac{n}{2}[130 + (n-1)(-5)] = 0$$

$$\frac{n}{2}[130 + 5n + 5] = 0$$

$$135n - 5n^2 = 0$$

$$n(135 - 5n) = 0$$

$$5n = 135$$

$$n = 27$$

118. How many terms of the AP $18, 16, 14 .....$ be taken so that their sum is zero?

Ans : [Board Term-2 Delhi 2016]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

Here $a = 18, d = -2, S_n = 0$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

e163

Let sum of n term be zero, then we have

$$\frac{n}{2}[36+(n-1)(-2)] = 0$$

$$n(38-2n) = 0$$

$$n = 19$$

119. How many terms of the AP $27, 24, 21....$ should be taken so that their sum is zero?

Ans : [Board Term-2 Delhi 2016]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

e164

Here $a = 27, d = -3, S_n = 0$

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

Let sum of n term be zero, then we have

$$\frac{n}{2}[54+(n-1)(-3)] = 0$$

$$n(-3n+57) = 0$$

$$n = 19$$

120. In an AP, if $S_5 + S_7 = 167$ and $S_{10} = 235$, then find the AP, where S_n denotes the sum of first n terms.

Ans : [Board Term-2 OD 2015]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

e165

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$S_5 + S_7 = 167$$

$$\frac{5}{2}(2a+4d)+\frac{7}{2}(2a+6d) = 167$$

$$5a+10d+7a+21d = 167$$

$$12a+31d = 167 \quad ...(1)$$

Now we have $S_{10} = 235$, thus

$$\frac{10}{2}[2a+(10-1)d] = 235$$

$$5(2a+9d) = 235$$

$$2a+9d = 47 \quad (2)$$

Solving (1) and (2), we get

$$a = 1, d = 5$$

Thus AP is 1, 6, 11....

121. Find the sum of sixteen terms of an AP $-1, -5, -9, ...$

Ans : [Board Term-2 2012]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

Here, $a_1 = -1, a_2 = -5$ and $d = -4$

e166

Now $S_n = \frac{n}{2}[2a+(n-1)d]$

$$S_{16} = \frac{16}{2}[2\times(-1)+(16-1)(-4)]$$

$$= 8[-2-60] = 8(-62)$$

$$= -496$$

122. If the n^{th} term of an AP is $7-3n$, find the sum of twenty five terms.

Ans : [Board Term-2 2012]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

Here $n = 25, a_n = 7-3n$

Taking $n = 1,2,3,....$ we have

e167

$$a_1 = 7-3\times1 = 4$$

$$a_2 = 7-3\times2 = 1$$

$$a_3 = 7-3\times3 = -2$$

Thus required AP is $4, 1, -2, ...$

Here, $a = 4, d = 1-4 = -3$

Now, $S_n = \frac{n}{2}[2a+(n-1)d]$

$$= \frac{25}{2}[2\times4+(25-1)(-3)]$$

$$= \frac{25}{2}[8+24(-3)]$$

$$= \frac{25}{2}(8-72) = -800$$

123. If the 1^{st} term of a series is 7 and 13^{th} term is 35. Find the sum of 13 terms of the sequence.

Ans : [Board Term-2 2012]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

Here $a = 7, a_{13} = 35$

$$a_n = a+(n-1)d$$
$$a_{13} = a+12d$$
$$35 = 7+12d \Rightarrow d = \frac{7}{3}$$

e168

Now $$S_n = \frac{n}{2}[2a+(n-1)d]$$
$$S_{13} = \frac{13}{2}\left[2\times7+12\times\left(\frac{7}{3}\right)\right]$$
$$= \frac{13}{2}[14+28]$$
$$= \frac{13}{2}\times42 = 273$$

124. If the n^{th} term of a sequence is $3-2n$. Find the sum of fifteen terms.

Ans : [Board Term-2 2012]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

Here, $a_n = 3-2n$

Taking $n=1$, $a_1 = 3-2 = 1$

15th term, $a_{15} = 3-2\times15 = 3-30 = -27$

Now $$S_n = \frac{n}{2}(a+1)$$
$$S_{15} = \frac{15}{2}[1+(-27)]$$

e169

$$= \frac{15}{2}[-26]$$
$$= 15\times(-13) = -195$$

125. If S_n denotes the sum of n terms of an AP whose common difference is d and first term is a, find $S_n - 2S_{n-1} + S_{n-2}$.

Ans : [Board Term-2 2011]

e170

We have $$a_n = S_n - S_{n-1}$$
$$a_{n-1} = S_{n-1} - S_{n-2}$$
$$S_n - 2S_{n-1} + S_{n-2} = S_n - S_{n-1} - S_{n-1} + S_{n-2}$$
$$= (S_n - S_{n-1}) - (S_{n-1} - S_{n-2})$$
$$= a_n - a_{n-1} = d$$

126. The sum of first n terms of an AP is $5n - n^2$. Find the n^{th} term of the AP

Ans : [Board Term-2 Foreign 2014]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We have, $S_n = 5n - n^2$

e171

Now, n^{th} term of AP,

$$a_n = S_n - S_{n-1}$$
$$= (5n-n^2) - [5(n-1)-(n-1)^2]$$
$$= 5n - n^2 - [5n-5-(n^2+1-2n)]$$
$$= 5n - n^2 - (5n-5-n^2-1+2n)$$
$$= 5n - n^2 - 7n + 6 + n^2$$
$$= -2n+6$$
$$a_n = -2(n-3)$$

Thus n^{th} term is $= -2(n-3)$

127. The first and last term of an AP are 5 and 45 respectively. If the sum of all its terms is 400, find its common difference.

Ans : [Board Term-2 2012]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We have $a = 5, a_n = 45$

Now $$45 = 5+(n-1)d$$
$$(n-1)d = 40 \qquad ...(1)$$

Given, $$S_n = 400$$

Now $$S_n = \frac{n}{2}(a+a_n)$$

e172

$$400 = \frac{n}{2}(5+45)$$
$$800 = 50n$$
$$n = 16$$

Substituting this value of n in (1) we have

$$(n-1)d = 40$$
$$15d = 40$$
$$d = \frac{40}{15} = \frac{8}{3}$$

128. If the sum of the first 7 terms of an AP is 49 and that of the first 17 terms is 289, find the sum of its first n terms.

Ans : [Board Term-2 Foreign 2012]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

e173

Now $$S_7 = \frac{7}{2}(2a+6d) = 49$$

$$a+3d = 7 \qquad ...(1)$$

and $$S_{17} = \frac{17}{2}(2a+16d) = 289$$

$$a+8d = 17$$

Subtracting (1) from (2), we get

$$5d = 10 \Rightarrow d = 2$$

Substituting this value of d in (1) we have

$$a = 1$$

Now $$S_n = \frac{n}{2}[2\times 1+(n-1)2]$$

$$= \frac{n}{2}[2+2n-2] = n^2$$

Hence, sum of n terms is n^2.

129. How many terms of the AP $-6, \frac{-11}{2}, -5, -\frac{9}{2}....$ are needed to give their sum zero.

Ans : [Board Term-2 OD Compt. 2017]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We have $a = -6, d = -\frac{11}{2}-(-6) = \frac{1}{2}$

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

e174

Let sum of n term be zero, then we have

$$\frac{n}{2}\left[2\times -6+(n-1)\frac{1}{2}\right] = 0$$

$$\frac{n}{2}\left[-12+\frac{n}{2}-\frac{1}{2}\right] = 0$$

$$\frac{n}{2}\left[\frac{n}{2}-\frac{25}{2}\right] = 0$$

$$n^2-25n = 0$$

$$n(n-25) = 0$$

$$n = 25$$

Hence 25 terms are needed.

130. Which term of the AP $3, 12, 21, 30, ...$ will be 90 more than its 50^{th} term.

Ans : [Board Term-2 Compt. 2017]

Let the first term be a, common difference be d and nth term be a_n.

We have $a = 3, d = 9$

e175

Now $$a_n = a+(n-1)d$$

$$a_{50} = 3+49\times 9 = 444$$

Now, $$a_n - a_{50} = 90$$

$$3+(n-1)9-444 = 90$$

$$(n-1)9 = 90+441$$

$$(n-1) = \frac{531}{9} = 49$$

$$n = 49+1 = 50$$

131. The 10^{th} term of an AP is -4 and its 22^{nd} term is -16. Find its 38^{th} term.

Ans : [Board Term-2 Delhi Compt. 2017]

Let the first term be a, common difference be d and nth term be a_n.

$$a_{10} = a+9d = -4 \qquad (1)$$

and $$a_{22} = a+21d = -16 \qquad (2)$$

Subtracting (2) from (1) we have

$$12d = -12 \Rightarrow d = -16$$

e176

Substituting this value of d in (1) we get

$$a = 5$$

Thus $$a_{38} = 5+37\times -1 = -32$$

Hence, $a_{38} = -32$

132. Find how many integers between 200 and 500 are divisible by 8.

Ans : [Board Term-2 Delhi Compt. 2017]

Number divisible by 8 are 208, 2016, 224, ... 496. It is an AP

Let the first term be a, common difference be d and nth term be a_n.

We have $a = 208, d = 8$ and $a_n = 496$

e177

Now $$a+(n-1)d = a_n$$

$$208+(n-1)d = 496$$

$$(n-1)8 = 496-208$$

$$n-1 = \frac{288}{8} = 36$$

$$n = 36+1 = 37$$

Hence, required numbers divisible by 8 is 37.

133. The fifth term of an AP is 26 and its 10^{th} term is 51. Find the AP

Ans : [Board Term-2 OD Compt. 2017]

Let the first term be a, common difference be d and nth term be a_n.

$$a_5 = a + 4d = 26 \quad ...(1)$$

$$a_{10} = a + 9d = 51 \quad ...(2)$$

Subtracting (1) from (2) we have

$$5d = 25 \Rightarrow d = 5$$

Substituting this value of d in equation (1) we get

e178

$$a = 6$$

Hence, the AP is 6, 11, 16, ...

134. Find the AP whose third term is 5 and seventh term is 9.

Ans : [Board Term-2 Delhi Compt. 2017]

Let the first term be a, common difference be d and nth term be a_n.

Now $\quad a_3 = a + 2d = 5 \quad ...(1)$

and $\quad a_7 = a + 6d = 9 \quad ...(2)$

Subtracting (2) from (1) we have

$$4d = 4 \Rightarrow d = 1$$

Substituting this value of d in (1) we get

e179

$$a = 3$$

Hence AP is 3, 4, 5, 6, ...

135. Find whether -150 is a term of the AP $11, 8, 5, 2, ...$

Ans : [Board Term-2 Delhi Compt. 2017]

Let the first term be a, common difference be d and nth term be a_n.

Let the n^{th} term of given AP $11, 8, 5, 2, ...$ be -150

Hence $a = 11$, $d = 8 - 11 = -3$ and $a_n = -150$

$$a + (n-1)d = a_n$$

$$11 + (n-1)(-3) = -150$$

$$(n-1)(-3) = -161$$

$$(n-1) = \frac{-161}{-3} = 53\frac{2}{3}$$

e180

which is not a whole number. Hence -150 is not a term of given AP.

136. If seven times the 7^{th} term of an AP is equal to eleven times the 11^{th} term, then what will be its 18^{th} term.

Ans : [Board Term-2 Foreign 2017]

Let the first term be a, common difference be d and nth term be a_n.

$$7a_7 = 11a_{11}$$

e181

Now $\quad 7(a + 6d) = 11(a + 10d)$

$$7a + 42d = 11a + 110d$$

$$11a - 7a = 42d - 110d$$

$$4a = -68d$$

$$4a + 68d = 0$$

$$4(a + 17d) = 0$$

$$a + 17d = 0$$

Hence, $\quad a_{18} = 0$

137. In an AP of 50 terms, the sum of the first 10 terms is 210 and the sum of its last 15 terms is 2565. Find the AP

Ans : [Board Term-2 Foreign 2017]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

$$S_{10} = 210$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

e182

$$210 = \frac{10}{2}(2a + 9d)$$

$$42 = 2a + 9d \quad (1)$$

Now $\quad a_{36} = a + 35d$

$$a_{50} = a + 49d$$

Sum of last 15 terms,

$$S_{36-50} = \frac{n}{2}(a_{36} + a_{50})$$

$$2565 = \frac{15}{2}(a + 35d + a + 49d)$$

$$171 = \frac{1}{2}(2a + 84d)$$

$$171 = a + 42d \quad (2)$$

Solving (1) and (2) we get $a = 3$ and $d = 4$

Hence, AP is 3, 7, 11, ...

THREE MARKS QUESTIONS

138. The sum of four consecutive number in AP is 32 and the ratio of the product of the first and last term to the product of two middle terms is 7 : 15. Find the numbers.

Ans : [Board 2020 Delhi Standard, 2018]

Let the four consecutive terms of AP be $(a-3d)$, $(a-d)$, $(a+d)$ and $(a+3d)$.

As per question statement we have

$$a-3d+a-d+a+d+a+3d = 32$$

$$4a = 32 \Rightarrow a = 8$$

and
$$\frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{7}{15}$$

e308

$$\frac{a^2-9d^2}{a^2-d^2} = \frac{7}{15}$$

$$\frac{64-9d^2}{64-d^2} = \frac{7}{15}$$

$$960-135d^2 = 448-7d^2$$

$$7d^2-135d^2 = 448-960$$

$$-128d^2 = -512$$

$$d^2 = 4 \Rightarrow d = \pm 2$$

Hence, the number are 2, 6, 10 and 14 or 14, 10, 6 and 2.

139. The sum of the first 7 terms of an AP is 63 and that of its next 7 terms is 161. Find the AP.

Ans : [Board 2020 Delhi Standard]

We have $S_7 = 63$

Now $S_n = \frac{n}{2}[2a+(n-1)d]$

e309

$$63 = \frac{7}{2}[2a+6d]$$

$$9 = a+3d \quad ...(1)$$

Now, sum of next 7 terms,

$$S_{8-14} = 161$$

$$S_{8-14} = \frac{7}{2}(a_8+a_{14})$$

$$161 = \frac{7}{2}(a+7d+a+13d)$$

$$161 = \frac{7}{2}(2a+20d)$$

$$23 = a+10d \quad ...(2)$$

Subtracting equation (1) from (2) we have

$$14 = 7d \Rightarrow d = 2$$

Substituting the value of d in (1), we get

$$a = 3$$

Hence, the AP is 3, 5, 7, 9, ...

140. Which term of the AP 20, $19\frac{1}{4}$, $18\frac{1}{2}$, $17\frac{3}{4}$, ... is the first negative term.

Ans : [Board 2020 OD Standard]

Here, $a = 20$

and $d = \frac{77}{4}-20 = -\frac{3}{4}$

e313

Let a_n is the first negative term, thus $a_n < 0$.

Now $a_n = a+(n-1)d$

$$20+(n-1)\left(-\frac{3}{4}\right) < 0$$

$$80-3n+3 < 0$$

$$83-3n < 0$$

$$n > \frac{83}{3} \quad n > 27.6$$

$$n = 28$$

Hence, the first negative term is 28th term.

141. Find the middle term of the AP 7, 13, 19, ..., 247.

Ans : [Board 2020 OD Standard]

In this AP, $a = 7$

$$d = 13-7 = 6$$

e314

$$a_n = a+(n-1)d$$

$$247 = 7+(n-1)6$$

$$6(n-1) = 240$$

$$n-1 = 40 \Rightarrow n = 41$$

Hence, the middle term$= \frac{n+1}{2} = \frac{41+1}{2} = \frac{42}{2} = 21.$

$$a_{21} = 7+(21-1)6 = 127$$

142. Show that the sum of all terms of an AP whose first term is a, the second term is b and last term is c, is equal to $\dfrac{(a+c)(b+c-2a)}{2(b-a)}$

Ans : [Board 2020 OD Standard]

Given, first term, $A = a$

and second term $A_2 = b$

Common difference, $D = b-a$

Last term, $A_n = c$

e310

$$A+(n-1)d = c$$

$$a+(n-1)(b-a) = c$$

$$(b-a)(n-1) = c-a$$

$$n-1 = \frac{c-a}{b-a}$$

$$n = \frac{c-a}{b-a}+1$$

$$= \frac{c-a+b-a}{b-a}$$

$$n = \frac{b+c-2a}{b-a}$$

Now sum of all terms

$$S_n = \frac{n}{2}[A+A_n] = \frac{(b+c-2a)}{2(b-a)}[a+c]$$

$$= \frac{(a+c)(b+c-2a)}{2(b-a)}$$ Hence Proved

143. If in an AP, the sum of first m terms is n and the sum of its first n terms is m, then prove that the sum of its first $(m+n)$ terms is $-(m+n)$.

Ans : [Board 2020 OD Standard]

Let 1st term of series be a and common difference be d, then we have

e311

$$S_m = n$$

and $$S_n = m$$

$$\frac{m}{2}[2a+(m-1)d] = n \quad ...(1)$$

$$\frac{n}{2}[2a+(n-1)d] = m \quad ...(2)$$

Subtracting we have

$$a(m-n)+\frac{d}{2}[m(m-1)-n(n-1)] = n-m$$

$$2a(m-n)+d[m^2-n^2-(m-n)] = 2(n-m)$$

$$2a(m-n)+d(m-n)[(m+n)-1] = 2(n-m)$$

$$2a+d[(m+n)-1] = -2$$

Now, $$S_{m+n} = \frac{m+n}{2}[2a+(m+n-1)d]$$

$$= \frac{m+n}{2}(-2)$$

$$= -(m+n)$$

144. The 17th term of an AP is 5 more than twice its 8th term. If 11th term of AP is 43, then find its n^{th} term.

Ans : [Board 2020 OD Basic]

Let a be the first term and d be the common difference.

n^{th} term of an AP,

$$a_n = a+(n-1)d$$

Since 17th term of an AP is 5 more than twice of its 8th term, thus

e301

$$a+(17-1)d = 5+2[a+(8-1)d]$$

$$a+16d = 5+2(a+7d)$$

$$a+16d = 5+2a+14d$$

$$2d-a = 5 \quad ...(1)$$

Since 11th term of AP is 43,

$$a+(11-1)d = 43$$

$$a+10d = 43 \quad ...(2)$$

Solving equation (1) and (2), we have

$$a = 3 \text{ and } d = 4$$

Hence, n^{th} term would be

$$a_n = 3+(n-1)4 = 4n-1$$

145. How many terms of the AP 24, 21, 18, ... must be taken so that their sum is 78?

Ans : [Board 2020 Delhi Basic]

Given : 24, 21, 18, ... are in AP.

e302

Here, $a = 24,\ d = 21-24 = -3$

Sum of n term, $$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$78 = \frac{n}{2}[2\times 24+(n-1)(-3)]$$

$$156 = n(48-3n+3)$$

$$156 = n(51-3n)$$
$$3n^2 - 51n + 156 = 0$$
$$n^2 - 17n + 52 = 0$$
$$n^2 - 13n - 4n + 52 = 0$$
$$(n-4)(n-13) = 0 \Rightarrow n = 4, 13$$

When $n = 4$, $S_4 = \frac{4}{2}[2\times 24 + (4-1)(-3)]$

$$= 2(48-9) = 2\times 39 = 78$$

When $n = 13$, $S_{13} = \frac{13}{2}[2\times 24 + (13-1)(-3)]$

$$= \frac{13}{2}[48 + (-36)] = 78$$

Hence, the number of terms $n = 4$ or $n = 13$.

146. Find the 20^{th} term of an AP whose 3^{rd} term is 7 and the seventh term exceeds three times the 3^{rd} term by 2. Also find its n^{th} term (a_n).

Ans : [Board Term-2 2012]

Let the first term be a, common difference be d and nth term be a_n.

We have $a_3 = a + 2d = 7$ (1)

$$a_7 = 3a_3 + 2$$

$$a + 6d = 3\times 7 + 2 = 23 \quad (2)$$

Solving (1) and (2) we have

$$4d = 16 \Rightarrow d = 4$$
$$a + 8 = 7 \Rightarrow a = -1$$
$$a_{20} = a + 19d = -1 + 19\times 4 = 75$$
$$a_n = a + (n-1)d$$
$$= -1 + 4n - 4$$
$$= 4n - 5.$$

e140

Hence n^{th} term is $4n-5$.

147. If 7^{th} term of an AP is $\frac{1}{9}$ and 9^{th} term is $\frac{1}{7}$, find 63^{rd} term.

Ans : [Board Term-2 Delhi 2014]

Let the first term be a, common difference be d and nth term be a_n.

We have $a_7 = \frac{1}{9} \Rightarrow a + 6d = \frac{1}{9}$ (1)

$$a_9 = \frac{1}{7} \Rightarrow a + 8d = \frac{1}{7} \quad (2)$$

Subtracting equation (1) from (2) we get

$$2d = \frac{1}{7} - \frac{1}{9} = \frac{2}{63} \Rightarrow d = \frac{1}{63}$$

e141

Substituting the value of d in (2) we get

$$a + 8\times\frac{1}{63} = \frac{1}{7}$$

$$a = \frac{1}{7} - \frac{8}{63} = \frac{9-8}{63} = \frac{1}{63}$$

Thus $a_{63} = a + (63-1)d$

$$= \frac{1}{63} + 62\times\frac{1}{63} = \frac{1+62}{63}$$

$$= \frac{63}{63} = 1$$

Hence, $a_{63} = 1$.

148. The ninth term of an AP is equal to seven times the second term and twelfth term exceeds five times the third term by 2. Find the first term and the common difference.

Ans : [Board SQP 2016]

Let the first term be a, common difference be d and nth term be a_n.

Now $a_9 = 7a_2$

e142

$$a + 8d = 7(a+d)$$
$$a + 8d = 7a + 7d$$
$$-6a + d = 0 \quad (1)$$

and $a_{12} = 5a_3 + 2$

$$a + 11d = 5(a+2d) + 2$$
$$a + 11d = 5a + 10d + 2$$
$$-4a + d = 2 \quad ...(2)$$

Subtracting (2) from (1), we get

$$-2a = -2$$
$$a = 1$$

Substituting this value of a in equation (1) we get

$$-6 + d = 0$$
$$d = 6$$

Hence first term is 1 and common difference is 6.

149. Determine an AP whose third term is 9 and when fifth term is subtracted from 8^{th} term, we get 6.

Ans : [Board Term-2 2015]

Let the first term be a, common difference be d and

nth term be a_n.

We have $a_3 = 9$

$$a + 2d = 9 \quad ...(1)$$

and $a_8 - a_5 = 6$

$$(a + 7d) - (a + 4d) = 6$$

$$3d = 6$$

$$d = 2$$

Substituting this value of d in (1), we get

$$a + 2(2) = 9$$

$$a = 5$$

So, AP is 5, 7, 9, 11, ...

150. Divide 56 in four parts in AP such that the ratio of the product of their extremes (1^{st} and 4^{rd}) to the product of means (2^{nd} and 3^{rd}) is 5:6.

Ans : [Board Term-2 Foreign 2016]

Let the four numbers be $a - 3d, a - d, a + d, a + 3d$

Now $a - 3d + a - d + a + d + a + 3d = 56$

$$4a = 56 \Rightarrow a = 14$$

Hence numbers are $14 - 3d, 14 - d, 14 + d, 14 + 3d$

Now, according to question, we have

$$\frac{(14 - 3d)(14 + 3d)}{(14 - d)(14 + d)} = \frac{5}{6}$$

$$\frac{196 - 9d^2}{196 - d^2} = \frac{5}{6}$$

$$6(196 - 9d^2) = 5(196 - d^2)$$

$$6 \times 196 - 54d^2 = 5 \times 196 - 5d^2$$

$$(6 - 5) \times 196 = 49d^2$$

$$d^2 = \frac{196}{49} = 4$$

$$d = \pm 2$$

Thus numbers are $a - 3d = 14 - 3 \times 2 = 8$

$$a - d = 14 - 2 = 12$$

$$a + d = 14 + 2 = 16$$

$$a + 3d = 14 + 3 \times 2 = 20$$

Thus required AP is $8, 12, 16, 20$.

151. [illegible] are a, b and c respectively, Show that $a(q - r) + b(r - p) + c(p - q) = 0$.

Ans : [Board Term-2 Foreign 2016]

Let the first term be A and the common difference be D.

$$a = A + (p - 1)D$$

$$b = A + (q - 1)D$$

$$c = A + (r - 1)D$$

Now $a(q - r) = [A + (p - 1)D][q - r]$

$$b(r - p) = [A + (q - 1)D][r - p]$$

and $c[p - q] = [A + (r - 1)D][p - q]$

$$a(q - r) + b(r - p) + c(p - q)$$

$$= [A + (p - 1)D][q - r] + $$
$$+ [A + (q - 1)D][r - p] + $$
$$+ [A + (r - 1)D][p - q] + $$

$$= A[p - q + q - p + q - r] + $$
$$+ D(p - 1)(q - r) + $$
$$+ D(q - 1)(r - p) + $$
$$+ D(r - 1)(p - q)$$

$$= A[0] + $$
$$+ D[p(q - r) - (q - r)]$$
$$+ D[q(r - p) - (r - p)]$$
$$+ D[r(p - q) - (p - q)]$$

$$= D[p(q - r) + q(r - p) + r(p - q)] + $$
$$- D[(q - r) + (r - p) + (p - q)]$$

$$= D[pq - pr + qr - qp + rp - rq] + 0$$

$$= D[0] = 0$$

152. The sum of n terms of an AP is $3n^2 + 5n$. Find the AP Hence find its 15^{th} term.

Ans : [Board Term-2 2013, 2012]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

Now $S_n = 3n^2 + 5n$

$$S_{n-1} = 3(n - 1)^2 + 5(n - 1)$$

$$= 3(n^2 + 1 - 2n) + 5n - 5$$

$$= 3n^2 + 3 - 6n + 5n - 5$$

$$= 3n^2 - n - 2$$

$$a_n = S_n - S_{n-1}$$

$$= 3n^2 + 5n - (3n^2 - n - 2)$$

$$= 6n + 2$$

Thus AP is 8, 14, 20, ...

Now $a_{15} = a + 14d = 8 + 14(6) = 92$

153. For what value of n, are the n^{th} terms of two APs 63, 65, 67, ... and 3, 10, 17, ... equal?

Ans :

Let a, d and A, D be the 1^{st} term and common difference of the 2 APs respectively.

n is same

For 1st AP, $a = 63,\ d = 2$

e148

For 2nd AP, $A = 3, D = 7$

Since nth term is same,

$$a_n = A_n$$

$$a + (n-1)d = A + (n-1)D$$

$$63 + (n-1)2 = 3 + (n-1)7$$

$$63 + 2n - 2 = 3 + 7n - 7$$

$$61 + 2n = 7n - 4$$

$$65 = 5n \Rightarrow n = 13$$

When n is 13, the n^{th} terms are equal i.e., $a_{13} = A_{13}$

154. In an AP the sum of first n terms is $\frac{3n^2}{2} + \frac{13n}{2}$. Find the 25^{th} term.

Ans : [Board Term-2 SQP 2015]

We have $S_n = \dfrac{3n^2 + 13n}{2}$

e183

$$a_n = S_n - S_{n-1}$$

$$a_{25} = S_{25} - S_{24}$$

$$= \frac{3(25)^2 + 13(25)}{2} - \frac{3(24)^2 + 13(24)}{2}$$

$$= \frac{1}{2}\{3(25^2 - 24^2) + 13(25 - 24)\}$$

$$= \frac{1}{2}(3 \times 49 + 13) = 80$$

155. The sum of first n terms of three arithmetic progressions are S_1, S_2 and S_3 respectively. The first term of each AP is 1 and common differences are 1, 2 and 3 respectively. Prove that $S_1 + S_3 = 2S_2$.

Ans : [Board Term-2 OD 2016]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We have $S_1 = 1 + 2 + 3 + ...n$

$S_2 = 1 + 3 + 5 + ...$ up to n terms

$S_3 = 1 + 4 + 7 + ...$ upto n terms

Now $S_n = \dfrac{n(n+1)}{2}$

e184

$$S_2 = \frac{n}{2}[2 + (n-1)2] = \frac{n}{2}[2n] = n^2$$

and $S_3 = \dfrac{n}{2}[2 + (n-1)3] = \dfrac{n(3n-1)}{2}$

Now, $S_1 + S_3 = \dfrac{n(n+1)}{2} + \dfrac{n(3n-1)}{2}$

$$= \frac{n[n + 1 + 3n - 1]}{2} = \frac{n[4n]}{2}$$

$$= 2n^2 = 2s_2$$ Hence Proved

156. If S_n denotes, the sum of the first n terms of an AP prove that $S_{12} = 3(S_8 - S_4)$.

Ans : [Board Term-2 Delhi 2015]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

e185

$$S_{12} = 6[2a + 11d] = 12a + 66d$$

$$S_8 = 4[2a + 7d] = 8a + 28d$$

$$S_4 = 2[2a + 3d] = 4a + 6d$$

$$3(S_8 - S_4) = 3[(8a + 28d) - (4a + 6d)]$$

$$= 3[4a + 22d] = 12a + 66d$$

$$= 6[2a + 11d] = S_{12}$$ Hence Proved

157. The 14^{th} term of an AP is twice its 8^{th} term. If the 6^{th} term is -8, then find the sum of its first 20 terms.

Ans : [Board Term-2 OD 2015]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

Here, $a_{14} = 2a_8$ and $a_6 = -8$

e186

Now $a + 13d = 2(a + 7d)$

$$a+13d = 2a+14d$$

$$a = -d \qquad ...(1)$$

and $\quad a_6 = -8$

$$a+5d = -8 \qquad ...(2)$$

Solving (1) and (2), we get

$$a = 2, d = -2$$

Now $\quad S_{20} = \frac{20}{2}[2\times 2+(20-1)(-2)]$

$$= 10[4+19\times(-2)]$$

$$= 10(4-38)$$

$$= 10\times(-34) = -340$$

158. If the ratio of the sums of first n terms of two AP's is $(7n+1):(4n+27)$, find the ratio of their m^{th} terms.

Ans : [Board Term-2 OD 2016]

Let a, and A be the first term and d and D be the common difference of two AP's, then we have

$$\frac{S_n}{S'_n} = \frac{7n+1}{4n+27}$$

$$\frac{\frac{n}{2}[2a+(n-1)d]}{\frac{n}{2}[2A+(n-1)D]} = \frac{7n+1}{4n+27}$$

$$\frac{2a+(n-1)d}{2A+(n-1)D} = \frac{7n+1}{4n+27}$$

$$\frac{a+\left(\frac{n-1}{2}\right)d}{A+\left(\frac{n-1}{2}\right)D} = \frac{7n+1}{4n+27}$$

Substituting $\frac{n-1}{2} = m-1$ or $n = 2m-1$ we get

$$\frac{a+(m-1)d}{A+(m-1)D} = \frac{7(2m-1)+1}{4(2m-1)+27} = \frac{14m-6}{8m+23}$$

Hence, $\quad \frac{a_m}{A_m} = \frac{14m-6}{8m+23}$

e187

159. If the sum of the first n terms of an AP is $\frac{1}{2}[3n^2+7n]$, then find its n^{th} term. Hence write its 20^{th} term.

Ans : [Board Term-2 Delhi 2015]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

Sum of n term, $\quad S_n = \frac{1}{2}[3n^2+7n]$

e188

Sum of 1 term, $\quad S_1 = \frac{1}{2}[3\times(1)^2+7(1)]$

$$= \frac{1}{2}[3+7] = \frac{1}{2}\times 10 = 5$$

Sum of 2 term, $\quad S_2 = \frac{1}{2}[3(2)^2+7\times 2]$

$$= \frac{1}{2}[12+14] = \frac{1}{2}\times 26 = 13$$

Now $\quad a_1 = S_1 = 5$

$$a_2 = S_2 - S_1 = 13-5 = 8$$

$$d = a_2 - a_1 = 8-5 = 3$$

Now, AP is $5, 8, 11, ...$

n^{th} term, $\quad a_n = a+(n-1)d$

$$= 5+(n-1)3$$

$$= 5+(20-1)(3)$$

$$= 5+57$$

$$= 62$$

Hence, $\quad a_2 = 62$

160. In an AP, if the 12^{th} term is -13 and the sum of its first four terms is 24, find the sum of its first ten terms.

Ans : [Board Term-2 Foreign 2015]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

$$a_{12} = a+11d = -13 \qquad ...(1)$$

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

Now $\quad S_4 = 2[2a+3d] = 24$

$$2a+3d = 12 \qquad ...(2)$$

Multiplying (1) by 2 and subtracting (2) from it we get

$$(2a+22d)-(2a+3d) = -26-12$$

$$19d = -38$$

$$d = -2$$

e189

Substituting the value of d in (1) we get

$$a+11\times -2 = -13$$

$$a = -13 + 22$$

$$a = 9$$

Now, $$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{10} = \frac{10}{2}(2 \times 9 + 9 \times -2)$$

$$= 5 \times (18 - 18) = 0$$

Hence, $S_{10} = 0$

161. The tenth term of an AP, is -37 and the sum of its first six terms is -27. Find the sum of its first eight terms.

Ans : [Board Term-2 Foreign 2015]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

e190

$$a_n = a + (n-1)d$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$a + 9d = -37 \quad ...(1)$$

$$3(2a + 5d) = -27$$

$$2a + 5d = -9 \quad ...(2)$$

Multiplying (1) by 2 and subtracting (2) from it, we get

$$(2a + 18d) - (2a + 5d) = -74 + 9$$

$$13d = -65$$

$$d = -5$$

Substituting the value of d in (1) we get

$$a + 9 \times -5 = -37$$

$$a = -37 + 45$$

$$a = 8$$

Now $$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{8}{2}[2 \times 8 + (8-1)(-5)]$$

$$= 4[16 - 35]$$

$$= 4 \times -19 = -76$$

Hence, $S_n = -76$

162. Find the sum of first seventeen terms of AP whose 4^{th} and 9^{th} terms are -15 and -30 respectively.

Ans : [Board Term-2 2014]

Let the first term be a, common difference be d and nth term be a_n.

Now $$a_4 = a + 3d = -15 \quad ...(1)$$

$$a_9 = a + 8d = -30 \quad ...(2)$$

Subtracting eqn (1) from eqn (2), we obtain

$$(a + 8d) - (a + 3d) = -30 - (-15)$$

$$5d = -15 \Rightarrow d = \frac{-15}{5} = -3$$

Substituting the value of d in (1) we get

e191

$$a + 3d = -15$$

$$a + 3(-3) = -15$$

$$a = -15 + 9 = -6$$

Now $$S_{17} = \frac{17}{2}[2 \times (-6) + (17-1)(-3)]$$

$$= \frac{17}{2}[--12 + 16 \times (-3)]$$

$$= \frac{17}{2}[-12 - 48]$$

$$= \frac{17}{2}[-60] = 17 \times (-30)$$

$$= -510$$

Thus $S_{17} = -510$

163. The common difference of an AP is -2. Find its sum, if first term is 100 and last term is -10.

Ans : [Board Term-2 2014]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We have $$a = 100, d = -2, t_n = -10$$

Now $$a_n = a + (n-1)d$$

e192

$$-10 = 100 + (n-1)(-2)$$

$$-10 = 100 - 2n + 2$$

$$2n = 112$$

$$n = 56$$

Thus 56^{th} term is -10 and number of terms in AP are 56.

Now $$S_n = \frac{n}{2}(a + 1)$$

$$S_{56} = \frac{56}{2}(100 - 10)$$

$$= \frac{56}{2}(90) = 56 \times 45 = 2520$$

Thus $S_n = 2520$

164. The 16^{th} term of an AP is five times its third term. If its 10^{th} term is 41, then find the sum of its first fifteen terms.

Ans : [Board Term-2 OD 2015]

Let the first term be a, common difference be d. nth term be a_n and sum of n term be S_n.

e193

We have, $a_{16} = 5a_3$

$$a + 15d = 5(a + 2d)$$

$$4a = 5d \qquad ...(1)$$

and $a_{10} = 41$

$$a + 9d = 41 \qquad ...(2)$$

Solving (1) and (2), we get $a = 5, d = 4$

Now
$$S_{15} = \frac{15}{2}[2 \times 5 + (15 - 1) \times 4]$$

$$= \frac{15}{2}[10 + 56]$$

$$= \frac{15}{2} \times 66 = 15 \times 33 = 495$$

Thus $S_{15} = 495$

165. The 13^{th} term of an AP is four times its 3^{rd} term. If the fifth term is 16, then find the sum of its first ten terms.

Ans : [Board Term-2 OD 2015]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

Here $a_{13} = 4a_3$

$$a + 12d = 4(a + 2d)$$

$$3a = 4d \qquad ...(1)$$

and $a_5 = 16$

$$a + 4d = 16 \qquad ...(2)$$

Substituting the value of $a = \frac{4}{3}d$ in (2) we have

$$\frac{4}{3}d + 4d = 16$$

$$16d = 48 \Rightarrow d = 3$$

e194

Thus $a = 4$ and $d = 3$

Now
$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$S_{10} = \frac{10}{2}[2 \times 4 + (10 - 1)3]$$

$$= 5[8 + 27] = 5 \times 35 = 175$$

Thus $S_{10} = 175$

166. The n^{th} term of an AP is given by $(-4n + 15)$. Find the sum of first 20 terms of this AP.

Ans : [Board Term-2 2013]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We have
$$a_n = -4n + 15$$

e195

$$a_1 = -4 \times 1 + 15 = 11$$

$$a_2 = -4 \times 2 + 15 = 7$$

$$a_3 = -4 \times 3 + 15 = 3$$

$$d = a_2 - a_1 = 7 - 11 = -4$$

Now, we have $a = 11$, $d = -4$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$S_{20} = \frac{20}{2}[2 \times 11 + (20 - 1) \times (-4)]$$

$$= 10[22 - 76]$$

$$= 10 \times (-54) = -540$$

Thus $S_{20} = -540$

167. The sum of first 7 terms of an AP is 63 and sum of its next 7 terms is 161. Find 28^{th} term of AP

Ans : [Board Term-2 Foreign 2014]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

Now, $S_7 = 63$

e196

$$\frac{7}{2}[2a + 6d] = 63$$

$$2a + 6d = 18 \qquad ...(1)$$

Also, sum of next 7 terms,

$$S_{14} = S_{first7} + S_{next7} = 63 + 161$$

$$\frac{14}{2}[2a + 13d] = 224$$

$$2a + 13d = 32 \qquad (2)$$

Subtracting equation (1) form (2) we get

$$7d = 14 \Rightarrow d = 2$$

Substituting the value of d in (1) we get

$$a = 3$$

Now $$a_n = a + (n-1)d$$

$$a_{28} = 3 + 2 \times (27)$$

$$= 57$$

Thus 28^{th} term is 57.

168. The sum of first n terms of an AP is given by $S_n = 3n^2 - 4n$. Determine the AP and the 12^{th} term.

Ans : [Board Term-2 Delhi 2014, 2012]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

$$S_n = 3n^2 - 4n$$

$$S_1 = 3(1)^2 - 4(1) = -1$$

$$S_2 = 3(2)^2 - 4(2) = 4$$

$$a_1 = S_1 = -1$$

$$a_2 = S_2 - S_1 = 4 - (-1) = 5$$

$$d = a_2 - a_1 = 5 - (-1) = 6$$

e197

Thus AP is $-1, 5, 11, ...$

Now $$a_{12} = a + 11d$$

$$= -1 + 11 \times 6 = 65$$

169. Find the sum of all two digit natural numbers which are divisible by 4.

Ans : [Board Term-2 Delhi Compt. 2017]

First two digit multiple of 4 is 12 and last is 96

So, $a = 12, d = 4$. Let n^{th} term be last term $a_n = 96$

Now $$a + (n-1)d = a_n$$

$$12 + (n-1)4 = 96$$

$$(n-1)4 = 96 - 12 = 84$$

$$n - 1 = 21$$

$$n = 21 + 1 = 22$$

e198

Now, $$S_{22} = \frac{22}{2}[12 + 96]$$

$$= 11 \times 108$$

$$= 1188$$

170. Find the sum of the following series.

$5 + (-41) + 9 + (-39) + 13 + (-37) + 17 + ... + (-5) + 81 + (-3)$

Ans : [Board Term-2 Foreign 2017]

The given series can be written as sum of two series

$(5 + 9 + 13 + ... + 81) +$

$+ (-41) + (-39) + (-37) + (-35) ... (-5) + (-3)$

For the series $(5 + 9 + 13 .....81)$

$$a = 5,\ d = 4 \text{ and } a_n = 81$$

Now $$a_n = a + (n-1)d$$

$$81 = 5 + (n-1)4$$

$$81 = 5 + (n-1)4$$

$$(n-1)4 = 76 \Rightarrow n = 20$$

$$S_n = \frac{20}{2}(5 + 81) = 860$$

e199

For series $(-41) + (-39) + (-37) + ... + (-5) + (-3)$

$$a_n = -3,\ a = -41 \text{ and } d = 2$$

$$a_n = -41 + (n-1)(2)$$

$$-3 = -41 + 2n - 2 \Rightarrow n = 20$$

Now $$S_n = \frac{20}{2}[-41 + -3] = -440$$

$$\text{Sum of the series} = 860 - 440 = 420$$

171. Find the sum of n terms of the series

$\left(4 - \frac{1}{n}\right) + \left(4 - \frac{2}{n}\right) + \left(4 - \frac{3}{n}\right) + ...$

Ans : [Board Term-2 Delhi 2017]

Let sum of n term be S_n

$$s_n = \left(4 - \frac{1}{n}\right) + \left(4 - \frac{2}{n}\right) + \left(4 - \frac{3}{n}\right) + ... \text{ up to } n \text{term}$$

$$= (4 + 4 + 4 + ... \text{ up to } n \text{ terms}) +$$

$$+ \left(-\frac{1}{n} - \frac{2}{n} - \frac{3}{n} - ...... \text{ up to } n \text{ terms}\right)$$

$$= (4 + 4 + 4 + ... \text{ up to } n \text{ terms}) +$$

$$-\frac{1}{n}(1 + 2 + 3 + ... \text{ up to } n \text{ terms})$$

$$= 4n - \frac{1}{n} \times \frac{n(n+1)}{2}$$

$$= 4n - \frac{n+1}{2} = \frac{7n-1}{2}$$

e200

Hence, sum of n terms $= \frac{7n-1}{2}$

172. Find the number of multiple of 9 lying between 300 and 700.

Ans : [Board Term-2 OD Compt. 2017]

The numbers, multiple of 9 between 300 and 700 are 306, 315, 324, ... 693.

Let the first term be a, common difference be d and nth term be $a_n = 693$

$$a_n = 306 + (n-1)9$$

$$693 = 306 + (n-1)9$$

$$(n-1)9 = 693 - 306 = 387$$

$$n - 1 = \frac{387}{9} = 43$$

$$n = 43 + 1 = 44$$

e201

Hence there are 44 terms.

173. If the sum of the first 14 terms of an AP is 1050 and its first term is 10 find it 20^{th} term.

Ans : [Board Term-2 OD Compt. 2017]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We have $a = 10$, and $S_{14} = 1050$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{14} = \frac{14}{2}[2 \times 10 + (14-1)d]$$

$$1050 = 7[20 + 13d]$$

$$20 + 13d = \frac{1050}{7} = 150$$

$$13d = 130 \Rightarrow d = 10$$

$$a_{20} = a + (n-1)d$$

$$= 10 + 19 \times 10 = 200$$

e202

Hence $a_{20} = 200$

174. If the tenth term of an AP is 52 and the 17^{th} term is 20 more than the 13^{th} term, find AP

Ans : [Board Term-2 OD 2017]

Let the first term be a, common difference be d and nth term be a_n.

Now $a_{10} = 52$

$$a + 9d = 52 \qquad ...(1)$$

Also $a_{17} - a_{13} = 20$

$$a + 16d - (a + 12d) = 20$$

$$4d = 20$$

$$d = 5$$

e203

Substituting this valued d in (1), we get

$$a = 7$$

Hence AP is 7, 12, 17, 22, ...

175. Find the sum of all odd number between 0 and 50.

Ans : [Board Term-2 Delhi Compt 2017]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

Given AP is $1 + 3 + 5 + 7 + ... + 49$

Let total number of terms be n. Here $a = 1$, $d = 2$ and $a_n = 49$.

$$a_n = 1 + (n-1) \times 2$$

$$49 = 1 + 2n - 2$$

$$50 = 2n \Rightarrow n = 25$$

e204

Now $S_{25} = \frac{n}{2}(a + a_n)$

$$= \frac{25}{2}(1 + 49)$$

$$= 25 \times 25 = 625$$

Hence, Sum of odd number is 625

176. Find the sum of first 15 multiples of 8.

Ans : [Board Term-2 Delhi Compt 2017]

Let the first term be $a = 8$, common difference be $d = 8$, nth term be a_n and sum of n term be S_n.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{15} = \frac{15}{2}[2 \times 8 + (15-1)8]$$

$$= \frac{15}{2}[16 + 112]$$

$$= \frac{15}{2} \times 128 = 996$$

e205

Hence, the sum of 15 terms is 960.

177. If m^{th} term of an AP is $\frac{1}{n}$ and n^{th} term is $\frac{1}{m}$ find the sum of first mn terms.

Ans : [Board Term-2 2017]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

Now $$a_m = a+(m-1)d = \frac{1}{n} \quad ...(1)$$

$$a_n = a+(n-1)d = \frac{1}{m} \quad ...(2)$$

Subtracting (2) from (1) we get

$$(m-n)d = \frac{1}{n} - \frac{1}{m} = \frac{m-n}{mn}$$

e206

$$d = \frac{1}{mn}$$

Substituting this value of d in equation (1), we get

$$a = \frac{1}{mn}$$

Now, $$S_{mn} = \frac{mn}{2}\left(\frac{2}{mn} + (mn-1)\frac{1}{mn}\right)$$

$$= 1 + \frac{mn}{2} - \frac{1}{2} = \frac{1}{2} + \frac{mn}{2}$$

$$= \frac{1}{2}[mn+1]$$

Hence, the sum of mn term is $\frac{1}{2}[mn+1]$.

178. How many terms of an AP $9, 17, 25, ...$ must be taken to give a sum of 636?

Ans : [Board Term-2 Delhi Compt 2015]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We have $a = 9, d = 8, S_n = 636$

Now $$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$636 = \frac{n}{2}[18+(n-1)8]$$

e207

$$636 = n[9+(n-1)4]$$

$$636 = n(9+4n-4)$$

$$636 = n(5+4n)$$

$$636 = 5n+4n^2$$

$$4n^2+5n-636 = 0$$

$$4n^2-48n+53n-636 = 0$$

$$4n(n-12)+53(n-12) = 0$$

$$(4n+53)(n-12) = 0$$

Thus $$n = \frac{-53}{4} \text{ or } 12$$

As n is a natural number $n = 12$. Hence 12 terms are required to give sum 636.

179. Find the number of natural numbers between 101 and 999 which are divisible by both 2 and 5.

Ans : [Board Term-2 OD 2014]

The sequence goes like 110, 120, 130, ... 990

Since they have a common difference of 10, they form an AP. Let the first term be a, common difference be d, nth term be a_n.

Here $a = 110,\ a_n = 990,\ d = 10$

e227

$$a_n = a+(n-1)d$$

$$990 = 110+(n-1)\times 10$$

$$990-110 = 10(n-1)$$

$$880 = 10(n-1)$$

$$88 = n-1$$

$$n = 88+1 = 89$$

Hence, there are 89 terms between 101 and 999 divisible by both 2 and 5.

180. How many three digit natural numbers are divisible by 7?

Ans : [Board Term-2 2013]

Let AP is 105, 112, 119, ..., 994 which is divisible by 7.

Let the first term be a, common difference be d, n th term be a_n.

Here, $a = 105, d = 112-105 = 7, a_n = 994$ then

$$a_n = a+(n-1)d$$

$$994 = 105+(n-1)\times 7$$

e228

$$889 = (n-1)\times 7$$

$$n-1 = \frac{889}{7} = 127$$

$$n = 127+1 = 128$$

Hence, there are 128 terms divisible by 7 in AP.

181. How many two digit numbers are divisible by 7?

Ans : [Board Term-2 SQP 2016]

Two digit numbers which are divisible by 7 are 14, 21, 28, ... 98. It forms an AP

Let the first term be a, common difference b[e d, n] th term be a_n.

e229

Here $a = 14,\ d = 7, a_n = 98$

Now
$$a_n = a+(n-1)d$$
$$98 = 14+(n-1)7$$
$$98-14 = 7n-7$$
$$84+7 = 7n$$
$$7n = 91 \Rightarrow n = 13$$

182. If the ratio of the 11^{th} term of an AP to its 18^{th} term is 2 : 3, find the ratio of the sum of the first five term of the sum of its first 10 terms.

Ans : [Board Term-2 Delhi Compt. 2017]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

e242

Now
$$\frac{a_{11}}{a_{18}} = \frac{a+10d}{a+17d} = \frac{2}{3}$$
$$2(a+17d) = 3(a+10d)$$
$$a = 4d \quad ...(1)$$

Now,
$$\frac{S_5}{S_{10}} = \frac{\frac{5}{2}(2a+4d)}{\frac{10}{2}[2a+9d]} = \frac{(a+2d)}{[2a+9d]}$$

Substituting the value $a = 4d$ we have

or,
$$\frac{S_5}{S_{10}} = \frac{4d+2d}{8d+9d} = \frac{6}{17}$$

Hence $S_5 : S_{10} = 6 : 17$

183. How many three digit numbers are such that when divided by 7, leave a remainder 3 in each case?

Ans : [Board Term-2 2012]

When a three digit number divided by 7 and leave 3 as remainder are 101, 108, 115, ... 997

These are in AP. Let the first term be a, common difference be d, nth term be a_n.

Here $a = 101, d = 7, a_n = 997$

e230

Now
$$a_n = a+(n-1)d$$
$$997 = 101+(n-1)7$$
$$997-101 = 896 = (n-1)7$$
$$\frac{896}{7} = n-1$$
$$n = 128+1 = 129$$

Hence, 129 numbers are divided by 7 which leaves remainder is 3.

184. How many multiples of 4 lie between 11 and 266?

Ans : [Board Term-2 2012]

First multiple of 4 is 12 and last multiple of 4 is 264. It forms a AP. Let multiples of 4 be n.

Let the first term be a, common difference be d, n th term be a_n.

Here, $a = 12, a_n = 264, d = 4$

e231

$$a_n = a+(n-1)d$$
$$264 = 12+(n-1)4$$
$$n = \frac{264-12}{4}+1$$

Hence, there are 64 multiples of 4 that lie between 11 and 266.

185. Prove that the n^{th} term of an AP can not be n^2+1. Justify your answer.

Ans : [Board Term-2 2015]

Let n^{th} term of AP,
$$a_n = n^2+1$$

Substituting the value of $n = 1, 2, 3, ...$ we get
$$a_1 = 1^2+1 = 2$$
$$a_2 = 2^2+1 = 5$$
$$a_3 = 3^2+1 = 10$$

e232

The obtained sequence is 2, 5, 10, 17,......

Its common difference
$$a_2-a_1 = a_3-a_2 = a_4-a_3$$
$$5-2 \neq 10-5 \neq 17-10$$
$$3 \neq 5 \neq 7$$

Since the sequence has no. common difference, n^2+1 is not a form of n^{th} term of an AP

186. If the p^{th} term of an AP is $\frac{1}{q}$ and q^{th} term is $\frac{1}{p}$. Prove that the sum of first pq term of the AP is $\left[\frac{pq+1}{2}\right]$.

Ans : [Board Term-2 Delhi 2017]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

e241

$$a_p = a+(p-1)d = \frac{1}{q} \quad ...(1)$$

and
$$a_q = a+(q-1)d = \frac{1}{p} \quad ...(2)$$

Solving (1) and (2) we get

$$a = \frac{1}{pq} \text{ and } d = \frac{1}{p}$$

$$S_{pq} = \frac{pq}{2}\left[2 \times \frac{1}{pq} + (pq-1)\frac{1}{pq}\right] = \frac{pq+1}{2}$$

187. Find the sum of all two digits odd positive numbers.

Ans : [Board Term-2 2014]

The list of 2 digits odd positive numbers are 11, 13 ...... 99. It forms an AP.

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

Here $a = 11, d = 2, l = 99$

Now
$$a_n = a + (n-1)d$$
$$99 = 11 + (n-1)2$$
$$88 = (n-1)2$$
$$n = 44 + 1 = 45$$
$$S_n = \frac{n}{2}[a + a_n]$$
$$= \frac{45}{2}[11 + 99]$$
$$S_n = \frac{45 \times 110}{2} = 2475$$

e233

Hence the sum of given AP is $S_n = 2475$

188. Find the sum of the two digits numbers divisible by 6.

Ans : [Board Term-2 2013]

Series of two digits numbers divisible by 6 is 12, 18, 24, ...96. It forms an AP. Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

Here $a = 12, d = 18 - 12 = 6, a_n = 96$

$$a_n = a + (n-1)d$$
$$96 = 12 + (n-1) \times 6$$
$$84 = 6(n-1)$$
$$n = 14 + 1 = 15$$
$$S_n = \frac{n}{2}[a + a_n]$$
$$= \frac{15}{2}[12 + 96]$$
$$= \frac{15 \times 108}{2}$$
$$= 15 \times 54 = 810$$

e234

Hence the sum of given AP is 810.

189. Find the sum of the integers between 100 and 200 that are divisible by 6.

Ans : [Board Term-2 2012]

The series as per question is 102, 108, 114, ... 198. which is an AP.

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

Here $a = 102, d = 6$ and $l = 198$

Now
$$198 = 102 + (n-1)6$$
$$96 = (n-1)6$$
$$\frac{96}{6} = n - 1$$
$$n = 17$$

e235

Now
$$S_{17} = \frac{n}{2}(a + a_n)$$
$$= \frac{17}{2}[102 + 198]$$
$$= \frac{17}{2} \times 300 = 17 \times 150 = 2550$$

Hence the sum of given AP is 2550.

FOUR MARKS QUESTIONS

190. If the sum of first four terms of an AP is 40 and that of first 14 terms is 280. Find the sum of its first n terms.

Ans : [Board 2019 Delhi]

Let a be the first term and d be the common difference.

Sum of n terms of an AP,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Now $S_4 = 40$ and $S_{14} = 280$

e318

$$\frac{4}{2}[2a + (4-1)d] = 40$$
$$2[2a + 3d] = 40$$
$$2a + 3d = 20 \quad (1)$$

and
$$\frac{14}{2}[2a + (14-1)d] = 280$$
$$7[2a + 13d] = 280$$
$$2a + 13d = 40 \quad (2)$$

Solving equations (1) and (2), we get

$$a = 7 \text{ and } d = 2$$

Now $$S_n = \frac{n}{2}[2 \times 7 + (n-1)2]$$

$$= \frac{n}{2}[14 + 2n - 2]$$

$$= \frac{n}{2}(12 + 2n) = 6n + n^2$$

Hence, sum of n terms is $6n + n^2$.

191. The first term of an AP is 3, the last term is 83 and the sum of all its terms is 903. Find the number of terms and the common difference of the AP.

Ans : [Board 2019 Delhi]

First term, $a = 3$

Last term, $a_n = 83$

Sum of n terms, $S_n = 903$

Since, $$S_n = \frac{n}{2}(a + a_n)$$

e319

$$903 = \frac{n}{2}(3 + 83)$$

$$1806 = 86n$$

$$n = \frac{1806}{86} \Rightarrow n = 21$$

Now $$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$903 = \frac{21}{2}[2 \times 3 + (21-1)d]$$

$$1806 = 21(6 + 20d)$$

$$6 + 20d = 86$$

$$20d = 80 \Rightarrow d = 4$$

Hence, the common difference is 4.

192. Find the common difference of the Arithmetic Progression (AP) $\frac{1}{a}, \frac{3-a}{3a}, \frac{3-2a}{3a}, \ldots (a \neq 0)$

Ans : [Board 2019 OD]

Given AP is $\frac{1}{a}, \frac{3-a}{3a}, \frac{3-2a}{3a}, \ldots \ldots (a \neq 0)$

Here, first term, $a_1 = \frac{1}{a}$

Second term, $a_2 = \frac{3-a}{3a}$

e320

Third term, $a_3 = \frac{3-2a}{3a}$

Common difference,

$$d = a_2 - a_1$$

$$= \frac{3-a}{3a} - \frac{1}{a} = \frac{3-a-3}{3a}$$

$$= \frac{-a}{3a} = \frac{-1}{3}$$

Here, common difference d of given AP is $\frac{-1}{3}$.

193. Which term of the Arithmetic Progression $-7, -12, -17, -22, \ldots$ will be -82? Is -100 any term of the AP ? Given reason for your answer.

Ans : [Board 2019 OD]

Given AP is $-7, -12, -17, -22, \ldots$

Here,

First term, $a_1 = -7$

e321

Second term $a_2 = -12$

Third term, $a_3 = -17$

Common difference,

$$d = a_2 - a_1 = -12 - (-7)$$

$$= -12 + 7 = -5$$

$$d = -5$$

Let a_n be the n^{th} term of AP and it will be -82.

Since, $$a_n = a_1 + (n-1)d$$

$$-82 = -7 + (n-1)(-5)$$

$$-82 = -7 - 5(n-1)$$

$$82 = 5n + 2$$

$$5n = 80 \Rightarrow n = 16$$

Hence, 16^{th} term of AP is -82. Since, these numbers are not factor of 5, hence -100 will not be a term in the given AP.

194. How many terms of the Arithmetic Progression 45, 39, 33, ... must be taken so that their sum is 180? Explain the double answer.

Ans : [Board 2019 OD]

Given AP is 45, 39, 33, ...

Here, $a = 45,\ d = 39 - 45 = -6$ and $S_n = 180$

Now
$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$180 = \frac{n}{2}[2 \times 45 + (n-1)(-6)]$$
$$360 = n(90 - 6n + 6)$$
$$360 = n(96 - 6n)$$
$$60 = n(16 - n)$$
$$n^2 - 16n + 60 = 0$$
$$n^2 - 6n - 10n + 60 = 0$$
$$n(n-6) - 10(n-6) = 0$$
$$(n-10)(n-6) = 0$$
$$n = 10 \text{ or } n = 6$$

Hence, 10 terms or 6 terms can be taken to get the sum of AP as 180.

Now, sum of 6 terms,
$$S_6 = \frac{6}{2}[2 \times 45 + (6-1)(-6)]$$
$$= 3(90 - 30)$$
$$= 3 \times 60 = 180 \qquad \text{Hence, verified.}$$

and sum of 10 terms,
$$S_{10} = \frac{10}{2}[2 \times 45 + (10-1)(-6)]$$
$$= 5(90 - 54)$$
$$= 5 \times 36 = 180 \qquad \text{Hence, verified.}$$

Here we have two values of n because d is negative. There will be negative terms after some positive terms. Thus first 6 term will give sum 180 and after 10 term it will be again 180 because negative term cancel positive term.

Series will be : 45, 39, 33, 27, 21, 15, 9, 3, -3, -9...

Here it may be easily seen that sum of initial 6 terms is 180. Sum of next 4 terms is zero. Thus sum of 10 terms is also 180.

195. The sum of three numbers in AP is 12 and sum of their cubes is 288. Find the numbers.

Ans : [Board Term-2 Delhi 2016]

Let the three numbers in AP be $a-d,\ a,\ a+d$.
$$a - d + a + a + d = 12$$
$$3a = 12$$
$$a = 4$$

Also, $(4-d)^3 + 4^3 + (4+d)^3 = 288$
$$64 - 48d + 12d^2 - d^3 + 64 + 64 + 48d + 12d^2 + d^3 = 288$$
$$24d^2 + 192 = 288$$
$$d^2 = 4$$
$$d = \pm 2$$

The numbers are 2, 4, 6 or 6, 4,2

196. Find the value of a, b and c such that the numbers $a, 7, b, 23$ and c are in AP

Ans : [Board Term-2 2015]

Let the common difference be d.

Since $a, 7, b, 23$ and c are in AP, we have
$$a + d = 7 \qquad ..(1)$$
$$a + 3d = 23 \qquad ...(2)$$

Form equation (1) and (2), we get
$$a = -1, d = 8$$
$$b = a + 2d = -1 + 2 \times 8 = -1 + 16 = 15$$
$$c = a + 4d = -1 + 4 \times 8 = -1 + 32 = 31$$

Thus $a = -1, b = 15, c = 31$

197. If S_n denotes the sum of first n terms of an AP, prove that, $S_{30} = 3(S_{20} - S_{10})$

Ans : [Board Term-2 Delhi 2015, Foreign 2014]

Let the first term be a, and common difference be d.

Now
$$S_{30} = \frac{30}{2}(2a + 29d) \qquad ...(1)$$
$$= 15(2a + 29d)$$
$$3(S_{20} - S_{10}) = 3[10(2a + 19d) - 5(2a + 9d)]$$
$$= 3[20a + 190d - 10a - 45d]$$
$$= 3[10a + 145d]$$
$$= 15[2a + 29d] \qquad ...(2)$$

Hence $S_{30} = 3(S_{20} - S_{10})$

198. The sum of first 20 terms of an AP is 400 and sum of first 40 terms is 1600. Find the sum of its first 10 terms.

Ans : [Board Term-2 2015]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We know $S_n = \frac{n}{2}[2a+(n-1)d]$

e211

Now $S_{20} = \frac{20}{2}(2a+19d)$

$400 = \frac{20}{2}(2a+19d)$

$400 = 10[2a+19d]$

$2a+19d = 40$ (1)

Also, $S_{40} = \frac{40}{2}(2a+39d)$

$1600 = 20[2a+39d]$

$2a+39d = 80$ (2)

Solving equation (1) and (2), we get $a = 1$ and $d = 2$.

Now $S_{10} = \frac{10}{2}[2\times 1+(10-1)(2)]$

$= 5[2+9\times 2]$

$= 5[2+18]$

$= 5\times 20 = 100$

199. Find $\left(4-\frac{1}{n}\right)+\left(7-\frac{2}{n}\right)+\left(10-\frac{3}{n}\right)+.....$ upto n terms.

Ans : [Board Term-2 2015]

Let sum of n term be S_n, then we have

$s_n = \left(4-\frac{1}{n}\right)+\left(7-\frac{2}{n}\right)+\left(40-\frac{3}{n}\right)+.....$ upto n terms.

$= (4+7+10+.....+n \text{ terms}) - \left(\frac{1}{n}+\frac{2}{n}+\frac{3}{n}.....+1\right)$

$= (4+7+10+....+n \text{ terms}) - \frac{1}{n}(1+2+3+...n)$

$= \frac{n}{2}[2\times 4+(n-1)(3)] - \frac{1}{n}\times\frac{n}{2}[2\times 1+(n-1)(1)]$

$= \frac{n}{2}[8+3n-3] - \frac{1}{2}[2+n-1]$

$= \frac{n}{2}(3n+5) - \frac{1}{2}(n+1)$

e212

$= \frac{3n^2+5n-n-1}{2} = \frac{3n^2+4n-1}{2}$

200. Find the 60^{th} term of the AP $8, 10, 12, ...,$ if it has a total of 60 terms and hence find the sum of its last 10 terms.

Ans : [Board Term-2 OD 2015]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

e213

We have $a = 8, d = 10-8 = 2$

$a_n = a+(n-1)d$

Now $a_{60} = 8+(60-1)2 = 8+59\times 2 = 126$

and $a_{51} = 8+50\times 2 = 8+100 = 108$

Sum of last 10 terms,

$S_{51-60} = \frac{n}{2}(a_{51}+a_{60})$

$= \frac{10}{2}(108+126)$

$= 5\times 234 = 1170$

Hence sum of last 10 terms is 1170.

201. An arithmetic progression $5, 12, 19, ...$ has 50 terms. Find its last term. Hence find the sum of its last 15 terms.

Ans : [Board Term-2 OD 2015]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We have $a = 5$, $d = 12-5 = 7$ and $n = 50$

$a_{50} = 5+(50-1)7$

$= 5+49\times 7 = 348$

Also the first term of the AP of last 15 terms be a_{36}

$a_{36} = 5+35\times 7$

$= 5+245 = 250$

e214

Now, sum of last 15 terms,

$S_{36-50} = \frac{15}{2}[a_{36}+a_{50}]$

$= \frac{15}{2}[250+348]$

$= \frac{15}{2}\times 598 = 4485$

Hence, sum of last 15 terms is 4485.

202. If the sum of first n term of an an AP is given by

$S_n = 3n^2 + 4n$. Determine the AP and the n^{th} term.

Ans : [Board Term-2 2014]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

e218

We have $\quad S_n = 3n^2 + 4n.$

$$a_1 = 3(1)^2 + 4(1) = 7$$

$$a_1 + a_2 = S_2 = 3(2)^2 + 4(2)$$

$$= 12 + 8 = 20$$

$$a_2 = S_2 - S_1 = 20 - 7 = 13$$

$$a + d = 13$$

or, $\quad 7 + d = 13$

Thus $\quad d = 13 - 7 = 6$

Hence AP is $7, 13, 19, ...$

Now, $\quad a_n = a + (n-1)d$

$$= 7 + (n-1)(6)$$

$$= 7 + 6n - 6$$

$$= 6n + 1$$

$$a_n = 6n + 1$$

203. The sum of the 3^{rd} and 7^{th} terms of an AP is 6 and their product is 8. Find the sum of first 20 terms of the AP.

Ans : [Board Term-2 2012]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

e219

We have $\quad a_3 + a_7 = 6$

$$a + 2d + a + 6d = 6$$

$$a + 4d = 3 \quad (1)$$

and $\quad a_3 \times a_7 = 8$

$$(a + 2d)(a + 6d) = 8 \quad (2)$$

Substituting the value $a = (3 - 4d)$ in (2) we get

$$(3 - 4d + 2d)(3 - 4d + 6d) = 8$$

$$(3 + 2d)(3 - 2d) = 8$$

$$9 - 4d^2 = 8$$

$$4d^2 = 1 \Rightarrow d^2 = \frac{1}{4} \Rightarrow d = \pm\frac{1}{2}$$

CASE 1 : Substituting $d = \frac{1}{2}$ in equation (1), $a = 1$.

$$S_{20} = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{20}{2}\left[2 + \frac{19}{2}\right] = 115$$

Thus $d = \frac{1}{2}, a = 1$ and $S_{20} = 115$

CASE 2 : Substituting $d = -\frac{1}{2}$ in equation (1) $a = 5$

$$S_{20} = \frac{20}{2}\left[2 \times 5 + 19 \times \left(-\frac{1}{2}\right)\right]$$

$$= 10\left[10 - \frac{19}{2}\right] = 15$$

Thus $d = -\frac{1}{2}, a = 5$ and $S_{20} = 15$

204. If the sum of first m terms of an AP is same as the sum of its first n terms $(m \neq n)$, show that the sum of its first $(m + n)$ terms is zero.

Ans : [Board Term-2 2012]

Let the first term be a, common difference be d, nth term be a_n, and sum of n term be S_n

Now $\quad S_m = S_n$

$$\frac{m}{2}[2a + (m-1)d] = \frac{n}{2}[2a + (n-1)d]$$

e223

$$2ma + m(m-1)d = 2na + n(n-1)d$$

$$2a(m-n) + [(m^2 - n^2) - m + n]\,d = 0$$

$$2a(m-n) + [(m-n)(m+n) - (m-n)]\,d = 0$$

$$(m-n)[2a + (m+n-1)d] = 0$$

$$2a + (m+n-1)d = 0 \qquad [m - n \neq 0]$$

$$S_{m+n} = \frac{m+n}{2}[2a + (m+n-1)d]$$

$$= \frac{m+n}{2} \times 0 = 0$$

205. If $1 + 4 + 7 + 10 ..... + n = 287$, Find the value of n.

Ans : [Board 2020 Std, Board Term-2 Foreign 2017]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.
We have $a = 1, d = 3$ and $S_n = 287$.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\frac{n}{2}[2 \times 1 + (n-1)3] = 287$$

$$\frac{n}{2}[2 + (3n - 3)] = 287$$

e225

$$3n^2 - n = 574$$

$$3n^2 - n - 574 = 0$$

$$3n^2 - 42n + 41n - 574 = 0$$

$$3n(n-14) + 41(n-14) = 0$$

$$(n-14)(3n+41) = 0$$

Since negative value is not possible, $n = 14$

$$a_{14} = a + (n-1)d$$

$$= 1 + 13 \times 3 = 40$$

206. Find the sum of first 24 terms of an AP whose n^{th} term is given by $a_n = 3 + 2n$.

Ans : [Board Term-2 OD Comptt. 2017]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

We have $a_n = 3 + 2n$

$$a_1 = 3 + 2 \times 1 = 5$$

$$a_2 = 3 + 2 \times 2 = 7$$

$$a_3 = 3 + 2 \times 3 = 9$$

e226

Thus the series is 5, 7, 9, ... in which

$$a = 5 \text{ and } d = 2$$

Now $S_n = \frac{n}{2}[2a + (n-1)d]$

$$S_{24} = \frac{24}{2}(2 \times 5 + 23 \times 2)$$

$$= 12 \times 56$$

Hence, $S_{24} = 672$.

207. Find the number of terms of the AP $-12, -9, -6, ..., 21$. If 1 is added to each term of this AP, then find the sum of all the terms of the AP thus obtained.

Ans : [Board Term-2 2013]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

We have $a = -12, d = -9 - (-12) = 3$

$$a_n = a + (n-1)d$$

$$21 = -12 + (n-1) \times 3$$

$$21 + 12 = (n-1) \times 3$$

$$33 = (n-1) \times 3$$

$$n - 1 = 11$$

e236

$$n = 11 + 1 = 12$$

Now, if 1 is added to each term we have a new AP with $-12+1, -a+1, -6+1......21+1$

Now we have $a = -11, d = 3$ and $a_n = 22$ and $n = 12$

Sum of this obtained AP,

$$S_{12} = \frac{12}{2}[-11 + 22]$$

$$= 6 \times 11 = 66$$

Hence the sum of new AP is 66.

208. How many terms of the AP $-6, \frac{11}{2}, -5, ...$ are needed to given the sum -25? Explain the double answer.

Ans : [Board Term-2 2012]

AP is $-6, -\frac{11}{2}, -5 ....$

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

Here we have $a = -6$

$$d = -\frac{11}{2} + \frac{6}{1} = \frac{1}{2}$$

$$S_n = -25$$

e237

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$-25 = \frac{n}{2}\left[-12 + (n-1) \times \frac{1}{2}\right]$$

$$-50 = n\left[\frac{-24 + (n-1)}{2}\right]$$

$$-100 = n[n - 25]$$

$$n^2 - 25n + 100 = 0$$

$$(n-20)(n-5) = 0$$

$$n = 20, 5$$

or, $S_{20} = S_5$

Here we have got two answers because two value of n sum of AP is same. Since a is negative and d is positive; the sum of the terms from 6^{th} to 20^{th} is zero.

209. If S_1, S_2, S_3 be the sum of $n, 2n, 3n$ terms respectively of an AP, prove that $S_3 = 3(S_2 - S_1)$.

Ans : [Board Term-2 2012]

Let the first term be a, and common difference be d.

Now $S_1 = \frac{n}{2}[2a+(n-1)d]$

$$S_2 = \frac{2n}{2}[2a+(2n-1)d]$$

e238

$$S_3 = \frac{3n}{2}[2a+(3n-1)d]$$

$$3(S_2 - S_1) = 3\left[\frac{2n}{2}[2a+(2n-1)d] - \frac{n}{2}[2a+(n-1)d]\right]$$

$$= 3\left[\frac{n}{2}[4a+2(2n-1)d] - [2a+(n-1)d]\right]$$

$$= 3\left[\frac{n}{2}(4a+4nd-2d-2a-nd+d)\right]$$

$$= 3\left[\frac{n}{2}(2a+3nd-d)\right]$$

$$= \frac{3n}{2}[2a+(3n-1)d] = S_3$$

210. An AP consists of 37 terms. The sum of the three middle most terms is 225 and the sum of the past three terms is 429. Find the AP.

Ans : [Board Term-2 SQP 2017]

Let the middle most terms of the AP be $(x-d)$, x and $(x+d)$.

We have $x-d+x+x+d = 225$

$$3x = 225 \Rightarrow x = 75$$

e243

and the middle term $= \frac{37+1}{2} = 19^{th}$ term

Thus AP is

$(x-18d), \ldots.(x-2d), (x-d), x, (x+d), (x+2d), \ldots\ldots.$

$(x-18d)$

Sum of last three terms,

$$(x+18d)+(x+17d)+(x+16d) = 429$$

$$3x+51d = 429$$

, $225+51d = 429 \Rightarrow d = 4$

First term $a_1 = x-18d = 75-18\times 4 = 3$

$$a_2 = 3+4 = 7$$

Hence AP $= 3, 7, 11, \ldots\ldots., 147$.

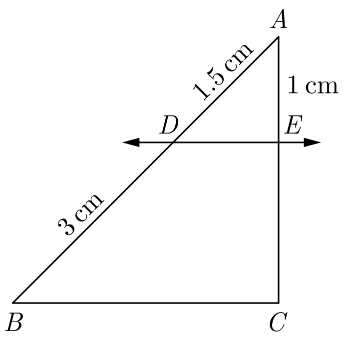
CHAPTER 6

TRIANGLES

ONE MARK QUESTIONS

Multiple Choice Questions

1. In the given figure, $DE \parallel BC$. The value of EC is



(a) 1.5 cm (b) 3 cm
(c) 2 cm (d) 1 cm

Ans :

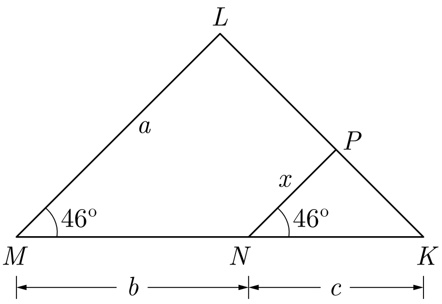
Since, $DE \parallel BC$

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{1.5}{3} = \frac{1}{EC} \Rightarrow EC = 2\,\text{cm}$$

Thus (c) is correct option.

2. In the given figure, x is



(a) $\frac{ab}{a+b}$ (b) $\frac{ac}{b+c}$
(c) $\frac{bc}{b+c}$ (d) $\frac{ac}{a+c}$

Ans :

In ΔKPN and ΔKLM, $\angle K$ is common and we have

$$\angle KNP = \angle KML = 46°$$

Thus by $A - A$ criterion of similarity,

$$\Delta KNP \sim \Delta KML$$

Thus $$\frac{KN}{KM} = \frac{NP}{ML}$$

$$\frac{c}{b+c} = \frac{x}{a} \Rightarrow x = \frac{ac}{b+c}$$

Thus (b) is correct option.

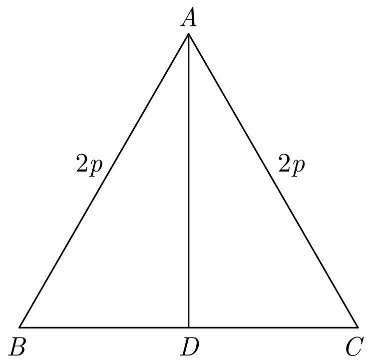
3. ΔABC is an equilateral triangle with each side of length $2p$. If $AD \perp BC$ then the value of AD is

(a) $\sqrt{3}$ (b) $\sqrt{3}\,p$
(c) $2p$ (d) $4p$

Ans :

We have $AB = BC = CA = 2p$

and $AD \perp BC$



In ΔADB, $AB^2 = AD^2 + BD^2$

$$(2p)^2 = AD^2 + p^2$$

$$AD^2 = \sqrt{3}\,p$$

Thus (b) is correct option.

4. Which of the following statement is false?
(a) All isosceles triangles are similar.
(b) All quadrilateral are similar.
(c) All circles are similar.
(d) None of the above

f200

Ans :

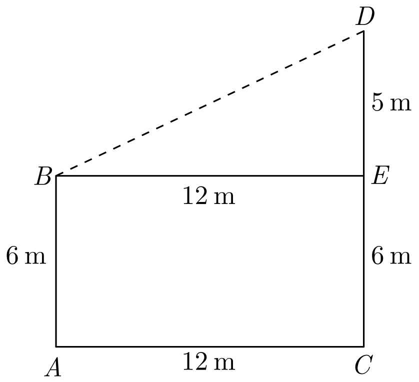
Isosceles triangle is a triangle in which two side of equal length. Thus two isosceles triangles may not be similar. Hence statement given in option (a) is false.
Thus (a) is correct option.

5. Two poles of height 6 m and 11 m stand vertically upright on a plane ground. If the distance between their foot is 12 m, then distance between their tops is
(a) 12 m (b) 14 m
(c) 13 m (d) 11 m

f201

Ans :

Let AB and CD be the vertical poles as shown below.



We have $AB = 6\text{ m},\ CD = 11\text{ m}$

and $AC = 12\text{ m}$

$$DE = CD - CE$$
$$= (11-6)\text{ m} = 5\text{ m}$$

In right angled, ΔBED,

$$BD^2 = BE^2 + DE^2 = 12^2 + 5^2 = 169$$
$$BD = \sqrt{169}\text{ m} = 13\text{ m}$$

Hence, distance between their tops is 13 m.
Thus (c) is correct option.

6. In a right angled ΔABC right angled at B, if P and Q are points on the sides AB and BC respectively, then
(a) $AQ^2 + CP^2 = 2(AC^2 + PQ^2)$
(b) $2(AQ^2 + CP^2) = AC^2 + PQ^2$
(c) $AQ^2 + CP^2 = AC^2 + PQ^2$
(d) $AQ + CP = \frac{1}{2}(AC + PQ)$

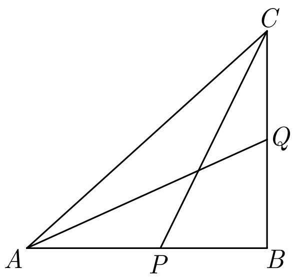
f204

Ans :

In right angled ΔABQ and ΔCPB,

$$CP^2 = CB^2 + BP^2$$

and $$AQ^2 = AB^2 + BQ^2$$



$$CP^2 + AQ^2 = CB^2 + BP^2 + AB^2 + BQ^2$$
$$= CB^2 + AB^2 + BP^2 + BQ^2$$
$$= AC^2 + PQ^2$$

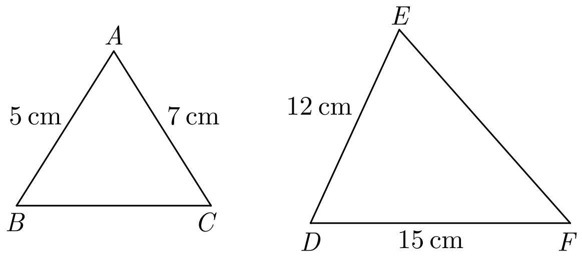
Thus (c) is correct option.

7. It is given that, $\Delta ABC \sim \Delta EDF$ such that $AB = 5$ cm, $AC = 7$ cm, $DF = 15$ cm and $DE = 12$ cm then the sum of the remaining sides of the triangles is
(a) 23.05 cm (b) 16.8 cm
(c) 6.25 cm (d) 24 cm

f205

Ans :

We have $\Delta ABC \sim \Delta EDF$



Now $$\frac{5}{12} = \frac{7}{EF} = \frac{BC}{15}$$

Taking first and second ratios, we get

$$\frac{5}{12} = \frac{7}{EF} \Rightarrow EF = \frac{7 \times 12}{5}$$

$$= 16.8 \text{ cm}$$

Taking first and third ratios, we get

$$\frac{5}{12} = \frac{BC}{15} \Rightarrow BC = \frac{5 \times 15}{12}$$

$$= 6.25 \text{ cm}$$

Now, sum of the remaining sides of triangle,

$$EF + BC = 16.8 + 6.25 = 23.05 \text{ cm}$$

Thus (a) is correct option.

8. The area of a right angled triangle is 40 sq cm and its perimeter is 40 cm. The length of its hypotenuse is

(a) 16 cm (b) 18 cm

(c) 17 cm (d) data insufficient

Ans : (b) 18 cm

Let c be the hypotenuse of the triangle, a and b be other sides.

Now $c = \sqrt{a^2 + b^2}$

We have, $a + b + c = 40$ and $\frac{1}{2}ab = 40 \Rightarrow ab = 80$

$$c = 40 - (a + b) \text{ and } ab = 80$$

f206

Squaring $c = 40 - (a + b)$ we have

$$c^2 = [40 - (a + b)]^2$$

$$a^2 + b^2 = 1600 - 2 \times 40(a + b) + (a + b)^2$$

$$a^2 + b^2 = 1600 - 2 \times 40(a + b) + a^2 + 2ab + b^2$$

$$0 = 1600 - 2 \times 40(a + b) + 2 \times 80$$

$$0 = 20 - (a + b) + 2$$

$$a + b = 22$$

$$c = 40 - (a + b) = 40 - 22 = 18 \text{ cm}$$

Thus (b) is correct option.

9. Assertion : In the $\triangle ABC$, $AB = 24$ cm, $BC = 10$ cm and $AC = 26$ cm, then $\triangle ABC$ is a right angle triangle.

Reason : If in two triangles, their corresponding angles are equal, then the triangles are similar.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

f207

We have,

$$AB^2 + BC^2 = (24)^2 + (10)^2$$

$$= 576 + 100 = 676 = AC^2$$

Thus $AB^2 + BC^2 = AC^2$ and ABC is a right angled triangle.

Also, two triangle are similar if their corresponding angles are equal.

Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

Thus (b) is correct option.

Fill in the Blank Questions

10. A line drawn through the mid-point of one side of a triangle parallel to another side bisects the .... side.

Ans :

third

f210

11. .... theorem states that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Ans :

Pythagoras

f211

12. Line joining the mid-points of any two sides of a triangle is .... to the third side.

Ans :

parallel

f212

13. All squares are ....

Ans :

similar

f213

14. Two triangles are said to be .... if corresponding angles of two triangles are equal.

Ans :

equiangular

f214

15. All similar figures need not be ....

Ans :

congruent

16. All circles are ....

Ans :

similar

17. If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the .... side.

Ans :

third

18. If a line divides any two sides of a triangle in the same ratio, then the line is .... to the third side.

Ans :

parallel

19. All congruent figures are similar but the similar figures need .... be congruent.

Ans :

not

20. Two figures are said to be .... if they have same shape but not necessarily the same size.

Ans :

similar

21. .... theorem states that if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

Ans :

Basic proportionality

22. All .... triangles are similar.

Ans :

equilateral

23. Two figures having the same shape and size are said to be ....

Ans :

congruent

24. Two triangles are similar if their corresponding sides are ....

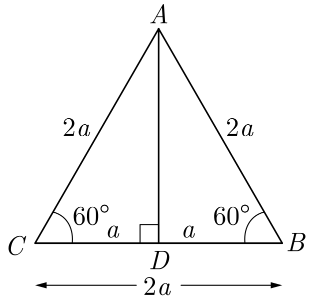
Ans : [Board 2020 OD Standard]

in the same ratio.

25. ΔABC is an equilateral triangle of side $2a$, then length of one of its altitude is ....

Ans : [Board 2020 Delhi Standard]

ΔABC is an equilateral triangle as shown below, in which $AD \perp BC$.



Using Pythagoras theorem we have

$$AB^2 = (AD)^2 + (BD)^2$$

$$(2a)^2 = (AD)^2 + (a)^2$$

$$4a^2 - a^2 = (AD)^2$$

$$(AD)^2 = 3a^2$$

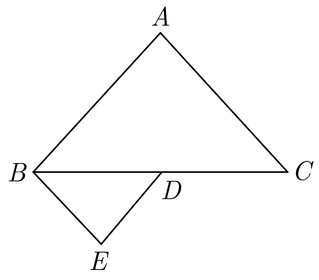
$$AD = a\sqrt{3}$$

Hence, the length of attitude is $a\sqrt{3}$.

26. ΔABC and ΔBDE are two equilateral triangle such that D is the mid-point of BC. Ratio of the areas of triangles ABC and BDE is ....

Ans : [Board 2020 Delhi Standard]

From the given information we have drawn the figure as below.



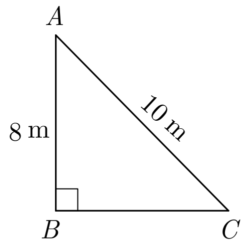
$$\frac{ar(\Delta ABC)}{ar(\Delta BDE)} = \frac{\frac{\sqrt{3}}{4}(BC)^2}{\frac{\sqrt{3}}{4}(BD)^2} = \frac{(BC)^2}{(\frac{1}{2}BC)^2}$$

$$= \frac{4BC^2}{BC^2} = \frac{4}{1} = 4:1$$

27. A ladder 10 m long reaches a window 8 m above the ground. The distance of the foot of the ladder from the base of the wall is .... m.

Ans : [Board 2020 Delhi Standard]

Let AB be the height of the window above the ground and BC be a ladder.



Here, $AB = 8$ m

and $AC = 10$ m

In right angled triangle ABC,

$$AC^2 = AB^2 + BC^2$$

$$10^2 = 8^2 + BC^2$$

$$BC^2 = 100 - 64 = 36$$

$$BC = 6\text{ m}$$

28. In ΔABC, $AB = 6\sqrt{3}$ cm, $AC = 12$ cm and $BC = 6$ cm, then $\angle B =$ ....

Ans : [Board 2020 OD Standard]

We have $AB = 6\sqrt{3}$ cm,

$AC = 12$ cm and

$BC = 6$ cm

Now $AB^2 = 36 \times 3 = 108$

$AC^2 = 144$

and $BC^2 = 36$

In can be easily observed that above values satisfy Pythagoras theorem,

$$AB^2 + BC^2 = AC^2$$

$$108 + 36 = 144\text{ cm}$$

Thus $\angle B = 90^\circ$

29. The perimeters of two similar triangles are 25 cm and 15 cm respectively. If one side of the first triangle is 9 cm, then the corresponding side of second triangle is ....

Ans : [Board 2020 Delhi Basic]

Ratio of the perimeter of two similar triangles is equal to the ratio of corresponding sides.

Thus $\dfrac{25}{15} = \dfrac{9}{\text{side}}$

$$\text{side} = \frac{9 \times 15}{25} = 5.4\text{ cm}$$

Very Short Answer Questions

30. ΔABC is isosceles with $AC = BC$. If $AB^2 = 2AC^2$, then find the measure of $\angle C$.

Ans : [Board 2020 Delhi Basic]

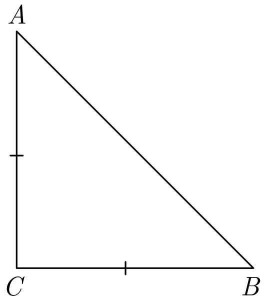
We have $AB^2 = 2AC^2$

$$AB^2 = AC^2 + AC^2$$

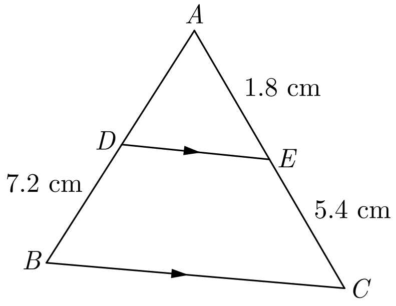
$$AB^2 = BC^2 + AC^2$$

$(BC = AC)$

It satisfies the Pythagoras theorem. Thus according to converse of Pythagoras theorem, ΔABC is a right angle triangle and $\angle C = 90^\circ$.



31. In Figure, $DE || BC$. Find the length of side AD, given that $AE = 1.8$ cm, $BD = 7.2$ cm and $CE = 5.4$ cm.



Ans : [Board 2019 OD]

Since $DE || BC$ we have

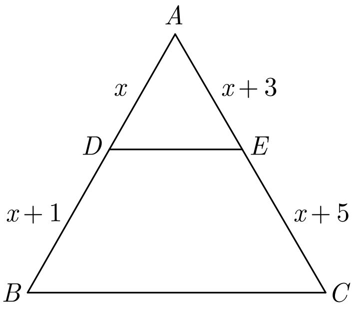
$$\frac{AD}{DB} = \frac{AE}{EC}$$

Substituting the values, we get

$$\frac{AD}{7.2} = \frac{1.8}{5.4}$$

$$AD = \frac{1.8 \times 7.2}{5.4} = \frac{12.96}{5.4} = 2.4\text{ cm}$$

32. In $\Delta ABC, DE || BC$, find the value of x.



Ans : [Board Term-1 2016]

In the given figure $DE || BC$, thus

f101

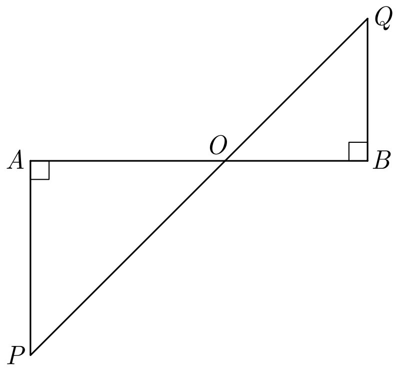
$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{x}{x+1} = \frac{x+3}{x+5}$$

$$x^2 + 5x = x^2 + 4x + 3$$

$$x = 3$$

33. In the given figure, if $\angle A = 90^o, \angle B = 90^o$, $OB = 4.5$ cm $OA = 6$ cm and $AP = 4$ cm then find QB.



Ans : [Board Term-1, 2015]

In ΔPAO and ΔQBO we have

f102

$$\angle A = \angle B = 90^o$$

Vertically opposite angle,

$$\angle POA = \angle QOB$$

Thus $\Delta PAO \sim \Delta QBO$

$$\frac{OA}{OB} = \frac{PA}{QB}$$

$$\frac{6}{4.5} = \frac{4}{QB}$$

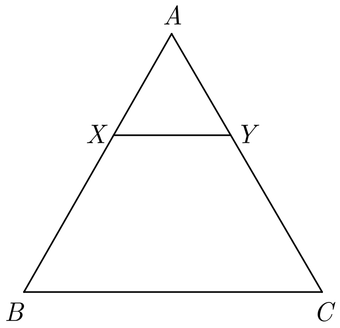
$$QB = \frac{4 \times 4.5}{6} = 3 \text{ cm}$$

Thus $QB = 3$ cm

34. In ΔABC, if X and Y are points on AB and AC respectively such that $\frac{AX}{XB} = \frac{3}{4}$, $AY = 5$ and $YC = 9$, then state whether XY and BC parallel or not.

Ans : [Board Term-1 2016, 2015]

As per question we have drawn figure given below.



In this figure we have

$$\frac{AX}{XB} = \frac{3}{4}, AY = 5 \text{ and } YC = 9$$

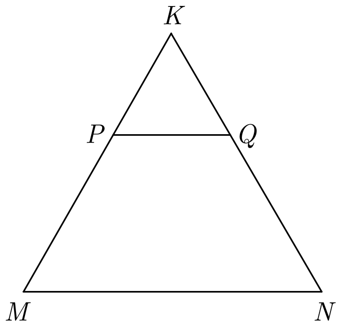
Now $$\frac{AX}{XB} = \frac{3}{4} \text{ and } \frac{AY}{YC} = \frac{5}{9}$$

f103

Since $$\frac{AX}{XB} \neq \frac{AY}{YC}$$

Hence XY is not parallel to BC.

35. In the figure, PQ is parallel to MN. If $\frac{KP}{PM} = \frac{4}{13}$ and $KN = 20.4$ cm then find KQ.



Ans :

In the given figure $PQ || MN$, thus

$$\frac{KP}{PM} = \frac{KQ}{QN} \quad \text{(By BPT)}$$

$$\frac{KP}{PM} = \frac{KQ}{KN - KQ}$$

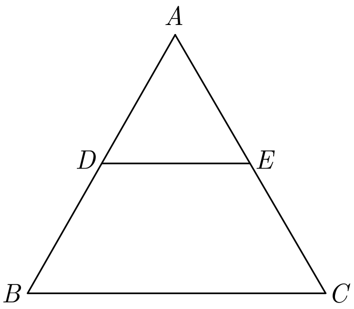
$$\frac{4}{13} = \frac{KQ}{20.4 - KQ}$$

$$4 \times 20.4 - 4KQ = 13KQ$$

$$17KQ = 4 \times 20.4$$

$$KQ = \frac{20.4 \times 4}{17} = 4.8 \text{ cm}$$

36. In given figure $DE || BC$. If $AD = 3c, DB = 4c$ cm and $AE = 6$ cm then find EC.



Ans : [Board Term-1 2016]

In the given figure $DE \parallel BC$, thus

$$\frac{AD}{BD} = \frac{AE}{EC}$$

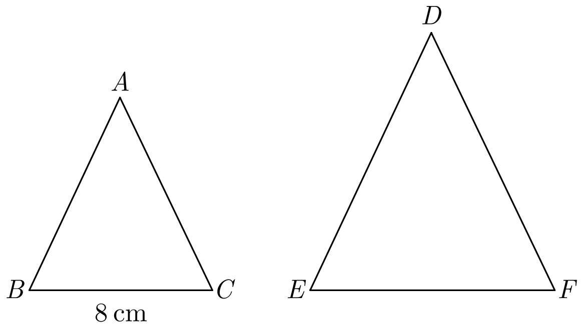
$$\frac{3}{4} = \frac{6}{EC}$$

$$EC = 8 \text{ cm}$$

37. If triangle ABC is similar to triangle DEF such that $2AB = DE$ and $BC = 8$ cm then find EF.

Ans :

As per given condition we have drawn the figure below.



Here we have $2AB = DE$ and $BC = 8$ cm

Since $\Delta ABC \sim \Delta DEF$, we have

$$\frac{AB}{BC} = \frac{DE}{EF}$$

$$\frac{AB}{8} = \frac{2AB}{EF}$$

$$EF = 2 \times 8 = 16 \text{ cm}$$

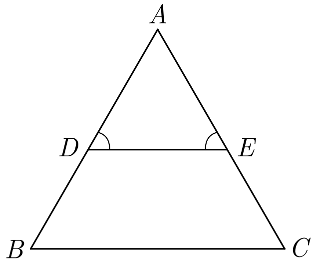
38. Are two triangles with equal corresponding sides always similar?

Ans : [Board Term-1 2015]

Yes, Two triangles having equal corresponding sides are are congruent and all congruent Δs have equal angles, hence they are similar too.

TWO MARKS QUESTIONS

39. In Figure $\angle D = \angle E$ and $\frac{AD}{DB} = \frac{AE}{EC}$, prove that ΔBAC is an isosceles triangle.



Ans : [Board 2020 Delhi Standard]

We have, $\angle D = \angle E$

and $\frac{AD}{DB} = \frac{AE}{EC}$

By converse of BPT, $DE \parallel BC$

Due to corresponding angles we have

$$\angle ADE = \angle ABC \text{ and}$$

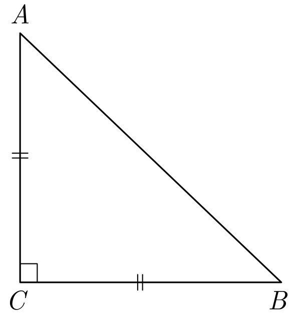
$$\angle AED = \angle ACB$$

Given $\angle ADE = \angle AED$

Thus $\angle ABC = \angle ACB$

Therefore BAC is an isosceles triangle.

40. In Figure, ABC is an isosceles triangle right angled at C with $AC = 4$ cm, Find the length of AB.



f241

Ans : [Board 2019 OD]

Since ABC is an isosceles triangle right angled at C,

$$AC = BC = 4\text{ cm}$$

$$\angle C = 90^\circ$$

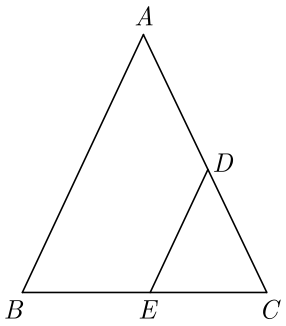
Using Pythagoras theorem in ΔABC we have,

$$AB^2 = BC^2 + AC^2$$

$$= 4^2 + 4^2 = 16 + 16 = 32$$

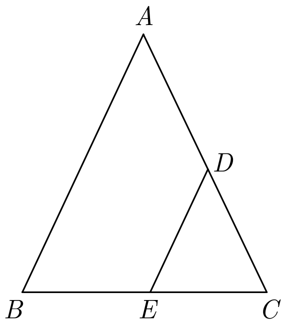
$$AB = 4\sqrt{2}\text{ cm.}$$

41. In the figure of ΔABC, the points D and E are on the sides CA, CB respectively such that $DE || AB$, $AD = 2x, DC = x + 3, BE = 2x - 1$ and $CE = x$. Then, find x.



OR

In the figure of ΔABC, $DE || AB$. If $AD = 2x$, $DC = x + 3, BE = 2x - 1$ and $CE = x$, then find the value of x.



Ans : [Board Term-1 2015, 2016]

We have
$$\frac{CD}{AD} = \frac{CE}{BE}$$

$$\frac{x+3}{2x} = \frac{x}{2x-1}$$

f110

$$5x = 3 \text{ or, } x = \frac{3}{5}$$

Alternative Method :

In ABC, $DE \,||\, AB$, thus

$$\frac{CD}{CA} = \frac{CE}{CB}$$

$$\frac{CD}{CA - CD} = \frac{CE}{CB - CE}$$

$$\frac{CD}{AD} = \frac{CE}{BE}$$

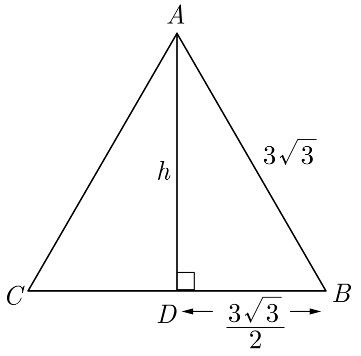
$$\frac{x+3}{2x} = \frac{x}{2x-1}$$

$$5x = 3 \text{ or, } x = \frac{3}{5}$$

42. In an equilateral triangle of side $3\sqrt{3}$ cm find the length of the altitude.

Ans : [Board Term-1 2016]

Let ΔABC be an equilateral triangle of side $3\sqrt{3}$ cm and AD is altitude which is also a perpendicular bisector of side BC. This is shown in figure given below.



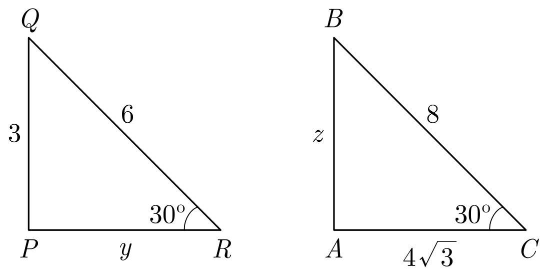
Now
$$(3\sqrt{3})^2 = h^2 + \left(\frac{3\sqrt{3}}{2}\right)^2$$

$$27 = h^2 + \frac{27}{4}$$

$$h^2 = 27 - \frac{27}{4} = \frac{81}{4}$$

$$h = \frac{9}{2} = 4.5 \text{ cm}$$

43. In the given figure, $\Delta ABC \sim \Delta PQR$. Find the value of $y + z$.



Ans : [Board Term-1 2010]

In the given figure $\Delta ABC \sim \Delta PQR$,

Thus
$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

$$\frac{z}{3} = \frac{8}{6} = \frac{4\sqrt{3}}{y}$$

$$\frac{z}{3} = \frac{8}{6} \text{ and } \frac{8}{6} = \frac{4\sqrt{3}}{y}$$

$$z = \frac{8 \times 3}{6} \text{ and } y = \frac{4\sqrt{3} \times 6}{8}$$

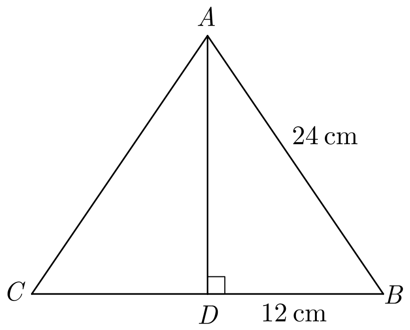
$$z = 4 \text{ and } y = 3\sqrt{3}$$

Thus
$$y + z = 3\sqrt{3} + 4$$

44. In an equilateral triangle of side 24 cm, find the length of the altitude.

Ans : [Board Term-1 2015]

Let ΔABC be an equilateral triangle of side 24 cm and AD is altitude which is also a perpendicular bisector of side BC. This is shown in figure given below.



Now
$$BD = \frac{BC}{2} = \frac{24}{2} = 12 \text{ cm}$$

$$AB = 24 \text{ cm}$$

$$AD = \sqrt{AB^2 - BD^2}$$

$$= \sqrt{(24)^2 - (12)^2}$$

$$= \sqrt{576 - 144}$$

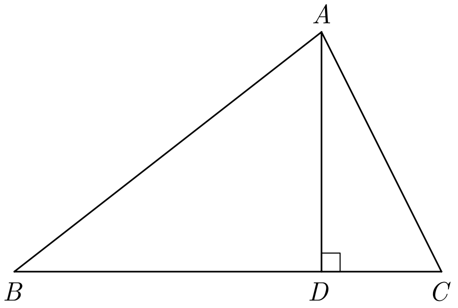
$$= \sqrt{432} = 12\sqrt{3}$$

Thus $AD = 12\sqrt{3}$ cm.

45. In $\Delta ABC, AD \perp BC$, such that $AD^2 = BD \times CD$. Prove that ΔABC is right angled at A.

Ans : [Board Term-1 2015]

As per given condition we have drawn the figure below.



We have $AD^2 = BD \times CD$

$$\frac{AD}{CD} = \frac{BD}{AD}$$

f114

Since $\angle D = 90^\circ$, by SAS we have

$$\Delta ADC \sim \Delta BDA$$

and $\angle BAD = \angle ACD$;

Since corresponding angles of similar triangles are equal

$$\angle DAC = \angle DBA$$

$$\angle BAD + \angle ACD + \angle DAC + \angle DBA = 180^\circ$$

$$2\angle BAD + 2\angle DAC = 180^\circ$$

$$\angle BAD + \angle DAC = 90^\circ$$

$$\angle A = 90^\circ$$

Thus ΔABC is right angled at A.

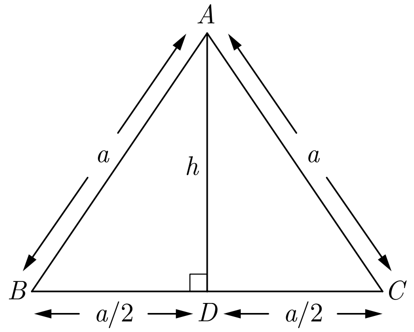
46. In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitudes. [Board 2020 SQP Standard]

or

Find the altitude of an equilateral triangle when each of its side is a cm.

Ans : [Board Term-1 2016]

Let ΔABC be an equilateral triangle of side a and AD is altitude which is also a perpendicular bisector of side BC. This is shown in figure given below.



In ΔABD, $a^2 = \left(\frac{a}{2}\right)^2 + h^2$

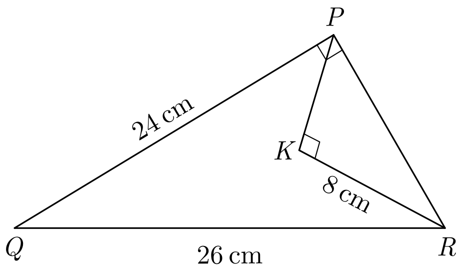
f115

$$h^2 = a^2 - \frac{a^2}{4} = \frac{3a^2}{4}$$

Thus $h = \frac{\sqrt{3}a}{2}$

Thus $4h^2 = 3a^2$ Hence Proved

47. In the given triangle $PQR, \angle QPR = 90^\circ, PQ = 24$ cm and $QR = 26$ cm and in $\Delta PKR, \angle PKR = 90^\circ$ and $KR = 8$ cm, find PK.



Ans : [Board Term-1 2012]

In the given triangle we have

$$\angle QPR = 90^0$$

Thus $QR^2 = QP^2 + PR^2$

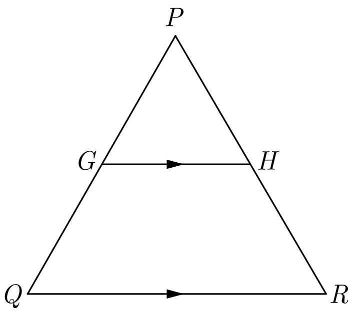
f118

$$PR = \sqrt{26^2 - 24^2}$$
$$= \sqrt{100} = 10 \text{ cm}$$

Now $\angle PKR = 90^0$

Thus $PK = \sqrt{10^2 - 8^2} = \sqrt{100 - 64}$
$$= \sqrt{36} = 6 \text{ cm}$$

48. In the given figure, G is the mid-point of the side PQ of ΔPQR and $GH || QR$. Prove that H is the mid-point of the side PR or the triangle PQR.



Ans : [Board Term-1 2012]

Since G is the mid-point of PQ we have

f121

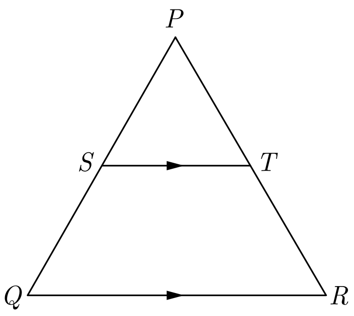
$$PG = GQ$$
$$\frac{PG}{GQ} = 1$$

We also have $GH || QR$, thus by BPT we get

$$\frac{PG}{GQ} = \frac{PH}{HR}$$
$$1 = \frac{PH}{HR}$$
$$PH = HR.$$ Hence proved.

Hence, H is the mid-point of PR.

49. In the given figure, in a triangle PQR, $ST || QR$ and $\frac{PS}{SQ} = \frac{3}{5}$ and $PR = 28$ cm, find PT.



Ans : [Board Term-1 2011]

We have $\frac{PS}{SQ} = \frac{3}{5}$

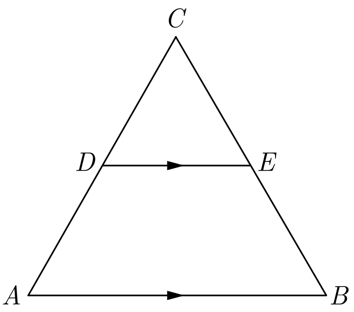
f122

$$\frac{PS}{PS + SQ} = \frac{3}{3+5}$$
$$\frac{PS}{PQ} = \frac{3}{8}$$

We also have, $ST || QR$, thus by BPT we get

$$\frac{PS}{PQ} = \frac{PT}{PR}$$
$$PT = \frac{PS}{PQ} \times PR$$
$$= \frac{3 \times 28}{8} = 10.5 \text{ cm}$$

50. In the given figure, $\angle A = \angle B$ and $AD = BE$. Show that $DE || AB$.



Ans : [Board Term-1, 2012, set-63]

In ΔCAB, we have

$$\angle A = \angle B \quad (1)$$

By isosceles triangle property we have

$$AC = CB$$

But, we have been given

$$AD = BE \quad (2)$$

Dividing equation (2) by (1) we get,

f123

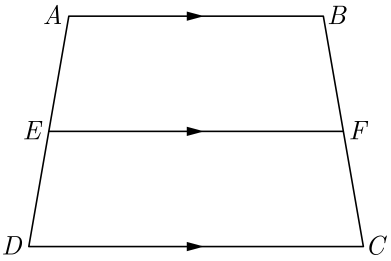
$$\frac{CD}{AD} = \frac{CE}{BE}$$

By converse of BPT,

$$DE || AB.$$ Hence Proved

51. In the given figure, if $ABCD$ is a trapezium in which

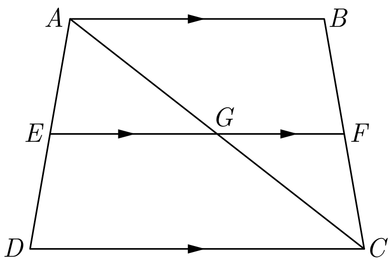
$AB \| CD \| EF$, then prove that $\frac{AE}{ED} = \frac{BF}{FC}$



Ans : [Board Term-1 2012]

We draw, AC intersecting EF at G as shown below.

f124



In $\Delta\, CAB$, $GF \| AB$, thus by BPT we have

$$\frac{AG}{CG} = \frac{BF}{FC} \quad ...(1)$$

In $\Delta\, ADC$, $EG \| DC$, thus by BPT we have

$$\frac{AE}{ED} = \frac{AG}{CG} \quad ...(2)$$

From equations (1) and (2),

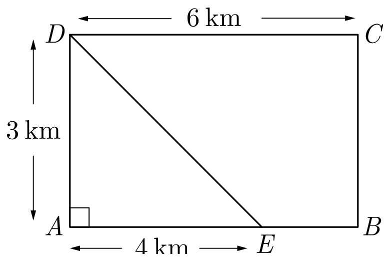
$$\frac{AE}{ED} = \frac{BF}{FC}.$$

Hence Proved.

52. In a rectangle $ABCD, E$ is a point on AB such that $AE = \frac{2}{3}AB$. If $AB = 6$ km and $AD = 3$ km, then find DE.

Ans : [Board Term-1 2016]

As per given condition we have drawn the figure below.



We have $AE = \frac{2}{3}AB = \frac{2}{3} \times 6 = 4$ km

In right triangle ADE,

f125

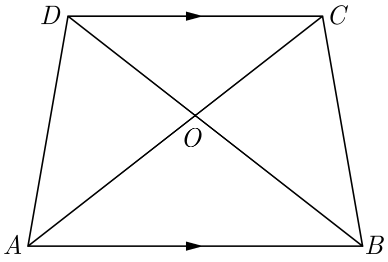
$$DE^2 = (3)^2 + (4)^2 = 25$$

Thus $DE = 5$ km

53. $ABCD$ is a trapezium in which $AB \| CD$ and its diagonals intersect each other at the point O. Show that $\frac{AO}{BO} = \frac{CO}{DO}$.

Ans : [Board Term-1 2012]

As per given condition we have drawn the figure below.



In $\Delta\, AOB$ and $\Delta\, COD$, $AB \| CD$,

f126

Thus due to alternate angles

$$\angle OAB = \angle DCO$$

and $$\angle OBA = \angle ODC$$

By AA similarity we have

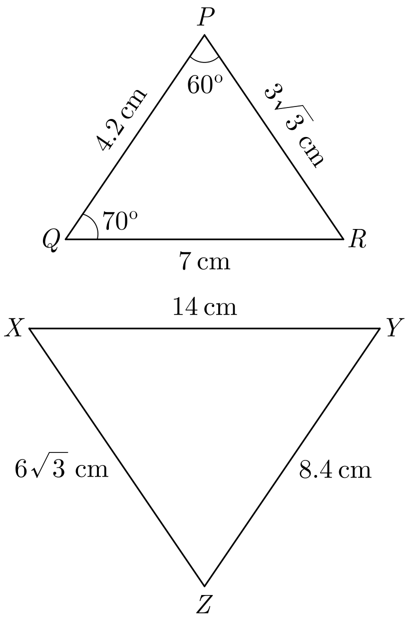
$$\Delta AOB \sim \Delta COD$$

For corresponding sides of similar triangles we have

$$\frac{AO}{CO} = \frac{BO}{DO}$$

$$\frac{AO}{BO} = \frac{CO}{DO}.$$ Hence Proved

54. In the given figures, find the measure of $\angle X$.



Ans : [Board Term-1 2012]

From given figures,

$$\frac{PQ}{ZY} = \frac{4.2}{8.4} = \frac{1}{2},$$

$$\frac{PR}{ZX} = \frac{3\sqrt{3}}{6\sqrt{3}} = \frac{1}{2}$$

and $$\frac{QR}{YX} = \frac{7}{14} = \frac{1}{2}$$

f127

Thus $$\frac{QP}{ZY} = \frac{PR}{ZX} = \frac{QR}{YX}$$

By SSS criterion we have

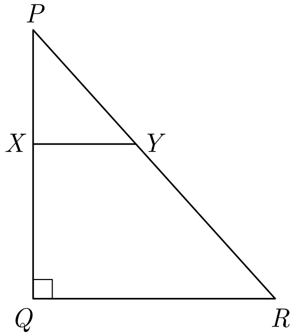
$$\Delta PQR \sim \Delta ZYX$$

Thus $$\angle X = \angle R$$

$$= 180^\circ - (60^\circ + 70^\circ) = 50^\circ$$

Thus $\angle X = 50^\circ$

55. In the given figure, PQR is a triangle right angled at Q and $XY || QR$. If $PQ = 6$ cm, $PY = 4$ cm and $PX : XQ = 1 : 2$. Calculate the length of PR and QR.



Ans : [Board Term-1 2012]

Since $XY \,||\, OR$, by BPT we have

f128

$$\frac{PX}{XQ} = \frac{PY}{YR}$$

$$\frac{1}{2} = \frac{PY}{PR - PY}$$

$$= \frac{4}{PR - 4}$$

$$PR - 4 = 8 \Rightarrow PR = 12 \text{ cm}$$

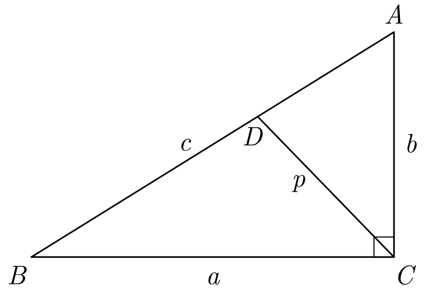
In right ΔPQR we have

$$QR^2 = PR^2 - PQ^2$$

$$= 12^2 - 6^2 = 144 - 36 = 108$$

Thus $QR = 6\sqrt{3}$ cm

56. ABC is a right triangle right angled at C. Let $BC = a$, $CA = b, AB = c$ $PQR, ST || QR$ and p be the length of perpendicular from C to AB. Prove that $cp = ab$.



Ans : [Board Term-1 2012]

In the given figure $CD \perp AB$, and $CD = p$

f129

Area, $$\Delta ABC = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times AB \times CD = \frac{1}{2} cp$$

Also, Area of $\Delta ABC = \frac{1}{2} \times BC \times AC = \frac{1}{2} ab$

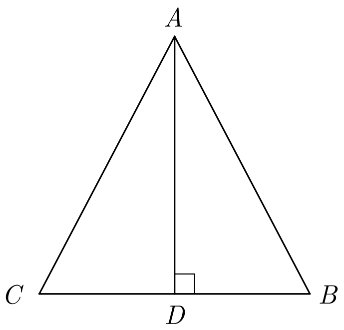
Thus $\frac{1}{2} cp = \frac{1}{2} ab$

$cp = ab$ Proved

57. In an equilateral triangle ABC, AD is drawn perpendicular to BC meeting BC in D. Prove that $AD^2 = 3BD^2$.

Ans : [Board Term-1 2012]

In ΔABD, from Pythagoras theorem,



$$AB^2 = AD^2 + BD^2$$

f130

Since $AB = BC = CA$, we get

$$BC^2 = AD^2 + BD^2,$$

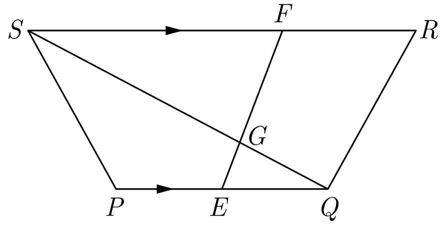
Since $\perp$ is the median in an equilateral Δ, $BC = 2BD$

$$(2BD)^2 = AD^2 + BD^2$$

$$4BD^2 - BD^2 = AD^2$$

$$3BD^2 = AD^2$$

58. In the figure, $PQRS$ is a trapezium in which $PQ || RS$. On PQ and RS, there are points E and F respectively such that EF intersects SQ at G. Prove that $EQ \times GS = GQ \times FS$.



Ans : [Board Term-1 2016]

In ΔGEQ and ΔGFS,

Due to vertical opposite angle,

$$\angle EGQ = \angle FGS$$

f131

Due to alternate angle,

$$\angle EQG = \angle FSG$$

Thus by AA similarity we have

$$\Delta GEQ \sim GFS$$

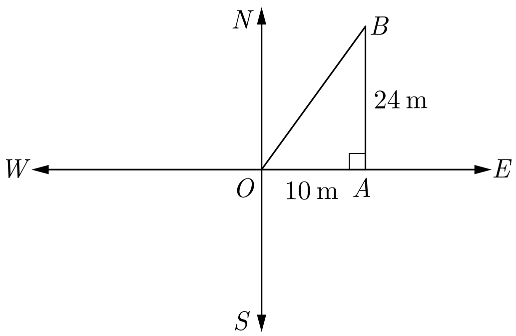
$$\frac{EQ}{FS} = \frac{GQ}{GS}$$

$$EQ \times GS = GQ \times FS$$

59. A man steadily goes 10 m due east and then 24 m due north.
(1) Find the distance from the starting point.
(2) Which mathematical concept is used in this problem?

Ans :

(1) Let the initial position of the man be at O and his final position be B. The man goes to 10 m due east and then 24 m due north. Therefore, ΔAOB is a right triangle right angled at A such that $OA = 10$ m and $AB = 24$ m. We have shown this condition in figure below.



By Pythagoras theorem,

f132

$$OB^2 = OA^2 + AB^2$$

$$= (10)^2 + (24)^2$$

$$= 100 + 576 = 676$$

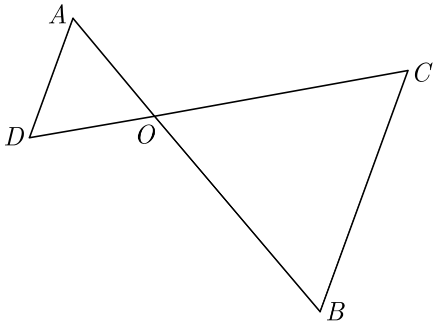
or, $OB = \sqrt{676} = 26$ m

Hence, the man is at a distance of 26 m from the starting point.

(2) Pythagoras Theorem

60. In the given figure, $OA \times OB = OC \times OD$, show that

$\angle A = \angle C$ and $\angle B = \angle D$.



Ans : [Board Term-1 2012]

We have $OA \times OB = OC \times OD$

$$\frac{OA}{OD} = \frac{OC}{OB}$$

f133

Due to the vertically opposite angles,

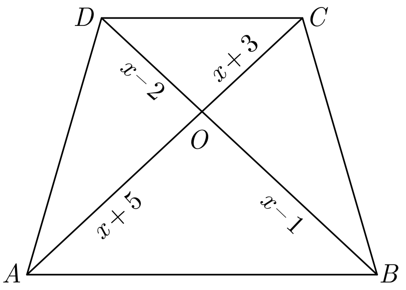
$$\angle AOD = \angle COB$$

Thus by SAS similarity we have

$$\Delta AOD \sim \Delta COB$$

Thus $\angle A = \angle C$ and $\angle B = \angle D$. because of corresponding angles of similar triangles.

61. In the given figure, if $AB || DC$, find the value of x.



Ans : [Board Term-1 2012]

We know that diagonals of a trapezium divide each other proportionally. Therefore

$$\frac{OA}{OC} = \frac{BO}{OD}$$

f134

$$\frac{x+5}{x+3} = \frac{x-1}{x-2}$$

$$(x+5)(x-2) = (x-1)(x+3)$$

$$x^2 - 2x + 5x - 10 = x^2 + 3x - x - 3$$

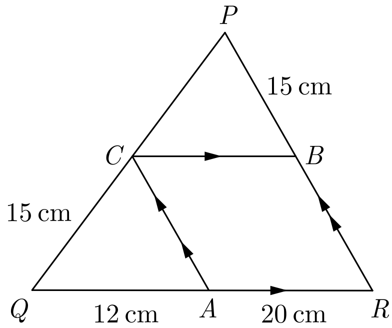
$$x^2 + 3x - 10 = x^2 + 2x - 3$$

$$3x - 10 = 2x - 3$$

$$3x - 2x = 10 - 3 \Rightarrow x = 7$$

Thus $x = 7$.

62. In the given figure, $CB || QR$ and $CA || PR$. If $AQ = 12$ cm, $AR = 20$ cm, $PB = CQ = 15$ cm, calculate PC and BR.



Ans : [Board Term-1 2012]

In ΔPQR, $CA || PR$

By BPT similarity we have

f135

$$\frac{PC}{CQ} = \frac{RA}{AQ}$$

$$\frac{PC}{15} = \frac{20}{12}$$

$$PC = \frac{15 \times 20}{12} = 25 \text{ cm}$$

In ΔPQR, $CB \parallel QR$

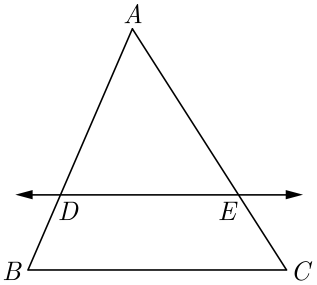
Thus $\frac{PC}{CQ} = \frac{PR}{BR}$

$$\frac{25}{15} = \frac{15}{BR}$$

$$BR = \frac{15 \times 15}{25} = 9 \text{ cm}$$

THREE MARKS QUESTIONS

63. In Figure, in ΔABC, $DE \parallel BC$ such that $AD = 2.4$ cm, $AB = 3.2$ cm and $AC = 8$ cm, then what is the length of AE?



Ans : [Board 2020 Delhi Basic]

We have $DE \parallel BC$

By BPT, $\frac{AD}{DB} = \frac{AE}{EC}$

f242

$$\frac{2.4}{AB - AD} = \frac{AE}{AC - AE}$$

$$\frac{2.4}{3.2 - 2.4} = \frac{AE}{8 - AE}$$

$$\frac{2.4}{0.8} = \frac{AE}{8 - AE}$$

$$3 = \frac{AE}{8 - AE}$$

$$\frac{3}{1+3} = \frac{AE}{8 - AE + AE}$$

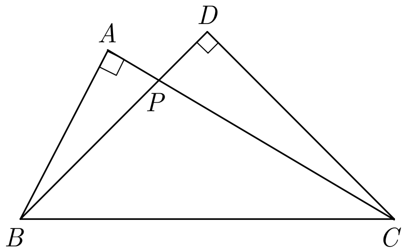
$$\frac{3}{4} = \frac{AE}{8} \Rightarrow AE = 6 \text{ cm}$$

64. Two right triangles ABC and DBC are drawn on the same hypotenuse BC and on the same side of BC. If AC and BD intersect at P, prove that $AP \times PC = BP \times DP$.

Ans : [Board 2019 OD]

Let ΔABC, and ΔDBC be right angled at A and D respectively.

As per given information in question we have drawn the figure given below.



In ΔBAP and ΔCDP we have

$$\angle BAP = \angle CDP = 90^\circ$$

f243

and due to vertical opposite angle

$$\angle BPA = \angle CPD$$

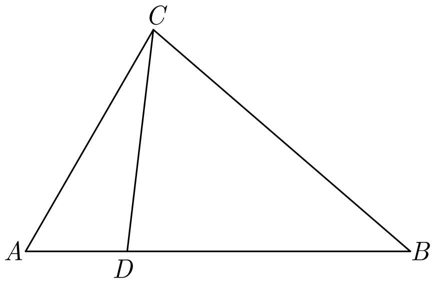
By AA similarity we have

$$\Delta BAP \sim \Delta CDP$$

Therefore $\frac{BP}{PC} = \frac{AP}{PD}$

$$AP \times PC = BP \times PD$$ Hence Proved

65. In the given figure, if $\angle ACB = \angle CDA$, $AC = 6$ cm and $AD = 3$ cm, then find the length of AB.



Ans : [Board 2020 SQP Standard]

In ΔABC and ΔACD we have

$\angle ACB = \angle CDA$ [given]

$\angle CAB = \angle CAD$ [common]

By AA similarity criterion we get

$$\Delta ABC \sim \Delta ACD$$

Thus $\frac{AB}{AC} = \frac{BC}{CD} = \frac{AC}{AD}$

f245

Now $\frac{AB}{AC} = \frac{AC}{AD}$

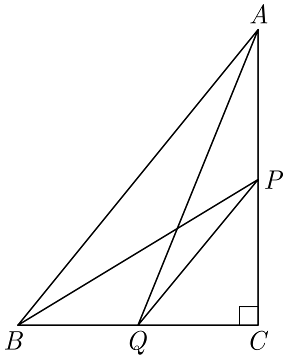
$$AC^2 = AB \times AD$$

$$6^2 = AB \times 3$$

$$AB = \frac{36}{3} = 12 \text{ cm}$$

66. If P and Q are the points on side CA and CB

respectively of ΔABC, right angled at C, prove that $(AQ^2 + BP^2) = (AB^2 + PQ^2)$



f246

Ans : [Board 2019 Delhi]

In right angled triangles ACQ and PCB

$$AQ^2 = AC^2 + CQ^2 \quad ...(1)$$

and $$BP^2 = PC^2 + CB^2 \quad ...(2)$$

Adding eq (1) and eq (2), we get

$$AQ^2 + BP^2 = (AC^2 + CQ^2) + (PC^2 + CB^2)$$

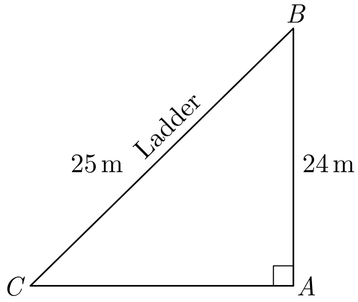
$$= (AC^2 + CB^2) + (PC^2 + CQ^2)$$

Thus $AQ^2 + BP^2 = AB^2 + PQ^2$ Hence Proved

67. A ladder 25 m long just reaches the top of a building 24 m high from the ground. What is the distance of the foot of ladder from the base of the building?

Ans : [Board 2020 OD Basic]

Let AB be the building and CB be the ladder. As per information given we have drawn figure below.



f247

Here $AB = 24$ m

$CB = 25$ m

and $\angle CAB = 90°$

By Pythagoras Theorem,

$$CB^2 = AB^2 + CA^2$$

or, $$CA^2 = CB^2 - AB^2$$

$$= 23^2 - 24^2$$

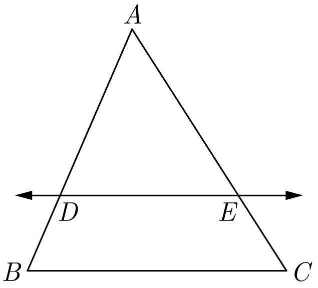
$$= 625 - 576 = 49$$

Thus $CA = 7$ m

Hence, the distance of the foot of ladder from the building is 7 m.

THREE MARKS QUESTIONS

68. In Figure, in ΔABC, $DE \parallel BC$ such that $AD = 2.4$ cm, $AB = 3.2$ cm and $AC = 8$ cm, then what is the length of AE?



Ans : [Board 2020 Delhi Basic]

We have $DE \parallel BC$

f242

By BPT, $$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{2.4}{AB - AD} = \frac{AE}{AC - AE}$$

$$\frac{2.4}{3.2 - 2.4} = \frac{AE}{8 - AE}$$

$$\frac{2.4}{0.8} = \frac{AE}{8 - AE}$$

$$3 = \frac{AE}{8 - AE}$$

$$\frac{3}{1+3} = \frac{AE}{8 - AE + AE}$$

$$\frac{3}{4} = \frac{AE}{8} \Rightarrow AE = 6 \text{ cm}$$

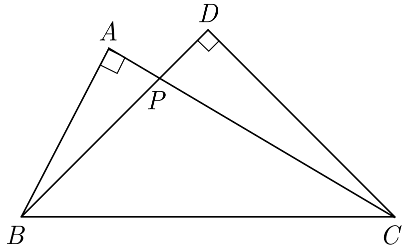
69. Two right triangles ABC and DBC are drawn on the same hypotenuse BC and on the same side of BC. If AC and BD intersect at P, prove that

$AP \times PC = BP \times DP$.

Ans : [Board 2019 OD]

Let ΔABC, and ΔDBC be right angled at A and D respectively.

As per given information in question we have drawn the figure given below.



In ΔBAP and ΔCDP we have

$$\angle BAP = \angle CDP = 90^\circ$$

and due to vertical opposite angle

$$\angle BPA = \angle CPD$$

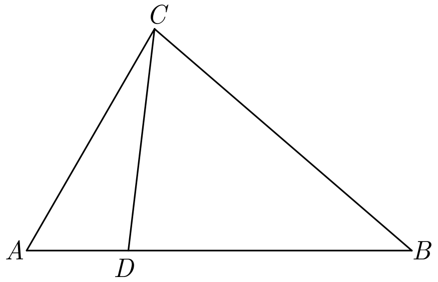
By AA similarity we have

$$\Delta BAP \sim \Delta CDP$$

Therefore $$\frac{BP}{PC} = \frac{AP}{PD}$$

$$AP \times PC = BP \times PD$$ Hence Proved

70. In the given figure, if $\angle ACB = \angle CDA$, $AC = 6$ cm and $AD = 3$ cm, then find the length of AB.



Ans : [Board 2020 SQP Standard]

In ΔABC and ΔACD we have

$$\angle ACB = \angle CDA \quad \text{[given]}$$

$$\angle CAB = \angle CAD \quad \text{[common]}$$

By AA similarity criterion we get

$$\Delta ABC \sim \Delta ACD$$

Thus $$\frac{AB}{AC} = \frac{BC}{CD} = \frac{AC}{AD}$$

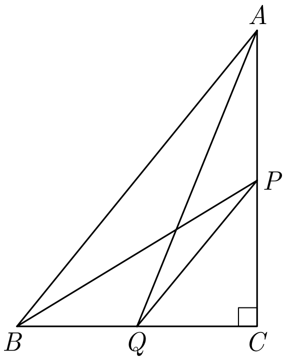
Now $$\frac{AB}{AC} = \frac{AC}{AD}$$

$$AC^2 = AB \times AD$$

$$6^2 = AB \times 3$$

$$AB = \frac{36}{3} = 12 \text{ cm}$$

71. If P and Q are the points on side CA and CB respectively of ΔABC, right angled at C, prove that $(AQ^2 + BP^2) = (AB^2 + PQ^2)$



Ans : [Board 2019 Delhi]

In right angled triangles ACQ and PCB

$$AQ^2 = AC^2 + CQ^2 \quad ...(1)$$

and $$BP^2 = PC^2 + CB^2 \quad ...(2)$$

Adding eq (1) and eq (2), we get

$$AQ^2 + BP^2 = (AC^2 + CQ^2) + (PC^2 + CB^2)$$

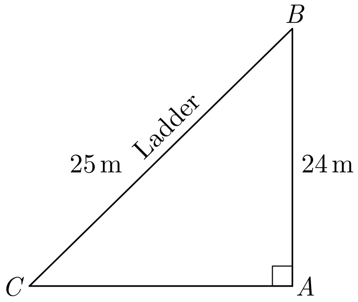
$$= (AC^2 + CB^2) + (PC^2 + CQ^2)$$

Thus $$AQ^2 + BP^2 = AB^2 + PQ^2$$ Hence Proved

72. A ladder 25 m long just reaches the top of a building 24 m high from the ground. What is the distance of the foot of ladder from the base of the building?

Ans : [Board 2020 OD Basic]

Let AB be the building and CB be the ladder. As per information given we have drawn figure below.



Here $$AB = 24 \text{ m}$$

$$CB = 25 \text{ m}$$

and $$\angle CAB = 90^\circ$$

By Pythagoras Theorem,

$$CB^2 = AB^2 + CA^2$$

or, $$CA^2 = CB^2 - AB^2$$

$$= 25^2 - 24^2$$

$$= 625 - 576 = 49$$

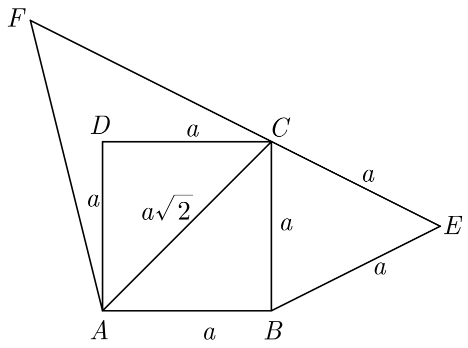
Thus $$CA = 7 \text{ m}$$

Hence, the distance of the foot of ladder from the building is 7 m.

73. Prove that area of the equilateral triangle described on the side of a square is half of this area of the equilateral triangle described on its diagonal.

Ans : [Board 2018, 2015]

As per given condition we have drawn the figure below. Let a be the side of square.



By Pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

$$= a^2 + a^2 = 2a^2$$

f138

$$AC = \sqrt{2}\,a$$

Area of equilateral triangle $\Delta\, BCE$,

$$\text{area}(\Delta\, BCE) = \frac{\sqrt{3}}{4}a^2$$

Area of equilateral triangle $\Delta\, ACF$,

$$\text{area}(\Delta\, ACF) = \frac{\sqrt{3}}{4}(\sqrt{2}\,a)^2 = \frac{\sqrt{3}}{2}a^2$$

Now, $$\frac{\text{area}(\Delta\, ACF)}{\text{area}(\Delta\, BCE)} = 2$$

$$\text{area}(\Delta\, ACF) = 2\text{area}(\Delta\, BEC)$$

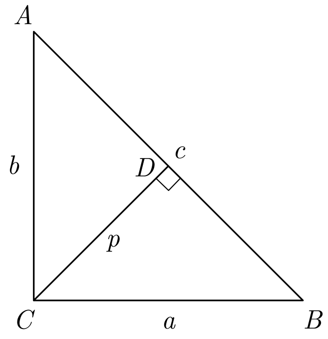
$$\text{area}(\Delta\, BEC) = \frac{1}{2}\text{area}(\Delta\, ACF)$$ Hence Proved.

74.

75. $\Delta\, ABC$ is right angled at C. If p is the length of the perpendicular from C to AB and a, b, c are the lengths of the sides opposite $\angle A, \angle B$ and $\angle C$ respectively, then prove that $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$.

Ans : [Board Term-1 2016]

As per given condition we have drawn the figure below.



In $\Delta\, ACB$ and $\Delta\, CDB$, $\angle B$ is common and

$$\angle ABC = \angle CDB = 90^0$$

Because of AA similarity we have

f140

$$\Delta\, ABC \sim \Delta\, CDB$$

Now $$\frac{b}{p} = \frac{c}{a}$$

$$\frac{1}{p} = \frac{c}{ab}$$

$$\frac{1}{p^2} = \frac{c^2}{a^2 b^2}$$

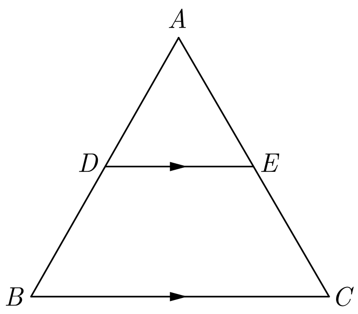
$$\frac{1}{p^2} = \frac{a^2 + b^2}{a^2 b^2} \qquad (c^2 = a^2 + b^2)$$

$$\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$$ Hence Proved

76. In $\Delta\, ABC, DE \,||\, BC$. If $AD = x + 2, DB = 3x + 16$, $AE = x$ and $EC = 3x + 5$, them find x.

Ans : [Board Term-1 2015]

As per given condition we have drawn the figure below.



In the give figure

$$DE \parallel BC$$

By BPT we have

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{x+2}{3x+16} = \frac{x}{x3+5}$$

$$(x+2)(3x+5) = x(3x+16)$$

$$3x^2+5x+6x+10 = 3x^2+16x$$

$$11x+10 = 16x$$

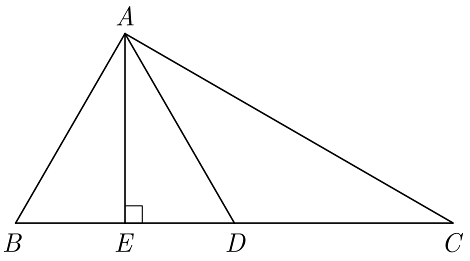
$$11x+10 = 10$$

$$5x = 10 \Rightarrow x = 2$$

77. If in ΔABC, AD is median and $AE \perp BC$, then prove that $AB^2 + AC^2 = 2AD^2 + \frac{1}{2}BC^2$.

Ans : [Board Term-1 2015]

As per given condition we have drawn the figure below.



In ΔABE, using Pythagoras theorem we have

$$AB^2 = AE^2 + BE^2$$
$$= AD^2 - DE^2 + (BD - DE)^2$$
$$= AD^2 - DE^2 + BD^2 + DE^2 - 2BD \times DE$$
$$= AD^2 + BD^2 - 2BD \times DE \quad ...(1)$$

In ΔAEC, we have

$$AC^2 = AE^2 + EC^2$$
$$= (AD^2 - ED^2) + (ED + DC)^2$$
$$= AD^2 - ED^2 + ED^2 + DC^2 + 2ED \times DC$$
$$= AD^2 + CD^2 + 2ED \times CD$$
$$= AD^2 + DC^2 + 2DC \times DE \quad ...(2)$$

Adding equation (1) and (2) we have

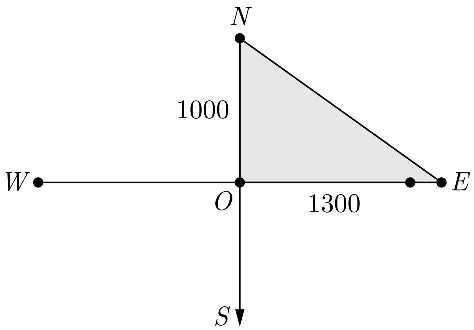
$$AB^2 + AC^2 = 2(AD^2 + BD^2) \quad (BD = DC)$$
$$= 2AD^2 + 2\left(\frac{1}{2}BC\right)^2 \quad \left(BD = \tfrac{1}{2}BC\right)$$
$$= 2AD^2 + \frac{1}{2}BC^2$$

Hence Proves

78. From an airport, two aeroplanes start at the same time. If speed of first aeroplane due North is 500 km/h and that of other due East is 650 km/h then find the distance between the two aeroplanes after 2 hours.

Ans : [Board Term-1 2015]

As per given condition we have drawn the figure below.



Distance covered by first aeroplane due North after two hours,

$$y = 500 \times 2 = 1,000 \text{ km.}$$

Distance covered by second aeroplane due East after two hours,

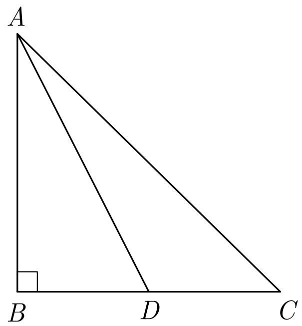
$$x = 650 \times 2 = 1,300 \text{ km.}$$

Distance between two aeroplane after 2 hours

$$NE = \sqrt{ON^2 + OE^2}$$
$$= \sqrt{(1000)^2 + (1300)^2}$$
$$= \sqrt{1000000 + 1690000}$$
$$= \sqrt{2690000}$$
$$= 1640.12 \text{ km}$$

79. In the given figure, ABC is a right angled triangle, $\angle B = 90^\circ$. D is the mid-point of BC. Show that

$AC^2 = AD^2 + 3CD^2$.



Ans : [Board Term-1 2016]

We have $BD = CD = \frac{BC}{2}$

$BC = 2BD$

Using Pythagoras theorem in the right ΔABC, we have

$$AC^2 = AB^2 + BC^2$$

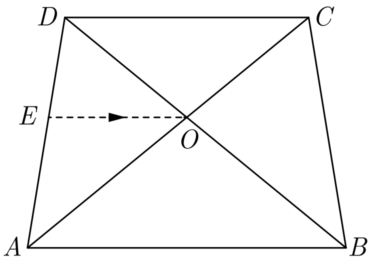
f145

$$= AB^2 + (2BD)^2$$
$$= AB^2 + 4BD^2$$
$$= (AB^2 + BD^2) + 3BD^2$$
$$AC^2 = AD^2 + 3CD^2$$

80. If the diagonals of a quadrilateral divide each other proportionally, prove that it is a trapezium.

Ans : [Board Term-1 2011]

As per given condition we have drawn quadrilateral $ABCD$, as shown below.



We have drawn $EO || AB$ on DA.

f146

In quadrilateral $ABCD$, we have

$$\frac{AO}{BO} = \frac{CO}{DO}$$

$$\frac{AO}{CO} = \frac{BO}{DO} \quad ...(1)$$

In ΔABD, $EO || AB$

By BPT we have

$$\frac{AE}{ED} = \frac{BO}{DO} \quad ...(2)$$

From equation (1) and (2), we get

$$\frac{AE}{ED} = \frac{AO}{CO}$$

In ΔADC, $\frac{AE}{ED} = \frac{AO}{CO}$

$EO || DC$ (Converse of BPT)

$EO || AB$ (Construction)

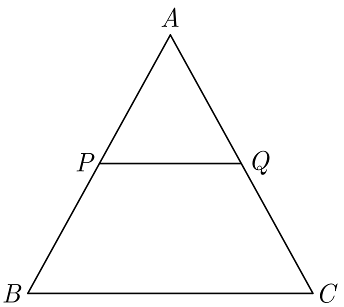
$AB || DC$

Thus in quadrilateral $ABCD$ we have

AB $AB \| CD$

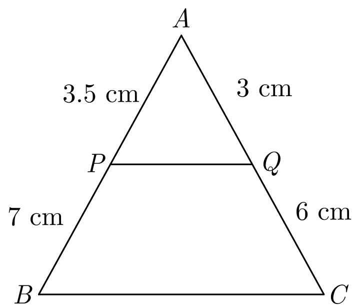
Thus $ABCD$ is a trapezium. Hence Proved

81. In the given figure, P and Q are the points on the sides AB and AC respectively of ΔABC, such that $AP = 3.5$cm, $PB = 7$ cm, $AQ = 3$ cm and $QC = 6$ cm. If $PQ = 4.5$ cm, find BC.



Ans : [Board Term-1 2011]

We have redrawn the given figure as below.



We have $\frac{AP}{AB} = \frac{3.5}{10.5} = \frac{1}{3}$

f147

and $\frac{AQ}{AC} = \frac{3}{9} = \frac{1}{3}$

In ΔABC, $\frac{AP}{AB} = \frac{AQ}{AC}$ and $\angle A$ is common.

Thus due to SAS we have

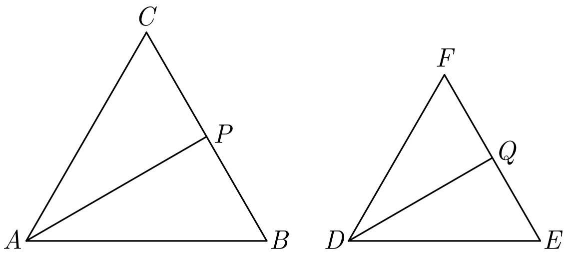
$$\Delta APQ \sim \Delta ABC$$

$$\frac{AP}{AB} = \frac{PQ}{BC}$$

$$\frac{1}{3} = \frac{4.5}{BC}$$

$$BC = 13.5 \text{ cm.}$$

82. In given figure $\Delta ABC \sim \Delta DEF$. AP bisects $\angle CAB$ and DQ bisects $\angle FDE$.



Prove that :

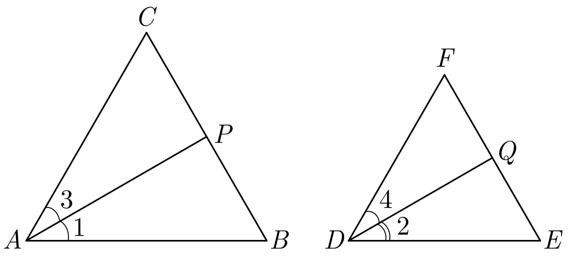
(1) $\frac{AP}{DQ} = \frac{AB}{DE}$

(2) $\Delta CAP \sim \Delta FDQ$.

f148

Ans : [Board Term-1 2016]

As per given condition we have redrawn the figure below.



(1) Since $\Delta ABC \sim \Delta DEF$

$$\angle A = \angle D \quad \text{(Corresponding angles)}$$

$$2\angle 1 = 2\angle 2$$

Also $\angle B = \angle E$ (Corresponding angles)

$$\frac{AP}{DQ} = \frac{AB}{DE} \quad \text{Hence Proved}$$

(2) Since $\Delta ABC \sim \Delta DEF$

$$\angle A = \angle D$$

and $\angle C = \angle F$

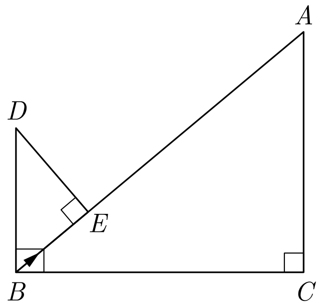
$$2\angle 3 = 2\angle 4$$

$$\angle 3 = \angle 4$$

By AA similarity we have

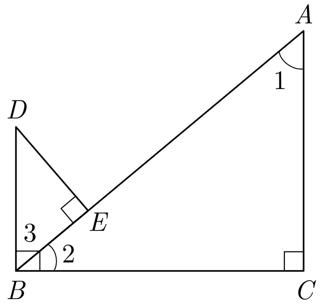
$$\Delta CAP \sim \Delta FDQ$$

83. In the given figure, $DB \perp BC, DE \perp AB$ and $AC \perp BC$. Prove that $\frac{BE}{DE} = \frac{AC}{BC}$.



Ans : [Board Term-1 2011]

As per given condition we have redrawn the figure below.



We have $DB \perp BC, DE \perp AB$ and $AC \perp BC$.

In ΔABC, $\angle C = 90°$, thus

$$\angle 1 + \angle 2 = 90°$$

f149

But we have been given,

$$\angle 2 + \angle 3 = 90^o$$

Hence $\angle 1 = \angle 3$

In ΔABC and ΔBDE,

$$\angle 1 = \angle 3$$

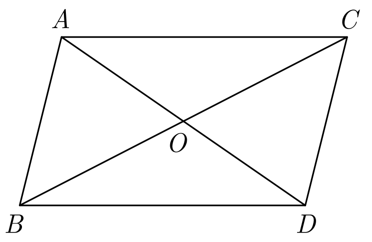
and $\angle ACB = \angle DEB = 90^o$

Thus by AA similarity we have

$$\Delta ABC \sim \Delta BDE$$

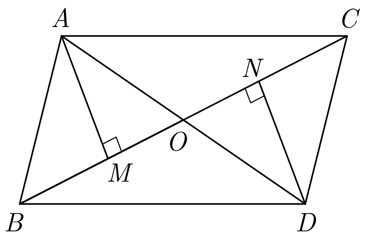
Thus $\dfrac{AC}{BC} = \dfrac{BE}{DE}$. Hence Proved

84. In the given figure, ΔABC and ΔABC and ΔDBC are on the same base BC. AD and BC intersect at O. Prove that $\dfrac{ar(\Delta ABC)}{ar(\Delta DBC)} = \dfrac{AO}{DO}$.



Ans : [Board 2020 OD Std, 2016, 2011]

As per given condition we have redrawn the figure below. Here we have drawn $AM \perp BC$ and $DN \perp BC$.



f150

In ΔAOM and ΔDON,

$$\angle AOM = \angle DON$$

(Vertically opposite angles)

$\angle AMO = \angle DNO = 90^o$ (Construction)

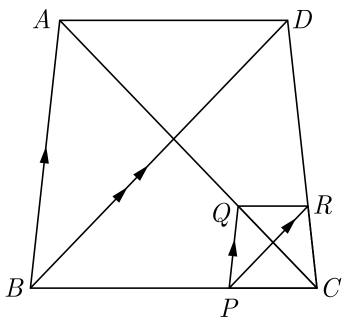
or, $\Delta AOM \sim \Delta DON$ (By AA similarity)

Thus $\dfrac{AO}{DO} = \dfrac{AM}{DN}$ ...(1)

Now, $\dfrac{ar(\Delta ABC)}{ar(\Delta DBC)} = \dfrac{\frac{1}{2} \times BC \times AM}{\frac{1}{2} \times BC \times DN}$

$= \dfrac{AM}{DN} = \dfrac{AO}{DO}$ From equation (1)

85. In the given figure, two triangles ABC and DBC lie on the same side of BC such that $PQ || BA$ and $PR || BD$. Prove that $QR || AD$.



f151

Ans : [Board Term-1 2011]

In ΔABC, we have $PQ || AB$ and $PR || BD$.

By BPT we have

$$\frac{BP}{PC} = \frac{AQ}{QC} \quad ...(1)$$

Again in ΔBCD, we have

$$PR || BD$$

By BPT we have

$$\frac{BP}{PC} = \frac{DR}{RC} \qquad \text{(by BPT) ...(2)}$$

$$\frac{AQ}{QC} = \frac{DR}{RC}$$

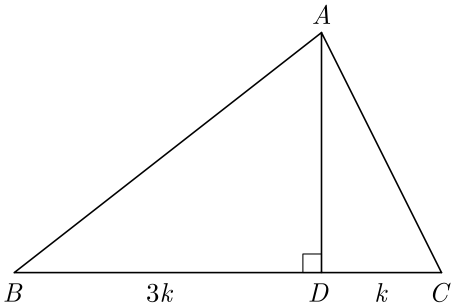
By converse of BPT,

$$PR \parallel AD \qquad \text{Hence proved}$$

86. The perpendicular AD on the base BC of a ΔABC intersects BC at D so that $DB = 3CD$. Prove that $2(AB)^2 = 2(AC)^2 + BC^2$.

Ans : [Board Term-1 2011, 2012, 2016]

As per given condition we have drawn the figure below.



f152

Here	DB	$= 3CD$
	BD	$= \frac{3}{4}BC$
	DC	$= \frac{1}{4}BC$

In ΔADB, we have

$$AB^2 = AD^2 + BD^2 \qquad ...(1)$$

In ΔADC, $\quad AC^2 = AD^2 + CD^2 \qquad ...(2)$

Subtracting equation (2) from (1), we get

$$AB^2 - AC^2 = BD^2 - CD^2$$

Since $DB = 3CD$ we get

$$AB^2 - AC^2 = \left(\frac{3}{4}BC\right)^2 - \left(\frac{1}{4}BC\right)^2$$

$$= \frac{9}{16}BC^2 - \frac{1}{16}BC^2 = \frac{BC^2}{2}$$

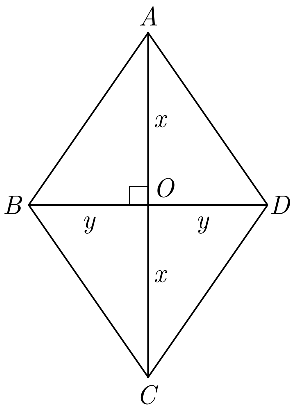
$$2(AB^2 - AC^2) = BC^2$$

$$2(AB)^2 = 2AC^2 + BC^2 \qquad \text{Hence Proved}$$

87. Prove that the sum of squares on the sides of a rhombus is equal to sum of squares of its diagonals.

Ans : [Board Term-1 2011]

Let, $ABCD$ is a rhombus and we know that diagonals of a rhombus bisect each other at 90°.



f155

Now $\quad AO = OC \Rightarrow AO^2 = OC^2$

$$BO = OD \Rightarrow BO^2 = OD^2$$

and $\quad \angle AOB = 90^0$

$$AB^2 = OA^2 + BO^2 = x^2 + y^2$$

Similarly, $\quad AD^2 = OA^2 + OD^2 = x^2 + y^2$

$$CD^2 = OC^2 + OD^2 = x^2 + y^2$$

$$CB^2 = OC^2 + OB^2 = x^2 + y^2$$

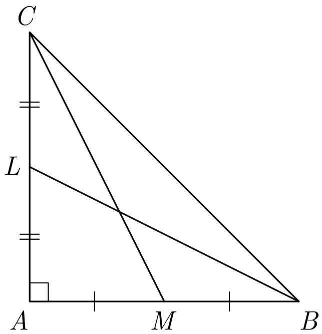
$$AB^2 + BC^2 + CD^2 + DA^2 = 4x^2 + 4y^2$$

$$= (2x)^2 + (2y)^2$$

$$AB^2 + BC^2 + CD^2 + AD^2 = AC^2 + BD^2$$

Hence Proved

88. In the given figure, BL and CM are medians of ΔABC, right angled at A. Prove that $4(BL^2 + CM^2) = 5BC^2$.



f156

Ans : [Board Term-1 2011]

We have a right angled triangle ΔABC at A where BL and CM are medians.

In ΔABL, $BL^2 = AB^2 + AL^2$

$$= AB^2 + \left(\frac{AC}{2}\right)^2 \quad (BL \text{ is median})$$

In ΔACM, $CM^2 = AC^2 + AM^2$

$$= AC^2 + \left(\frac{AB}{2}\right)^2 \quad (CM \text{ is median})$$

Now $BL^2 + CM^2 = AB^2 + AC^2 + \frac{AC^2}{4} + \frac{AB^2}{4}$

$$4(BL^2 + CM^2) = 5AB^2 + 5AC^2$$

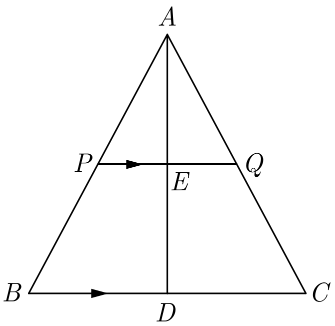
$$= 5(AB^2 + AC^2)$$

$$= 5BC^2$$ Hence Proved

89. In a ΔABC, let P and Q be points on AB and AC respectively such that $PQ || BC$. Prove that the median AD bisects PQ.

Ans : [Board Term-1 2011]

As per given condition we have drawn the figure below.



The median AD intersects PQ at E.

f157

We have, PQ || BE

$\angle ApE = \angle B$ and $\angle AQE = \angle C$

(Corresponding angles)

Thus in ΔAPE and ΔABD we have

$$\angle APE = \angle ABD$$

$$\angle PAE = \angle BAD \quad \text{(common)}$$

Thus $\Delta APE \sim \Delta ABD$

$$\frac{PE}{BD} = \frac{AE}{AD} \quad ...(1)$$

Similarly, $\Delta AQE \sim \Delta ACD$

or, $$\frac{QE}{CD} = \frac{AE}{AD} \quad ...(2)$$

From equation (1) and (2) we have

$$\frac{PE}{BD} = \frac{QE}{CD}$$

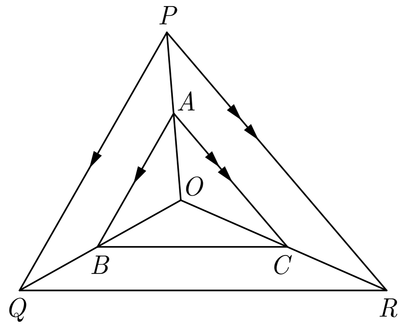
As $CD = BD$, we get

$$\frac{PE}{BD} = \frac{QE}{BD}$$

$$PE = QE$$

Hence, AD bisects PQ.

90. In the given figure A, B and C are points on OP, OQ and OR respectively such that $AB || PQ$ and $AC || PR$. Prove that $BC || QR$.



Ans : [Board Term-1 2012]

In ΔPOQ, $AB || PQ$

By BPT $$\frac{AO}{AP} = \frac{OB}{BQ} \quad ...(1)$$

In ΔOPR, $AC || PR$,

By BPT $$\frac{OA}{AP} = \frac{OC}{CR} \quad (2)$$

From equations (1) and (2), we have

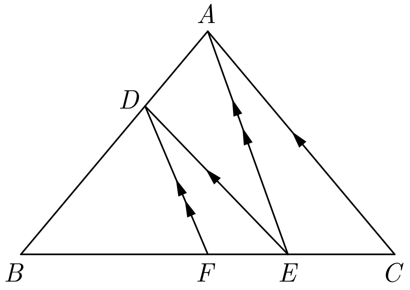
f158

$$\frac{OB}{BQ} = \frac{OC}{CR}$$

By converse of BPT we have

$$BC || QR$$ Hence Proved

91. In the given figure, $DE || AC$ and $DF || AE$. Prove that $\frac{BE}{FE} = \frac{BE}{EC}$.



Ans : [Board 2020 Delhi Std, 2012]

In ΔABC, $DE \parallel AC$, (Given)

By BPT $\frac{BD}{DA} = \frac{BE}{EC}$ ...(1)

In ΔABE, $DF \parallel AE$, (Given)

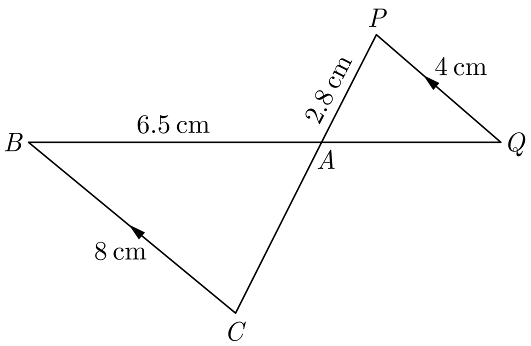
By BPT $\frac{BD}{DA} = \frac{BF}{FE}$ ...(2)

From (1) and (2), we have

$$\frac{BF}{FE} = \frac{BE}{EC}.$$

f160

92. In the given figure, $BC || PQ$ and $BC = 8$ cm, $PQ = 4$ cm, $BA = 6.5$ cm $AP = 2.8$ cm Find CA and AQ.



Ans : [Board Term-1 2012]

In ΔABC and ΔAPQ, $AB = 6.5$ cm, $BC = 8$ cm,

$PQ = 4$ cm and $AP = 2.8$ cm.

We have $BC \parallel PQ$

Due to alternate angles

$$\angle CBA = \angle AQP$$

Due to vertically opposite angles,

$$\angle BAC = \angle PAQ$$

Due to AA similarity,

$$\Delta ABC \sim \Delta AQP$$

$$\frac{AB}{AQ} = \frac{BC}{QP} = \frac{AC}{AP}$$

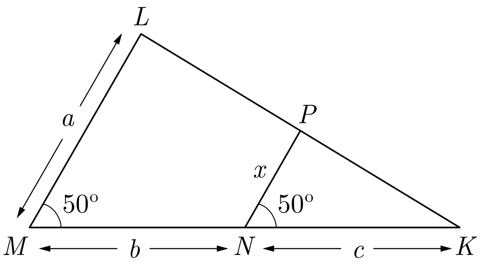
$$\frac{6.5}{AQ} = \frac{8}{4} = \frac{AC}{AP}$$

$$AQ = \frac{6.5}{2} = 3.25 \text{ cm}$$

$$AC = 2 \times 2.5 = 5.6 \text{ cm}$$

f161

93. In the given figure, find the value of x in terms of a, b and c.



Ans : [Board Term-1 2012]

In triangles LMK and PNK, $\angle K$ is common and

$$\angle M = \angle N = 50^{\circ}$$

Due to AA similarity,

$$\Delta LMK \sim \Delta PNK$$

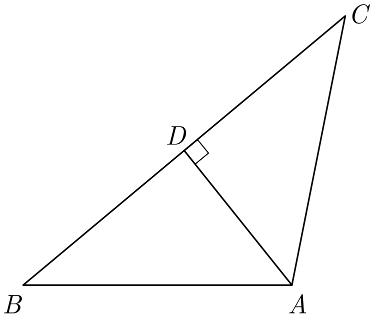
$$\frac{LM}{PN} = \frac{KM}{KN}$$

$$\frac{a}{x} = \frac{b+c}{c}$$

$$x = \frac{ac}{b+c}$$

f159

94. In the given figure, if $AD \perp BC$, prove that $AB^2 + CD^2 = BD^2 + AC^2$.



Ans : [Board 2020 OD Standard]

In right ΔADC,

$$AC^2 = AD^2 + CD^2 \quad ...(1)$$

In right ΔADB,

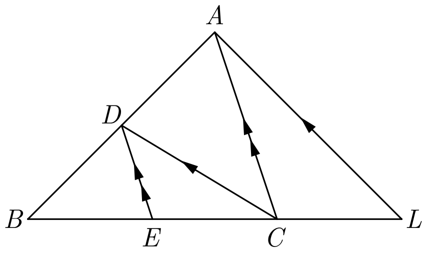
$$AB^2 = AD^2 + BD^2 \quad ...(2)$$

Subtracting equation (1) from (2) we have

$$AB^2 - AC^2 = BD^2 - CD^2$$

$$AB^2 + CD^2 = AC^2 + BD^2.$$

95. In the given figure, $CD \,||\, LA$ and $DE \,||\, AC$. Find the length of CL, if $BE = 4$ cm and $EC = 2$ cm.



Ans : [Board Term-1 2012]

In ΔABC, $DE \,||\, AC$, $BE = 4$ cm and $EC = 2$cm

By BPT $$\frac{BD}{DA} = \frac{BE}{EC} \quad ...(1)$$

In ΔABL, $DC \,||\, AL$

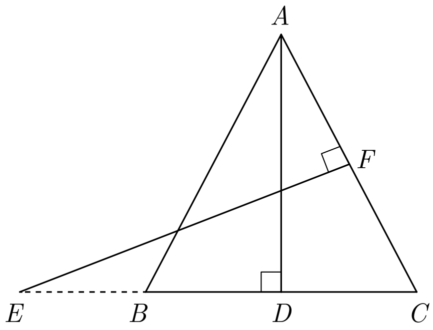
By BPT $$\frac{BD}{DA} = \frac{BC}{CL} \quad ...(2)$$

From equations (1) and (2),

$$\frac{BE}{EC} = \frac{BC}{CL}$$

$$\frac{4}{2} = \frac{6}{CL} \Rightarrow CL = 3 \text{ cm}$$

96. In the given figure, $AB = AC$. E is a point on CB produced. If AD is perpendicular to BC and EF perpendicular to AC, prove that ΔABD is similar to ΔCEF.



Ans : [Board Term-1 2012]

In ΔABD and ΔCEF, we have

$$AB = AC$$

Thus $$\angle ABC = \angle ACB$$

$$\angle ABD = \angle ECF$$

$$\angle ADB = \angle EFC \quad \text{(each } 90^\circ)$$

Due to AA similarity

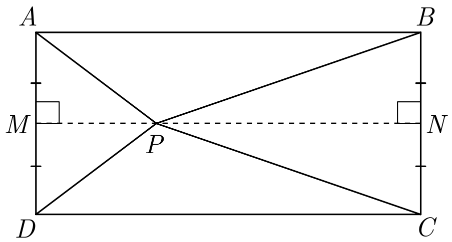
$$\Delta ABD \sim \Delta ECF \quad \text{Hence proved}$$

FOUR MARKS QUESTIONS

97. In a rectangle $ABCD$, P is any interior point. Then prove that $PA^2 + PC^2 = PB^2 + PD^2$.

Ans : [Board 2020 OD Basic]

As per information given we have drawn figure below.



Here P is any point in the interior of rectangle $ABCD$. We have drawn a line MN through point P and parallel to AB and CD.

We have to prove $PA^2 + PC^2 = PB^2 + PD^2$

Since $AB \parallel MN$, $AM \parallel BN$ and $\angle A = 90^\circ$, thus $ABNM$ is rectangle. $MNCD$ is also a rectangle.

Here, $PM \perp AD$ and $PN \perp BC$,

$$AM = BN \text{ and } MD = NC \quad ...(1)$$

Now, in ΔAMP, $$PA^2 = AM^2 + MP^2 \quad ...(2)$$

In ΔPMD, $$PD^2 = MP^2 + MD^2 \quad ...(3)$$

In ΔPNB, $$PB^2 = PN^2 + BN^2 \quad ...(4)$$

In ΔPNC, $$PC^2 = PN^2 + NC^2 \quad ...(5)$$

From equation (2) and (5) we obtain,

$$PA^2 + PC^2 = AM^2 + MP^2 + PN^2 + NC^2$$

Using equation (1) we have

$$PA^2 + PC^2 = BN^2 + MP^2 + PN^2 + MD^2$$

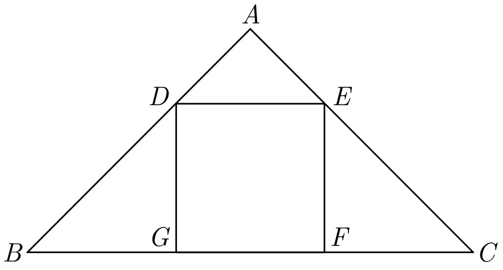
$$= (BN^2 + PN^2) + (MP^2 + MD^2)$$

Using equation (3) and (4) we have

$$PA^2 + PC^2 = PB^2 + PD^2$$

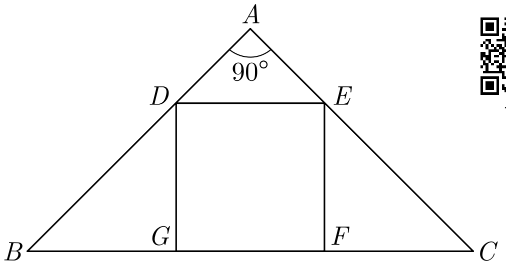
f248

98. In the given figure, $DEFG$ is a square and $\angle BAC = 90^\circ$. Show that $FG^2 = BG \times FC$.



Ans : [Board 2020 SQP Standard]

We have redrawn the given figure as shown below.



f249

In ΔADE and ΔGBD, we have

$$\angle DAE = \angle BGD \quad \text{[each } 90^\circ]$$

Due to corresponding angles we have

$$\angle ADE = \angle GDB$$

Thus by AA similarity criterion,

$$\Delta ADE \sim \Delta GBD$$

Now, in ΔADE and ΔFEC,

$$\angle EAD = \angle CFE \quad \text{[each } 90^\circ]$$

Due to corresponding angles we have

$$\angle AED = \angle FCE$$

Thus by AA similarity criterion,

$$\Delta ADE \sim \Delta FEC$$

Since $\Delta ADE \sim \Delta GBD$ and $\Delta ADE \sim \Delta FEC$ we have

$$\Delta GBD \sim \Delta FEC$$

Thus $$\frac{GB}{FE} = \frac{GD}{FC}$$

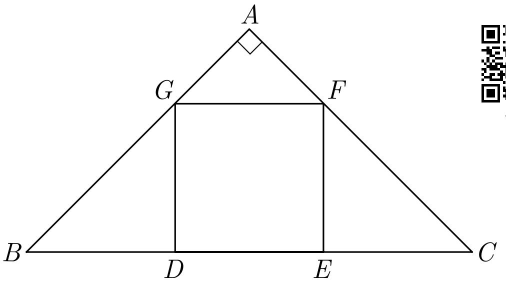
Since $DEFG$ is square, we obtain,

$$\frac{BG}{FG} = \frac{FG}{FC}$$

Therefore $$FG^2 = BG \times FC$$ Hence Proved

99. In Figure $DEFG$ is a square in a triangle ABC right angled at A. Prove that

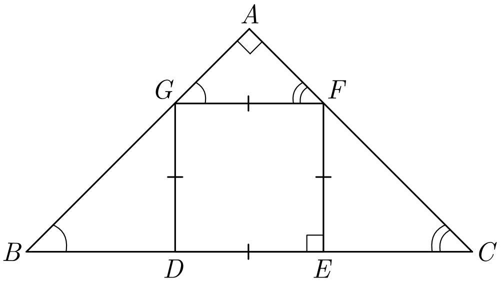
(i) $\Delta AGF \sim \Delta DBG$ (ii) $\Delta AGF \sim \Delta EFC$



f250

Ans : [Board 2020 Delhi, OD Basic]

We have redrawn the given figure as shown below.



Here ABC is a triangle in which $\angle BAC = 90^\circ$ and $DEFG$ is a square.

(i) In ΔAGF and ΔDBG

$$\angle GAF = \angle BDG \quad (\text{each } 90^\circ)$$

Due to corresponding angles,

$$\angle AGF = \angle GBD$$

Thus by AA similarity criterion,

$$\Delta AGF \sim \Delta DBG \qquad \text{Hence Proved}$$

(ii) In ΔAGF and ΔEFC,

$$\angle GAF = \angle CEF \qquad \text{(each } 90^\circ\text{)}$$

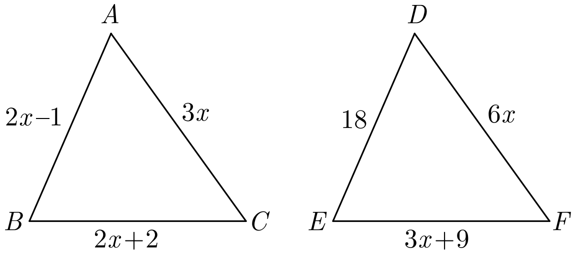
Due to corresponding angles,

$$\angle AFG = \angle FCE$$

Thus by AA similarity criterion,

$$\Delta AGF \sim \Delta EFC \qquad \text{Hence Proved}$$

100. In Figure, if $\Delta ABC \sim \Delta DEF$ and their sides of lengths (in cm) are marked along them, then find the lengths of sides of each triangle.



Ans : [Board 2020 OD Standard]

Since $\Delta ABC \sim \Delta DEF$, we have

$$\frac{AB}{BC} = \frac{DE}{EF}$$

$$\frac{2x-1}{2x+2} = \frac{18}{3x+9}$$

$$(2x-1)(3x+9) = 18(2x+2)$$

$$(2x-1)(x+3) = 6(2x+2)$$

$$2x^2 - x + 6x - 3 = 12x + 12$$

$$2x^2 + 5x - 12x - 15 = 0$$

$$2x^2 - 7x - 15 = 0$$

$$2x^2 - 10x + 3x - 15 = 0$$

$$2x(x-5) + 3(x-5) = 0$$

$$(x-5)(2x+3) = 0 \Rightarrow x = 5 \text{ or } x = \frac{-3}{2}$$

But $x = \frac{-3}{2}$ is not possible, thus $x = 5$.

Now in ΔABC, we get

$$AB = 2x - 1 = 2 \times 5 - 1 = 9$$

$$BC = 2x + 2 = 2 \times 5 + 2 = 12$$

$$AC = 3x = 3 \times 5 = 15$$

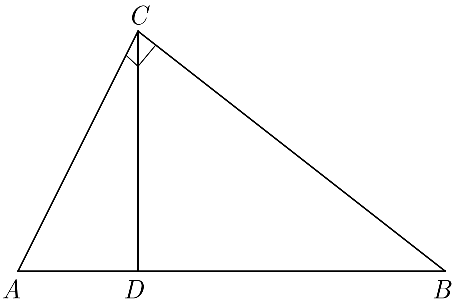
and in ΔDEF, we get

$$DE = 18$$

$$EF = 3x + 9 = 3 \times 5 + 9 = 24$$

$$DE = 6x = 6 \times 5 = 30.$$

101. In Figure, $\angle ACB = 90^\circ$ and $CD \perp AB$, prove that $CD^2 = BD \times AD$.



Ans : [Board 2019 Delhi]

In ΔACB we have

$$\angle ACB = 90^\circ$$

and $\qquad CD \perp AB$

Thus $\qquad AB^2 = CA^2 + CB^2 \qquad \text{...(1)}$

In ΔCAD, $\angle ADC = 90^\circ$, thus we have

$$CA^2 = CD^2 + AD^2 \qquad \text{...(2)}$$

and in ΔCDB, $\angle CDB = 90^\circ$, thus we have

$$CB^2 = CD^2 + BD^2 \qquad \text{...(3)}$$

Adding equation (2) and (3), we get

$$CA^2 + CB^2 = 2CD^2 + AD^2 + BD^2$$

Substituting AB^2 from equation (1) we have

$$AB^2 = 2CD^2 + AD^2 + BD^2$$

$$AB^2 - AD^2 = BD^2 + 2CD^2$$

$$(AB + AD)(AB - AD) = BD^2 + 2CD^2$$

$$(AB + AD)BD - BD^2 = 2CD^2$$

$$BD[(AB + AD) - BD] = 2CD^2$$

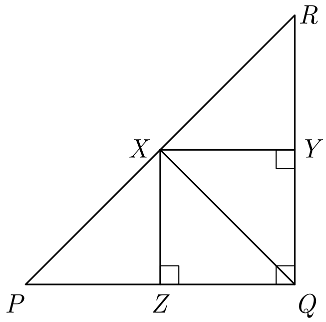
$$BD[AD + (AB - BD)] = 2CD^2$$

$$BD[AD + AD] = 2CD^2$$

$$BD \times 2AD = 2CD^2$$

$$CD^2 = BD \times AD \qquad \text{Hence Proved}$$

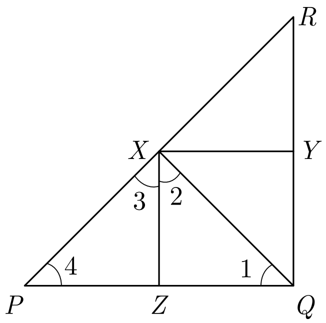
102. ΔPQR is right angled at Q. $QX \perp PR$, $XY \perp RQ$ and $XZ \perp PQ$ are drawn. Prove that $XZ^2 = PZ \times ZQ$.



Ans : [Board Term-1 2015]

We have redrawn the given figure as below.

f165



It may be easily seen that $RQ \perp PQ$ and $XZ \perp PQ$ or $XZ \parallel YQ$.

Similarly $XY \parallel ZQ$

Since $\angle PQR = 90°$, thus $XYQZ$ is a rectangle.

In ΔXZQ, $\angle 1 + \angle 2 = 90°$ ...(1)

and in ΔPZX, $\angle 3 + \angle 4 = 90°$ ...(2)

$XQ \perp PR$ or, $\angle 2 + \angle 3 = 90°$ ...(3)

From eq. (1) and (3), $\angle 1 = \angle 3$

From eq. (2) and (3), $\angle 2 = \angle 4$

Due to AA similarity,

$$\Delta PZX \sim \Delta XZQ$$

$$\frac{PZ}{XZ} = \frac{XZ}{ZQ}$$

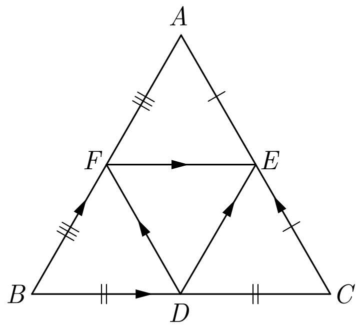
$$XZ^2 = PZ \times ZQ \qquad \text{Hence proved}$$

103. In ΔABC, the mid-points of sides BC, CA and AB are D, E and F respectively. Find ratio of $ar(\Delta DEF)$ to $ar(\Delta ABC)$.

Ans : [Board Term-1 2015]

As per given condition we have given the figure below. Here F, E and D are the mid-points of AB, AC and BC respectively.

f166



Hence, $FE \parallel BC, DE \parallel AB$ and $DF \parallel AC$

By mid-point theorem,

If $DE \parallel BA$ then $DE \parallel BF$

and if $FE \parallel BC$ then $FE \parallel BD$

Therefore $FEDB$ is a parallelogram in which DF is diagonal and a diagonal of parallelogram divides it into two equal Areas.

Hence $ar(\Delta BDF) = ar(\Delta DEF)$ ...(1)

Similarly $ar(\Delta CDE) = ar(\Delta DEF)$ ...(2)

$(\Delta AFE) = ar(\Delta DEF)$ ...(3)

$(\Delta DEF) = ar(\Delta DEF)$ ...(4)

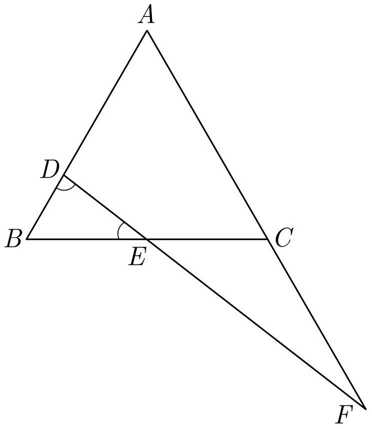
Adding equation (1), (2), (3) and (4), we have

$$ar(\Delta BDF) + ar(\Delta CDE) + ar(\Delta AFE) + ar(\Delta DEF) = 4ar(\Delta DEF)$$

$$ar(\Delta ABC) = 4ar(\Delta DEF)$$

$$\frac{ar(\Delta DEF)}{ar(\Delta ABC)} = \frac{1}{4}$$

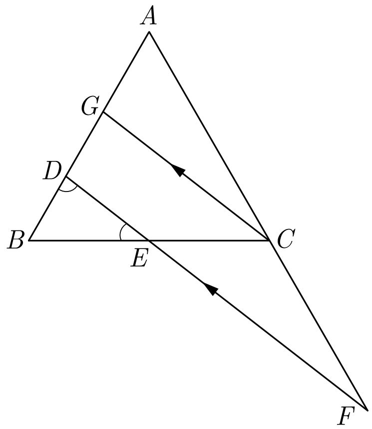
104. In the figure, $\angle BED = \angle BDE$ and E is the mid-point of BC. Prove that $\frac{AF}{CF} = \frac{AD}{BE}$.



Ans :

We have redrawn the given figure as below. Here $CG \,||\, FD$.

f168



We have $\angle BED = \angle BDE$

Since E is mid-point of BC,

$$BE = BD = EC \quad \text{...(1)}$$

In ΔBCG, $DE \,||\, FG$

From (1) we have

$$\frac{BD}{DG} = \frac{BE}{EC} = 1$$

$$BD = DG = EC = BE$$

In ΔADF, $CG \,||\, FD$

By BPT $\frac{AG}{GD} = \frac{AC}{CF}$

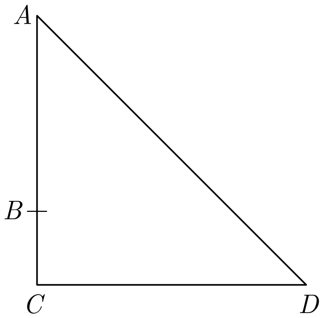
$$\frac{AG + GD}{GD} = \frac{AF + CF}{CF}$$

, $$\frac{AD}{GD} = \frac{AF}{CF}$$

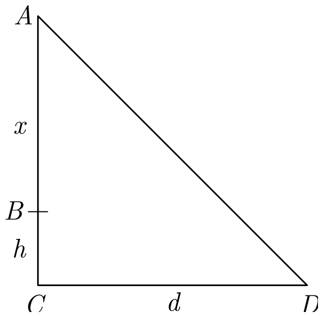
Thus $$\frac{AF}{CF} = \frac{AD}{BE}$$

105. In the right triangle, B is a point on AC such that $AB + AD = BC + CD$. If $AB = x, BC = h$ and $CD = d$, then find x (in term of h and d).

Ans : [Board Term-1 2015]



We have redrawn the given figure as below.



f170

We have $AB + AD = BC + CD$

$$AD = BC + CD - AB$$

$$AD = h + d - x$$

In right ΔACD, we have

$$AD^2 = AC^2 + DC^2$$

$$(h + d - x)^2 = (x + h)^2 + d^2$$

$$(h + d - x)^2 - (x + h)^2 = d^2$$

$$(h + d - x - x - h)(h + d - x + x + h) = d^2$$

$$(d - 2x)(2h + d) = d^2$$

$$2hd + d^2 - 4hx - 2xd = d^2$$

$$2hd = 4hx + 2xd$$

$$= 2(2h + d)x$$

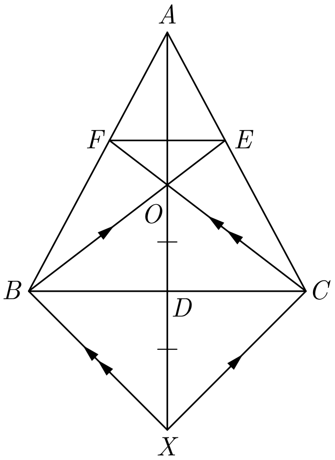
or, $$x = \frac{hd}{2h+d}$$

106. In ΔABC, AD is a median and O is any point on AD. BO and CO on producing meet AC and AB at E and F respectively. Now AD is produced to X such that $OD = DX$ as shown in figure.

Prove that :

(1) $EF || BC$

(2) $AO : AX = AF : AB$



Ans : [Board Term-1 2015]

Since BC and OX bisect each other, $BXCO$ is a parallelogram. Therefore $BE || XC$ and $BX || CF$.

In ΔABX, by BPT we get,

$$\frac{AF}{FB} = \frac{AO}{OX} \quad ..(1)$$

In ΔAXC, $$\frac{AE}{EC} = \frac{AO}{OX} \quad ...(2)$$

From (1) and (2) we get

$$\frac{AF}{FB} = \frac{AE}{EC}$$

f169

By converse of BPT we have

$$EF || BC$$

From (1) we get $$\frac{OX}{OA} = \frac{FB}{AF}$$

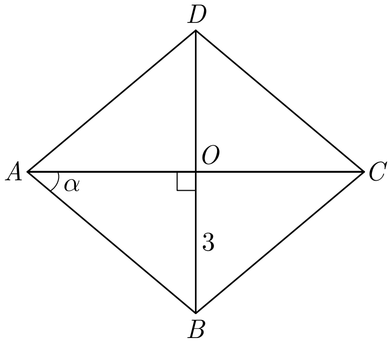
$$\frac{OX+OA}{OA} = \frac{FB+AF}{AF}$$

$$\frac{AX}{OA} = \frac{AB}{AF}$$

$$\frac{AO}{AX} = \frac{AF}{AB}$$

Thus $AO : AX = AF : AB$ Hence Proved

107. $ABCD$ is a rhombus whose diagonal AC makes an angle α with AB. If $\cos\alpha = \frac{2}{3}$ and $OB = 3$ cm, find the length of its diagonals AC and BD.



Ans : [Board Term-1 2013]

We have $\cos\alpha = \frac{2}{3}$ and $OB = 3$ cm

In ΔAOB, $$\cos\alpha = \frac{2}{3} = \frac{AO}{AB}$$

f171

Let $OA = 2x$ then $AB = 3x$

Now in right angled triangle ΔAOB we have

$$AB^2 = AO^2 + OB^2$$

$$(3x)^2 = (2x)^2 + (3)^2$$

$$9x^2 = 4x^2 + 9$$

$$5x^2 = 9$$

$$x = \sqrt{\frac{9}{5}} = \frac{3}{\sqrt{5}}$$

Hence, $$OA = 2x = 2\left(\frac{3}{\sqrt{5}}\right) = \frac{6}{\sqrt{5}} \text{ cm}$$

and $$AB = 3x = 3\left(\frac{3}{\sqrt{5}}\right) = \frac{9}{\sqrt{5}} \text{ cm}$$

Diagonal $$BD = 2 \times OB = 2 \times 3 = 6 \text{ cm}$$

and $$AC = 2AO$$

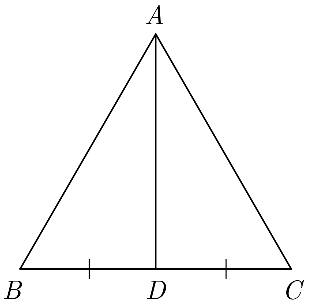
$$= 2 \times \frac{6}{\sqrt{5}} = \frac{12}{\sqrt{5}} \text{ cm}$$

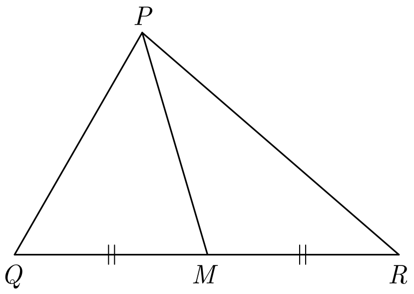
108. In ΔABC, AD is the median to BC and in ΔPQR, PM is the median to QR. If $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$. Prove that $\Delta ABC \sim \Delta PQR$.

Ans : [Board Term-1 2012, 2013]

As per given condition we have drawn the figure below.

f173





In ΔABC AD is the median, therefore

$$BC = 2BD$$

and in ΔPQR, PM is the median,

$$QR = 2QM$$

Given, $$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{BC}{QR}$$

or, $$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{2BD}{2QM}$$

In triangles ABD and PQM,

$$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{BD}{QM}$$

By SSS similarity we have

$$\Delta ABD \sim \Delta PQM$$

By CPST we have

$$\angle B = \angle Q,$$

In ΔABC and ΔPQR,

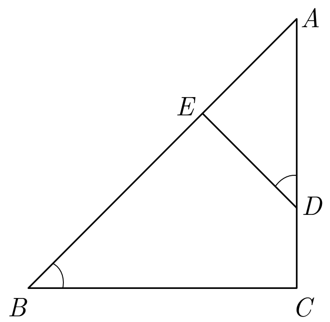
$$\frac{AB}{PQ} = \frac{BC}{QR}$$

By SAS similarity we have

$$\angle B = \angle Q,$$

Thus $\Delta ABC \sim \Delta PQR$. Hence Proved.

109. In ΔABC, if $\angle ADE = \angle B$, then prove that $\Delta ADE \sim \Delta ABC$.
Also, if $AD = 7.6$ cm, $AE = 7.2$ cm, $BE = 4.2$ cm and $BC = 8.4$ cm, then find DE.



Ans : [Board Term-1 2015]

In ΔADE and ΔABC, $\angle A$ is common.

and we have $\angle ADE = \angle ABC$

f167

Due to AA similarity,

$$\Delta ADE \sim \Delta ABC$$

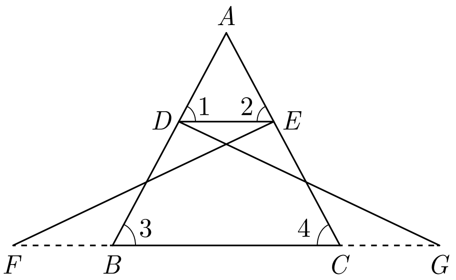
$$\frac{AD}{AB} = \frac{DE}{BC}$$

$$\frac{AD}{AE + BE} = \frac{DE}{BC}$$

$$\frac{7.6}{4.2 + 4.2} = \frac{DE}{8.4}$$

$$DE = \frac{7.6 \times 8.4}{11.4} = 5.6 \text{ cm}$$

110. In the following figure, $\Delta FEC \cong \Delta GBD$ and $\angle 1 = \angle 2$. Prove that $\Delta ADE \cong \Delta ABC$.



Ans : [Board Term-1 2012]

Since $\Delta FEC \cong \Delta GBD$

$$EC = BD \quad ...(1)$$

Since $\angle 1 = \angle 2$, using isosceles triangle property

$$AE = AD \quad ...(2)$$

From equation (1) and (2), we have

$$\frac{AE}{EC} = \frac{AD}{BD}$$

$$DE \,||\, BC, \quad \text{(Converse of BPT)}$$

Due to corresponding angles we have

$$\angle 1 = \angle 3 \text{ and } \angle 2 = \angle 4$$

f174

Thus in ΔADE and ΔABC,

$$\angle A = \angle A$$

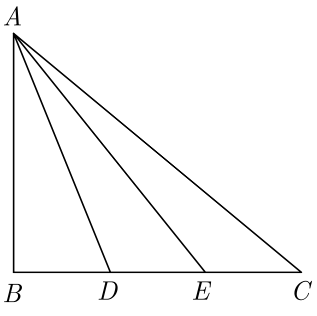
$$\angle 1 = \angle 3$$

$$\angle 2 = \angle 4$$

Sy by AAA criterion of similarity,

$$\Delta ADE \sim \Delta ABC \quad \text{Hence proved}$$

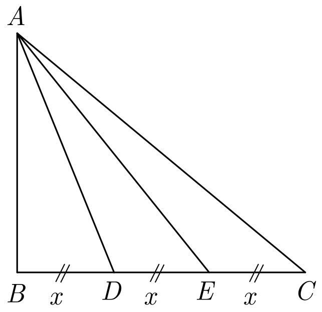
111. In the given figure, D and E trisect BC. Prove that $8AE^2 = 3AC^2 + 5AD^2$.



Ans : [Board Term-1 2013]

As per given condition we have drawn the figure below.

f175



Since D and E trisect BC, let $BD = DE = EC$ be x.

Then $BE = 2x$ and $BC = 3x$

In ΔABE, $AE^2 = AB^2 + BE^2 = AB^2 + 4x^2$ ...(1)

In ΔABC, $AC^2 = AB^2 + BC^2 = AB^2 + 9x^2$ ...(2)

In ΔADB, $AD^2 = AB^2 + BD^2 = AB^2 + x^2$ ...(3)

Multiplying (2) by 3 and (3) by 5 and adding we have

$$3AC^2 + 5AD^2 = 3(AB^2 + 9x^2) + (AB^2 + x^2)$$

$$= 3AB^2 + 27x^2 + 5AB^2 + 5x^2$$

$$= 8AB^2 + 32x^2$$

$$= 8(AB^2 + 4x^2) = 8AE^2$$

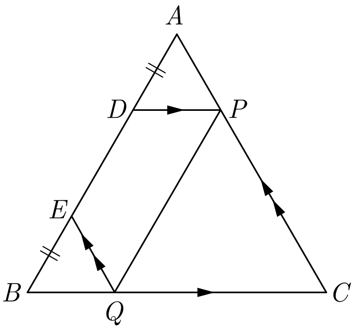
Thus $3AC^2 + 5AD^2 = 8AE^2$ Hence Proved

112. Let ABC be a triangle D and E be two points on side AB such that $AD = BE$. If $DP \,||\, BC$ and $EQ \,||\, AC$, then prove that $PQ \,||\, AB$.

Ans : [Board Term-1 2012]

As per given condition we have drawn the figure below.

f176



In ΔABC, $DP \,||\, BC$

By BPT we have $\dfrac{AD}{DB} = \dfrac{AP}{PC}$, ...(1)

Similarly, in ΔABC, $EQ \,||\, AC$

$$\frac{BQ}{QC} = \frac{BE}{EA} \qquad ...(2)$$

From figure,

$$EA = AD + DE$$

$$= BE + ED \qquad (BE = AD)$$

$$= BD$$

Therefore equation (2) becomes,

$$\frac{BQ}{QC} = \frac{AD}{BD} \qquad ...(3)$$

From (1) and (3), we have

$$\frac{AP}{PC} = \frac{BQ}{QC}$$

By converse of BPT,

$$PQ \,||\, AB \qquad \text{Hence Proved}$$

113. Prove that in a right triangle, the square of the hypotenuse is equal to sum of squares of other two sides. [Board 2020 Delhi Basic, 2019 Delhi, 2018]

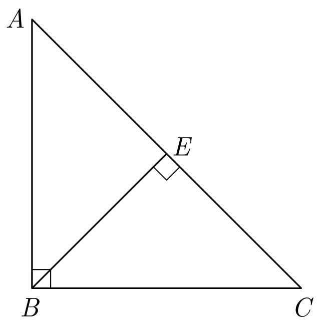
or

Prove that in a right triangle, the square of the hypotenuse is equal to sum of squares of other two sides. Using the above result, prove that, in rhombus $ABCD$, $4AB^2 = AC^2 + BD^2$.

Ans : [Board Term -2 SQP 2017, 2015]

(1) As per given condition we have drawn the figure below. Here $AB \perp BC$.

We have drawn $BE \perp AC$



In $\Delta\, AEB$ and $\Delta\, ABC$ $\angle A$ common and

$$\angle E = \angle B \qquad \text{(each } 90^{\circ})$$

By AA similarity we have

$$\Delta\, AEB \sim \Delta\, ABC$$

$$\frac{AE}{AB} = \frac{AB}{AC}$$

$$AB^2 = AE \times AC$$

Now, in $\Delta\, CEB$ and $\Delta\, CBA$, $\angle C$ is common and

$$\angle E = \angle B \qquad \text{(each } 90^{\circ})$$

By AA similarity we have

$$\Delta\, AEB \sim \Delta\, CBA$$

$$\frac{CE}{BC} = \frac{BC}{AC}$$

$$BC^2 = CE \times AC \qquad ...(2)$$

Adding equation (1) and (2) we have

$$AB^2 + BC^2 = AE \times AC + CE \times AC$$

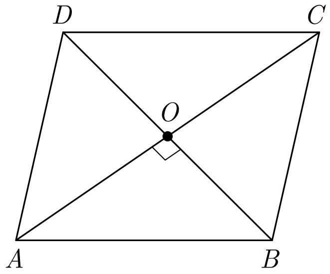
$$= AC(AE + CE)$$

$$= AC \times AC$$

Thus

$$AB^2 + BC^2 = AC^2 \qquad \text{Hence proved}$$

(2) As per given condition we have drawn the figure below. Here $ABCD$ is a rhombus.



We have drawn diagonal AC and BD.

$$AO = OC = \frac{1}{2}AC$$

and

$$BO = OD = \frac{1}{2}BD$$

$$AC \perp BD$$

Since diagonal of rhombus bisect each other at right angle,

$$\angle AOB = 90^{\circ}$$

$$AB^2 = OA^2 + OB^2$$

$$= \left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2$$

$$= \frac{AC^2}{4} + \frac{BD^2}{4}$$

or

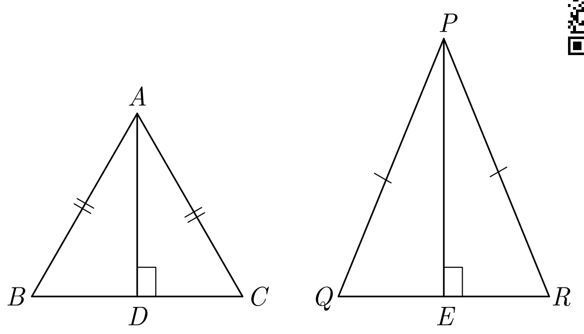
$$4AB^2 = AC^2 + BD^2 \qquad \text{Hence proved}$$

114. Vertical angles of two isosceles triangles are equal. If their areas are in the ratio 16 : 25, then find the ratio of their altitudes drawn from vertex to the opposite side.

Ans : [Board Term-1 2015]

As per given condition we have drawn the figure below.

f179



Here $\angle A = \angle P$ $\angle B = \angle C$ and $\angle Q = \angle R$

Let $\angle A = \angle P$ be x.

In ΔABC, $\angle A + \angle B + \angle C = 180^\circ$

$$x + \angle B + \angle B = 180^\circ \qquad (\angle B = \angle C)$$

$$2\angle B = 180^\circ - x$$

$$\angle B = \frac{180^\circ - x}{2} \qquad ...(1)$$

Now, in ΔPQR,

$$\angle P + \angle Q + \angle R = 180^\circ \qquad (\angle Q = \angle R)$$

$$x^2 + \angle Q + \angle Q = 180^\circ$$

$$2\angle Q = 180^\circ - x$$

$$\angle Q = \frac{180^\circ - x}{2}$$

In ΔABC and ΔPQR,

$$\angle A = \angle P \qquad \text{[Given]}$$

$$\angle B = \angle Q \qquad \text{[From eq. (1) and (2)]}$$

Due to AA similarity,

$$\Delta ABC \sim \Delta PQR$$

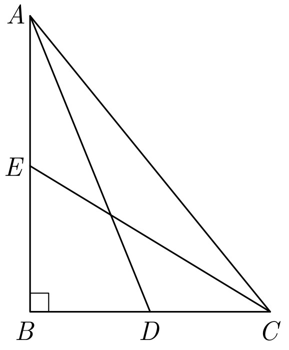
Now $$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AD^2}{PE^2}$$

$$\frac{16}{25} = \frac{AD^2}{PE^2}$$

$$\frac{4}{5} = \frac{AD}{PE}$$

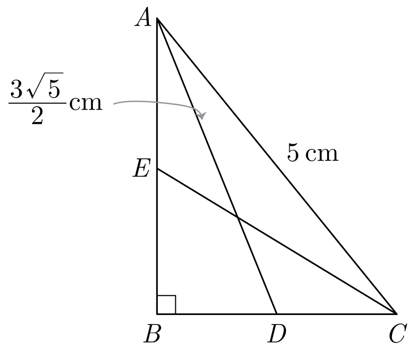
Thus $$\frac{AD}{PE} = \frac{4}{5}$$

115. In the figure, ABC is a right triangle, right angled at B. AD and CE are two medians drawn from A and C respectively. If $AC = 5$ cm and $AD = \frac{3\sqrt{5}}{2}$ cm, find the length of CE.



Ans : [Board Term-1 2013]

We have redrawn the given figure as below.



Here in ΔABC, $\angle B = 90^\circ$, AD and CE are two medians.

Also we have $AC = 5$ cm and $AD = \frac{3\sqrt{5}}{2}$.

By Pythagoras theorem we get

$$AC^2 = AB^2 + BC^2 = (5)^2 = 25 \qquad ...(1)$$

In ΔABD, $$AD^2 = AB^2 + BD^2$$

$$\left(\frac{3\sqrt{5}}{2}\right)^2 = AB^2 + \frac{BC^2}{4}$$

$$\frac{45}{4} = AB^2 + \frac{BC^2}{4} \qquad ...(2)$$

In ΔEBC, $$CE^2 = BC^2 + \frac{AB^2}{4} \qquad ..(3)$$

Subtracting equation (2) from equation (1),

$$\frac{3BC^2}{4} = 25 - \frac{45}{4} = \frac{55}{4}$$

$$BC^2 = \frac{55}{3} \quad ...(4)$$

From equation (2) we have

$$AB^2 + \frac{55}{12} = \frac{45}{4}$$

$$AB^2 = \frac{45}{4} - \frac{55}{12} = \frac{20}{3}$$

From equation (3) we get

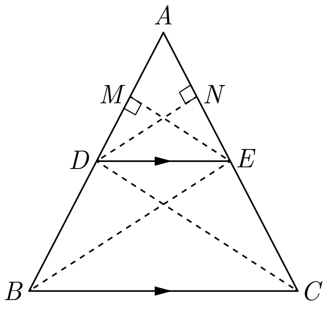
$$CE^2 = \frac{55}{3} + \frac{20}{3 \times 4} = \frac{240}{12} = 20$$

Thus $CE = \sqrt{20} = 2\sqrt{5}$ cm.

116. If a line drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio. Prove it.

Ans : [Board 2019 OD, SQP 2020 STD, 2012]

A triangle ABC is given in which $DE || BC$. We have drawn $DN \perp AE$ and $EM \perp AD$ as shown below. We have joined BE and CD.



In ΔADE,

$$\text{area } (\Delta ADE) = \frac{1}{2} \times AE \times DN \quad ...(1)$$

In ΔDEC,

$$\text{area } (\Delta DCE) = \frac{1}{2} \times CE \times DN \quad ...(2)$$

Dividing equation (1) by (2) we have,

$$\frac{\text{area}(\Delta ADE)}{\text{area}(\Delta DEC)} = \frac{\frac{1}{2} \times AE \times DN}{\frac{1}{2} \times CE \times DN}$$

or,
$$\frac{\text{area}(\Delta ADE)}{\text{area}(\Delta DEC)} = \frac{AE}{CE} \quad ...(3)$$

Now in ΔADE,

$$\text{area}(\Delta ADE) = \frac{1}{2} \times AD \times EM \quad ...(4)$$

and in ΔDEB,

$$\text{area}(\Delta DEB) = \frac{1}{2} \times EM \times BD \quad ...(5)$$

Dividing eqn. (4) by eqn. (5),

$$\frac{\text{area}(\Delta ADE)}{\text{area}(\Delta DEB)} = \frac{\frac{1}{2} \times AD \times EM}{\frac{1}{2} \times BD \times EM}$$

or,
$$\frac{\text{area}(\Delta ADE)}{\text{area}(\Delta DEB)} = \frac{AD}{BD} \quad ...(6)$$

Since ΔDEB and ΔDEC lie on the same base DE and between two parallel lines DE and BC.

$$\text{area}(\Delta DEB) = \text{area}(\Delta DEC)$$

From equation (3) we have

$$\frac{\text{area}(\Delta ADE)}{\text{area}(\Delta DEB)} = \frac{AE}{CE} \quad ...(7)$$

From equations (6) and (7) we get

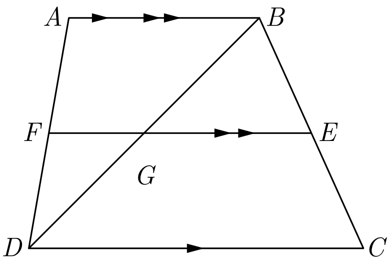
$$\frac{AE}{CE} = \frac{AD}{BD}.$$

Hence proved.

117. In a trapezium $ABCD, AB || DC$ and $DC = 2AB$. $EF = AB$, where E and F lies on BC and AD respectively such that $\frac{BE}{EC} = \frac{4}{3}$ diagonal DB intersects EF at G. Prove that, $7EF = 11AB$.

Ans : [Board Term-1 2012]

As per given condition we have drawn the figure below.



In trapezium $ABCD$,

$$AB || DC \text{ and } DC = 2AB.$$

Also,
$$\frac{BE}{EC} = \frac{4}{3}$$

Thus
$$EF || AB || CD$$

$$\frac{AF}{FD} = \frac{BE}{EC} = \frac{4}{3}$$

In ΔBGE and ΔBDC, $\angle B$ is common and due to corresponding angles,

$$\angle BEG = \angle BCD$$

Due to AA similarity we get

$$\Delta BGE \sim \Delta BDC$$

, $$\frac{EG}{CD} = \frac{BE}{BC} \qquad ...(1)$$

As, $$\frac{BE}{EC} = \frac{4}{3}$$

$$\frac{BE}{BE+EC} = \frac{4}{4+3} = \frac{4}{7}$$

$$\frac{BE}{BC} = \frac{4}{7} \qquad ...(2)$$

From (1) and (2) we have

$$\frac{EG}{CD} = \frac{4}{7}$$

, $$EG = \frac{4}{7}CD \qquad ...(3)$$

Similarly, $$\Delta DGF \sim \Delta DBA$$

$$\frac{DF}{DA} = \frac{FG}{AB}$$

$$\frac{FG}{AB} = \frac{3}{7}$$

$$FG = \frac{3}{7}AB \qquad ...(4)$$

$$\left[\frac{AF}{AD} = \frac{4}{7} = \frac{BE}{BC} \Rightarrow \frac{EC}{BC} = \frac{3}{7} = \frac{DE}{DA}\right]$$

Adding equation (3) and (4) we have

$$EG+FG = \frac{4}{7}DC + \frac{3}{7}AB$$

$$EF = \frac{4}{7}\times(2AB) + \frac{3}{7}AB$$

$$= \frac{8}{7}AB + \frac{3}{7}AB = \frac{11}{7}AB$$

$$7EF = 11AB$$ Hence proved.

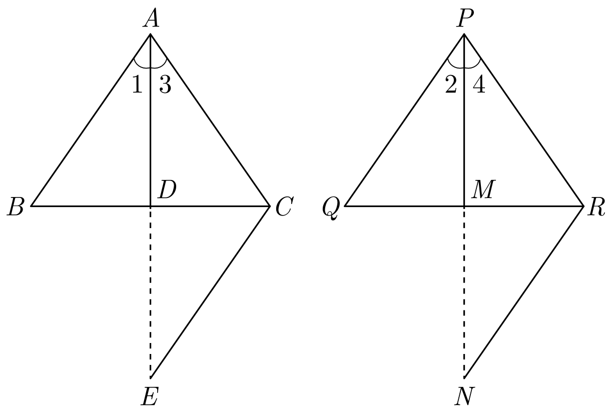
118. Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that $\Delta ABC \sim \Delta PQR$.

Ans : [Board Term-1 2012]

It is given that in ΔABC and ΔPQR, AD and PM are their medians,

such that $$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{AC}{PR}$$

We have produce AD to E such that $AD = DE$ and produce PM to N such that $PM = MN$. We join CE and RN. As per given condition we have drawn the figure below.



In ΔABD and ΔEDC,

$$AD = DE \qquad \text{(By construction)}$$

$$\angle ADB = \angle EDC \qquad \text{(VOA)}$$

$$BD = DC \qquad (AD \text{ is a median})$$

By SAS congruency

$$\Delta ABD \cong \Delta EDC$$

$$AB = CE \qquad \text{(By CPCT)}$$

Similarly, $$PQ = RN \text{ and } \angle A = \angle 2$$

$$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{AC}{PR} \qquad \text{(Given)}$$

, $$\frac{CE}{RN} = \frac{2AD}{2PM} = \frac{AC}{PR}$$

f183

$$\frac{CE}{RN} = \frac{AE}{PN} = \frac{AC}{PR}$$

By SSS similarity, we have

$$\Delta AEC \sim \Delta PNR$$

$$\angle 3 = \angle 4$$

$$\angle 1 = \angle 2$$

$$\angle 1 + \angle 3 = \angle 2 + \angle 4$$

By SAS similarity, we have

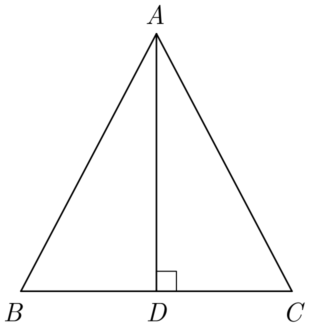
$$\Delta ABC \sim \Delta PQR$$ Hence Proved

119. In $\Delta ABC, AD \perp BC$ and point D lies on BC such that $2DB = 3CD$. Prove that $5AB^2 = 5AC^2 + BC^2$.

Ans : [Board Term-1 2015]

It is given in a triangle $\Delta ABC, AD \perp BC$ and point D lies on BC such that $2DB = 3CD$.

As per given condition we have drawn the figure below.



f184

Since $2DB = 3CD$

$$\frac{DB}{CD} = \frac{3}{2}$$

Let DB be $3x$, then CD will be $2x$ so $BC = 5x$.

Since $\angle D = 90°$ in ΔADB, we have

$$AB^2 = AD^2 + DB^2 = AD^2 + (3x)^2$$

$$= AD^2 + 9x^2$$

$$5AB^2 = 5AD^2 + 45x^2$$

$$5AD^2 = 5AB^2 - 45x^2 \quad ...(1)$$

and $$AC^2 = AD^2 + CD^2 = AD^2 + (2x)^2$$

$$= AD^2 + 4x^2$$

$$5AC^2 = 5AD^2 + 20x^2$$

$$5AD^2 = 5AC^2 - 20x^2 \quad ...(2)$$

Comparing equation (1) and (2) we have

$$5AB^2 - 45x^2 = 5AC^2 - 20x^2$$

$$5AB^2 = 5AC^2 - 20x^2 + 45x^2$$

$$= 5AC^2 + 25x^2$$

$$= 5AC^2 + (5x)^2$$

$$= 5AC^2 + BC^2 \quad [BC = 5x]$$

Therefore $5AB^2 = 5AC^2 + BC^2$ Hence proved

120. In a right triangle ABC, right angled at C. P and Q are points of the sides CA and CB respectively, which divide these sides in the ratio $2:1$.

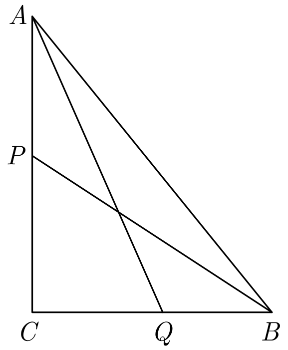
Prove that : $$9AQ^2 = 9AC^2 + 4BC^2$$

$$9BP^2 = 9BC^2 + 4AC^2$$

$$9(AQ^2 + BP^2) = 13AB^2$$

Ans :

As per given condition we have drawn the figure below.



f185

Since P divides AC in the ratio $2:1$

$$CP = \frac{2}{3}AC$$

and Q divides CB in the ratio $2:1$

$$QC = \frac{2}{3}BC$$

$$AQ^2 = QC^2 + AC^2$$

$$= \frac{4}{9}BC^2 + AC^2$$

or, $$9AQ^2 = 4BC^2 + 9AC^2 \quad ...(1)$$

Similarly, we get

$$9BP^2 = 9BC^2 + 4AC^2 \quad ...(2)$$

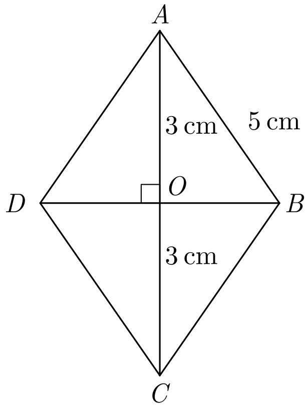
Adding equation (1) and (2), we get

$$9(AQ^2 + BP^2) = 13AB^2$$

121. Find the length of the second diagonal of a rhombus, whose side is 5 cm and one of the diagonals is 6 cm.

Ans :

As per given condition we have drawn the figure below.



We have $AB = BC = CD = AD = 5$ cm and $AC = 6$ cm

Since $AO = OC$, $AO = 3$ cm

Here ΔAOB is right angled triangle as diagonals of rhombus intersect at right angle.

By Pythagoras theorem,

$$OB = 4 \text{ cm.}$$

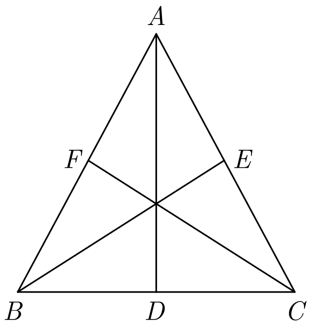
Since $DO = OB, BD = 8$ cm, length of the other diagonal $= 2(BO)$ where $BO = 4$ cm

Hence $BD = 2 \times BO = 2 \times 4 = 8$ cm

122. Prove that three times the sum of the squares of the sides of a triangle is equal to four times the sum of the squares of the medians of the triangle.

Ans :

As per given condition we have drawn the figure below.



In triangle sum of squares of any two sides is equal to twice the square of half of the third side, together with twice the square of median bisecting it.

If AD is the median,

$$AB^2 + AC^2 = 2\left\{AD^2 + \frac{BC^2}{4}\right\}$$

$$2(AB^2 + AC^2) = 4AD^2 + BC^2 \quad ...(1)$$

Similarly by taking BE and CF as medians,

$$2(AB^2 + BC^2) = 4BE^2 + AC^2 \quad ...(2)$$

and $$2(AC^2 + BC^2) = 4CF^2 + AB^2 \quad ...(3)$$

Adding, (1), (2) and (iii), we get

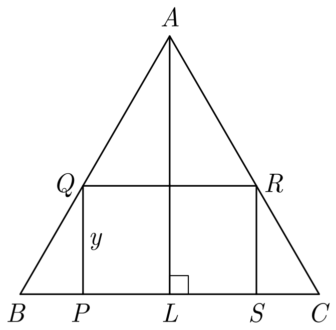
$$3(AB^2 + BC^2 + AC^2) = 4(AD^2 + BE^2 + CF^2)$$

Hence proved

123. ABC is an isosceles triangle in which $AB = AC = 10$ cm $BC = 12$ cm $PQRS$ is a rectangle inside the isosceles triangle. Given $PQ = SR = y$, $PS = PR = 2x$. Prove that $x = 6 - \frac{3y}{4}$.

Ans :

As per given condition we have drawn the figure below.



Here we have drawn $AL \perp BC$.

Since it is isosceles triangle, AL is median of BC,

$$BL = LC = 6 \text{ cm.}$$

In right ΔALB, by Pythagoras theorem,

$$AL^2 = AB^2 - BL^2$$

$$= 10^2 - 6^2 = 64 = 8^2$$

Thus $AL = 8$ cm.

In ΔBPQ and ΔBLA, angle $\angle B$ is common and

$$\angle BPQ = \angle BLA = 90^\circ$$

Thus by AA similarity we get

$$\Delta BPQ \sim \angle BLA$$

$$\frac{PB}{PQ} = \frac{BL}{AL}$$

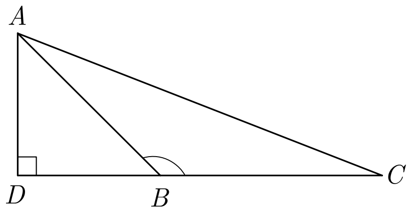
$$\frac{6-x}{y} = \frac{6}{8}$$

$$x = 6 - \frac{3y}{4}$$ Hence proved.

124. If ΔABC is an obtuse angled triangle, obtuse angled at B and if $AD \perp CB$. Prove that :
$AC^2 = AB^2 + BC^2 + 2BC \times BD$

Ans : [Board 2020 Delhi Basic]

As per given condition we have drawn the figure below.



f189

In ΔADB, by Pythagoras theorem

$$AB^2 = AD^2 + BD^2 \quad ...(1)$$

In ΔADC, By Pythagoras theorem,

$$AC^2 = AD^2 + CD^2$$
$$= AD^2 + (BC + BD)^2$$
$$= AD^2 + BC^2 + 2BC \times BD + BD^2$$
$$= (AD^2 + BD^2) + 2BC \times BD$$

Substituting $(AD^2 + BD^2) = AB^2$ we have

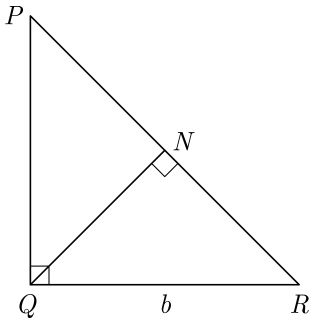
$$AC^2 = AB^2 + BC^2 + 2BC \times BD$$

125. If A be the area of a right triangle and b be one of the sides containing the right angle, prove that the length of the altitude on the hypotenuse is $\frac{2Ab}{\sqrt{b^4 + 4A^2}}$.

Ans :

As per given condition we have drawn the figure below.

f190



Let $QR = b$, then we have

$$A = ar(\Delta PQR)$$
$$= \frac{1}{2} \times b \times PQ$$
$$PQ = \frac{2 \cdot A}{b} \quad ...(1)$$

Due to AA similarity we have

$$\Delta PNQ \sim \Delta PQR$$
$$\frac{PQ}{PR} = \frac{NQ}{QR} \quad ...(2)$$

From ΔPQR

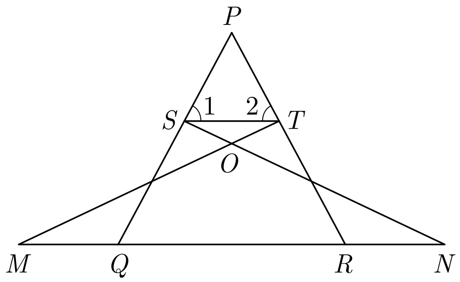
$$PQ^2 + QR^2 = PR^2$$
$$\frac{4A^2}{b^2} + b^2 = PR^2$$
$$PR = \sqrt{\frac{4A^2 + b^4}{b^2}}$$

Equation (2) becomes

$$\frac{2A}{b \times PR} = \frac{NQ}{b}$$
$$NQ = \frac{2A}{PR}$$

Altitude, $$NQ = \frac{2Ab}{\sqrt{4A^2 + b^4}}$$ Hence Proved.

126. In given figure $\angle 1 = \angle 2$ and $\Delta NSQ \sim \Delta MTR$, then prove that $\Delta PTS \sim \Delta PRO$.



Ans : [Board Term-1 SQP 2017]

We have $\Delta NSQ \cong \Delta MTR$

By CPCT we have

f191

$$\angle SQN = \angle TRM$$

From angle sum property we get

$$\angle P + \angle 1 + \angle 2 = \angle P + \angle PQR + \angle PRQ$$

$$\angle 1 + \angle 2 = \angle PQR + \angle PRQ$$

Since $\angle 1 = \angle 2$ and $\angle PQR = \angle PRQ$ we get

$$2\angle 1 = 2\angle PQR$$

$$\angle 1 = \angle PQR$$

Also $\angle 2 = \angle QPR$ (common)

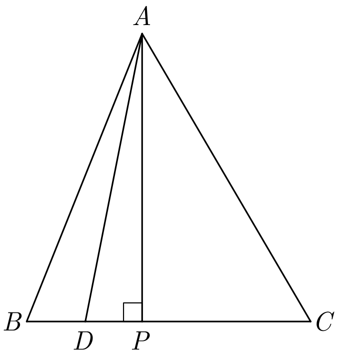
Thus by AAA similarity,

$$\Delta PTS \sim \Delta PRQ$$

127. In an equilateral triangle ABC, D is a point on the side BC such the $BD = \frac{1}{3}BC$. Prove that $9AD^2 = 7AB^2$.

Ans : [Board 2018, SQP 2017]

As per given condition we have shown the figure below. Here we have drawn $AP \perp BC$.



Here $AB = BC = CA$ and $BD = \dfrac{1}{3}BC$.

f192

In ΔADP,

$$AD^2 = AP^2 + DP^2$$

$$= AP^2 + (BP - BD)^2$$

$$= AP^2 + BP^2 + BD^2 + 2BP \cdot BD$$

From ΔAPB using $AP^2 + BP^2 = AB^2$ we have

$$AD^2 = AB^2 + \left(\frac{1}{3}BC\right)^2 - 2\left(\frac{BC}{2}\right)\left(\frac{BC}{3}\right)$$

$$= AB^2 + \frac{AB^2}{9} - \frac{AB^2}{3} = \frac{7}{9}AB^2$$

$$9AD^2 = 7AB^2$$

Hence Proved

CHAPTER 7

COORDINATE GEOMETRY

ONE MARK QUESTIONS

Multiple Choice Questions

1. The point P on x-axis equidistant from the points $A(-1,0)$ and $B(5,0)$ is

(a) (2, 0) (b) (0, 2)

(c) (3, 0) (d) $(-3,5)$

Ans : [Board 2020 OD Standard]

Let the position of the point P on x-axis be $(x,0)$, then

$$PA^2 = PB^2$$

$$(x+1)^2+(0)^2 = (5-x)^2+(0)^2$$

$$x^2+2x+1 = 25+x^2-10x$$

$$2x+10x = 25-1$$

$$12x = 24 \Rightarrow x = 2$$

g215

Hence, the point $P(x,0)$ is (2, 0).

Thus (a) is correct option.

Alternative :

You may easily observe that both point $A(-1,0)$ and $B(5,0)$ lies on x-axis because y ordinate is zero. Thus point P on x-axis equidistant from both point must be mid point of $A(-1,0)$ and $B(5,0)$.

$$x = \frac{-1+5}{2} = 2$$

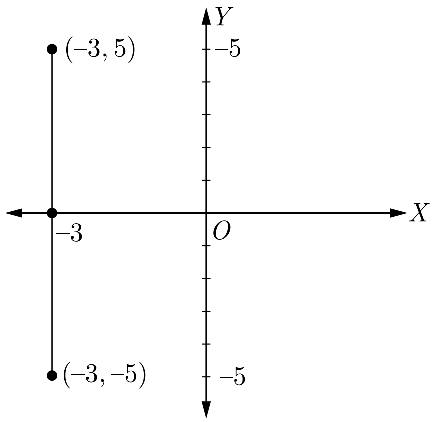
2. The co-ordinates of the point which is reflection of point $(-3,5)$ in x-axis are

(a) (3, 5) (b) (3, -5)

(c) $(-3,-5)$ (d) $(-3,5)$

Ans : [Board 2020 OD Standard]

The reflection of point $(-3,5)$ in x- axis is $(-3,-5)$.



g216

Thus (c) is correct option.

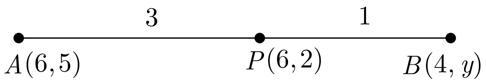
3. If the point P (6, 2) divides the line segment joining $A(6,5)$ and $B(4,y)$ in the ratio $3:1$ then the value of y is

(a) 4 (b) 3

(c) 2 (d) 1

Ans : [Board 2020 OD Standard]

As per given information in question we have drawn the figure below,



Here, $x_1 = 6,\ y_1 = 5$

and $x_2 = 4\ \ y_2 = y$

g217

Now $$y = \frac{my_2+ny_1}{m+n}$$

$$2 = \frac{3\times y+1\times 5}{3+1}$$

$$2 = \frac{3y+5}{4}$$

$$3y+5 = 8$$

$$3y = 8-5 = 3 \Rightarrow y = 1$$

Thus (d) is correct option.

4. The distance between the points $(a\cos\theta + b\sin\theta,\ 0)$, and $(0,\ a\sin\theta - b\cos\theta)$ is

(a) $a^2 + b^2$ (b) $a^2 - b^2$

(c) $\sqrt{a^2+b^2}$ (d) $\sqrt{a^2-b^2}$

Ans : [Board 2020 Delhi Standard]

We have $x_1 = a\cos\theta + b\sin\theta$ and $y_1 = 0$

and $x_2 = 0$ and $y_2 = a\sin\theta - b\cos\theta$

g218

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$
$$= (0 - a\cos\theta - b\sin\theta)^2 + (a\sin\theta - b\cos\theta - 0)^2$$
$$= (-1)^2(a\cos\theta + b\sin\theta)^2 + (a\sin\theta - b\cos\theta)^2$$
$$= a^2\cos^2\theta + b^2\sin^2\theta + 2ab\cos\theta\sin\theta + $$
$$+ a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta$$
$$= a^2(\sin^2\theta + \cos^2\theta) + b^2(\sin^2\theta + \cos^2\theta)$$
$$= a^2 \times 1 + b^2 \times 1 = a^2 + b^2$$

Thus $d^2 = a^2 + b^2$

$$d = \sqrt{a^2+b^2}$$

Therefore (c) is correct option.

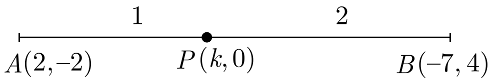
5. If the point $P(k,\ 0)$ divides the line segment joining the points $A(2,-2)$ and $B(-7,4)$ in the ratio 1 : 2, then the value of k is

(a) 1 (b) 2

(c) -2 (d) -1

Ans : [Board 2020 Delhi Standard]

As per question statement figure is shown below.



$$k = \frac{1(-7) + 2(2)}{1+2} \qquad \left(x = \frac{mx_2 + nx_1}{m+n}\right)$$

$$= \frac{-7+4}{3} = \frac{-3}{3} = -1$$

g219

Thus $k = -1$

Thus (d) is correct option.

6. The coordinates of a point A on y-axis, at a distance of 4 units from x-axis and below it are

(a) (4, 0) (b) (0, 4)

(c) $(-4, 0)$ (d) $(0,\ -4)$

g221

Ans :
[Board 2020 Delhi Basic]

Because the point is 4 units down the x-axis i.e., co-ordinate is -4 and on y-axis abscissa is 0. So, the coordinates of point A is $(0, -4)$.

Thus (d) is correct option.

7. The distance of the point $(-12,\ 5)$ from the origin is

(a) 12 (b) 5

(c) 13 (d) 169

g223

Ans :

The distance between the origin and the point (x, y) is $\sqrt{x^2+y^2}$.

Therefore, the distance between the origin and point $(-12, 5)$

$$d = \sqrt{(-12-0)^2 + (5-0)^2}$$
$$= \sqrt{144+25} = \sqrt{169}$$
$$= 13 \text{ units}$$

Thus (c) is correct option.

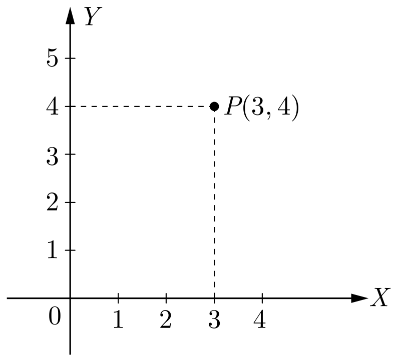
8. Distance of point $P(3,4)$ from x-axis is

(a) 3 units (b) 4 units

(c) 5 units (d) 1 units

Ans : [Board 2020 Delhi Basic]

Point $P(3,4)$ is 4 units from the x-axis and 3 units from the y-axis.



g224

Thus (b) is correct option.

9. The distance of the point $P(-3,-4)$ from the x-axis (in units) is

(a) 3 (b) -3

(c) 4 (d) 5

g225

Ans :
[Board 2020 SQP Standard]

Point $P(-3,-4)$ is 4 units from the x-axis and 3 units from the y-axis.

Thus (c) is correct option.

10. If $A(\frac{m}{3}, 5)$ is the mid-point of the line segment joining the points $Q(-6,7)$ and $R(-2,3)$, then the value of m is

(a) -12 (b) -4

(c) 12 (d) -6

g226

Ans : [Board 2020 SQP Standard]

Given points are $Q(-6,7)$ and $R(-2,3)$

$$\text{Mid point} \quad A(\tfrac{m}{3}, 5) = \left(\frac{-6-2}{2}, \frac{7+3}{2}\right)$$

$$= (-4, 5)$$

$$\text{Equating,} \qquad \frac{m}{3} = -4 \Rightarrow m = -12$$

Thus (a) is correct option.

11. The mid-point of the line-segment AB is $P(0,4)$, if the coordinates of B are $(-2, 3)$ then the co-ordinates of A are

(a) (2, 5) (b) $(-2,-5)$

(c) (2, 9) (d) $(-2,11)$

Ans : [Board 2020 OD Basic]

Let point A be (x, y).

Now using mid-point formula,

$$(0,4) = \left(\frac{x-2}{2}, \frac{y+3}{2}\right)$$

g227

$$\text{Thus} \qquad 0 = \frac{x-2}{2} \Rightarrow x = 2$$

$$\text{and} \qquad 4 = \frac{y+3}{2} \Rightarrow y = 5$$

Hence point A is (2, 5).

Thus (a) is correct option.

12. x-axis divides the line segment joining $A(2,-3)$ and $B(5,6)$ in the ratio

(a) 2 : 3 (b) 3 : 5

(c) 1 : 2 (d) 2 : 1

Ans : [Board 2020 OD Basic]

Let point $P(x, 0)$ on x-axis divide the segment joining points $A(2, -3)$ and $B(5, 6)$ in ratio $k : 1$, then

$$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

g228

$$0 = \frac{6k-3}{k+1}$$

$$6k = 3 \Rightarrow k = \frac{1}{2}$$

Therefore ratio is 1 : 2.

Thus (c) is correct option.

13. The point which divides the line segment joining the points $(8,-9)$ and $(2, 3)$ in the ratio 1 : 2 internally lies in the

(a) I quadrant (b) II quadrant

(c) III quadrant (d) IV quadrant

Ans : [Board 2020 SQP Standard]

We have $x_1 = 8$, $y_1 = -9$, $x_2 = 2$ and $y_2 = 3$.

$$\text{and} \qquad m_1 : m_2 = 1:2$$

Let the required point be $P(x, y)$

$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2} = \frac{1 \times 2 + 2 \times 8}{1+2} = 6$$

$$\text{and} \qquad y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} = \frac{1 \times 3 + 2(-9)}{1+2} = -5$$

Thus $(x, y) = (6, -5)$ and this point lies in IV quadrant.

Thus (d) is correct option.

g229

14. If the centre of a circle is (3, 5) and end points of a diameter are (4, 7) and $(2, y)$, then the value of y is

(a) 3 (b) -3

(c) 7 (d) 4

Ans : [Board 2020 Delhi Basic]

Since, centre is the mid-point of end points of the diameter.

$$(3, 5) = \left(\frac{4+2}{2}, \frac{7+y}{2}\right)$$

g230

Comparing both the sides, we get

$$5 = \frac{7+y}{2}$$

$$7 + y = 10 \Rightarrow y = 3$$

Thus (a) is correct option.

15. If the distance between the points $A(4, p)$ and $B(1,0)$ is 5 units then the value(s) of p is(are)

(a) 4 only (b) -4 only

(c) ± 4 (d) 0

Ans : [Board 2020 Delhi Basic]

Given, points are $A(4, p)$ and $B(1,0)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y^2 - y_1)^2}$$

$$5 = \sqrt{(1-4)^2 + (0-p)^2}$$
$$25 = 9 + p^2$$
$$p^2 = 25 - 9 = 16$$
$$p = \pm 4$$

g231

Thus (c) is correct option.

16. If the points $(a, 0)$, $(0, b)$ and $(1, 1)$ are collinear, then $\frac{1}{a} + \frac{1}{b}$ equals

(a) 1 (b) 2

(c) 0 (d) -1

g232

Ans :

Let the given points are $A(a, 0)$, $B(0, b)$ and $C(1, 1)$. Since, A, B, C are collinear.

Hence, $$\text{ar}\,(\Delta ABC) = 0$$

$$\frac{1}{2}[a(b-1) + 0(1-0) + 1(0-b)] = 0$$
$$ab - a - b = 0$$
$$a + b = ab$$
$$\frac{a+b}{ab} = 1$$
$$\frac{1}{a} + \frac{1}{b} = 1$$

Thus (a) is correct option.

17. If the points $A(4, 3)$ and $B(x, 5)$ are on the circle with centre $O(2, 3)$, then the value of x is

(a) 0 (b) 1

(c) 2 (d) 3

g233

Ans :

Since, A and B lie on the circle having centre O.

$$OA = OB$$
$$\sqrt{(4-2)^2 + (3-3)^2} = \sqrt{(x-2)^2 + (5-3)^2}$$
$$2 = \sqrt{(x-2)^2 + 4}$$
$$4 = (x-2)^2 + 4$$
$$(x-2)^2 = 0 \Rightarrow x = 2$$

Thus (c) is correct option.

18. The ratio in which the point $(2, y)$ divides the join of $(-4, 3)$ and $(6, 3)$, hence the value of y is

(a) $2:3, y = 3$ (b) $3:2, y = 4$

(c) $3:2, y = 3$ (d) $3:2, y = 2$

Ans :

Let the required ratio be $k:1$

g234

Then, $$2 = \frac{6k - 4(1)}{k+1}$$

or $$k = \frac{3}{2}$$

The required ratio is $\frac{3}{2}:1$ or $3:2$

Also, $$y = \frac{3(3) + 2(3)}{3+2} = 3$$

Thus (c) is correct option.

19. The point on the x-axis which is equidistant from the points $A(-2, 3)$ and $B(5, 4)$ is

(a) (0, 2) (b) (2, 0)

(c) (3, 0) (d) $(-2, 0)$

g235

Ans :

Let $P(x, 0)$ be a point on x-axis such that,

$$AP = BP$$
$$AP^2 = BP^2$$
$$(x+2)^2 + (0-3)^2 = (x-5)^2 + (0+4)^2$$
$$x^2 + 4x + 4 + 9 = x^2 - 10x + 25 + 16$$
$$14x = 28$$
$$x = 2$$

Hence required point is (2, 0).

Thus (b) is correct option.

20. C is the mid-point of PQ, if P is $(4, x)$, C is $(y, -1)$ and Q is $(-2, 4)$, then x and y respectively are

(a) -6 and 1 (b) -6 and 2

(c) 6 and -1 (d) 6 and -2

Ans :

Since, $C(y, -1)$ is the mid-point of $P(4, x)$ and $Q(-2, 4)$.

We have, $$\frac{4-2}{2} = y \Rightarrow y = 1$$

g236

and $$\frac{4+x}{2} = -1 \Rightarrow x = -6$$

Thus (a) is correct option.

21. If three points (0, 0), $(3, \sqrt{3})$ and $(3, \lambda)$ form an equilateral triangle, then λ equals

(a) 2 (b) -3

(c) -4 (d) None of these

Ans :

Let the given points are $A(0,0)$, $B(3,\sqrt{3})$ and $C(3,\lambda)$.

Since, ΔABC is an equilateral triangle, therefore

$$AB = AC$$

$$\sqrt{(3-0)^2+(\sqrt{3}-0)^2} = \sqrt{(3-0)^2+(\lambda-0)^2}$$

$$9+3 = 9+\lambda^2$$

$$\lambda^2 = 3 \Rightarrow \lambda = \pm\sqrt{3}$$

Thus (d) is correct option.

g237

22. If $x-2y+k=0$ is a median of the triangle whose vertices are at points $A(-1,3)$, $B(0,4)$ and $C(-5,2)$, then the value of k is

(a) 2 (b) 4

(c) 6 (d) 8

g239

Ans :

Coordinate of the centroid G of ΔABC

$$= \left(\frac{-1+0-5}{2}, \frac{3+4+2}{3}\right)$$

$$= (-2,3)$$

Since, G lies on the median, $x-2y+k = 0$, it must satisfy the equation,

$$-2-6+k = 0 \Rightarrow k=8$$

Thus (d) is correct option.

23. The centroid of the triangle whose vertices are $(3,-7)$, $(-8,6)$ and $(5, 10)$ is

(a) (0, 9) (b) (0, 3)

(c) (1, 3) (d) (3, 5)

g240

Ans :

Centroid is $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

i.e. $$\left(\frac{3+(-8)+5}{3}, \frac{-7+6+10}{3}\right) = \left(\frac{0}{3}, \frac{9}{3}\right)$$

$$= (0,3)$$

Thus (b) is correct option.

24. The distance of the point $P(2,3)$ from the x-axis is

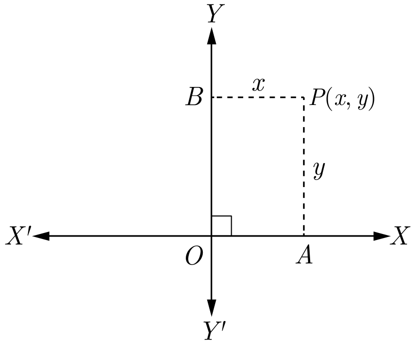
(a) 2 (b) 3

(c) 1 (d) 5

Ans :

We know that, if (x,y) is any point on the cartesian plane in first quadrant, then x is perpendicular distance from y-axis and y is perpendicular distance from x-axis.

Distance of the point $P(2,3)$ from the x-axis is 3.



g242

Thus (b) is correct option.

25. The distance between the points $A(0,6)$ and $B(0,-2)$ is

(a) 6 (b) 8

(c) 4 (d) 2

Ans :

Distance between the points (x_1,y_1) and (x_2,y_2) is given as,

$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Here, $x_1 = 0$, $y_1 = 6$ and $x_2 = 0$, $y_2 = -2$

Distance between $A(0,6)$ and $B(0,-2)$

$$AB = \sqrt{(0-0)^2+(-2-6)^2}$$

$$= \sqrt{0+(-8)^2} = \sqrt{8^2} = 8$$

g243

Thus (b) is correct option.

26. The distance of the point $P(-6,8)$ from the origin is

(a) 8 (b) $2\sqrt{7}$

(c) 10 (d) 6

Ans :

Distance between the points (x,y) and origin is given as,

$$d = \sqrt{x^2+y^2}$$

Distance between $P(-6,8)$ and origin is,

$$PO = \sqrt{(6)^2+(-8)^2} = \sqrt{36+64}$$

$$= \sqrt{100} = 10$$

g244

Thus (c) is correct option.

27. The distance between the points (0, 5) and $(-5, 0)$ is

(a) 5 (b) $5\sqrt{2}$

(c) $2\sqrt{5}$ (d) 10

Ans :

Distance between the points (x_1, y_1) and (x_2, y_2) is given as,

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Here, $x_1 = 0,\ y_1 = 5$ and $x_2 = -5,\ y_2 = 0$

Distance between the points (0, 5) and $(-5, 0)$

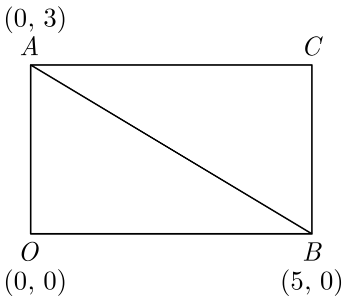
$$d = \sqrt{[-5-0]^2 + [0-(-5)]^2}$$

$$= \sqrt{5^2 + 5^2} = \sqrt{50} = 5\sqrt{2}$$

Thus (b) is correct option.

g245

28. If $AOBC$ is a rectangle whose three vertices are $A(0, 3)$, $O(0, 0)$ and $B(5, 0)$, then the length of its diagonal is



(a) 5 (b) 3

(c) $\sqrt{34}$ (d) 4

g246

Ans :

Length of the diagonal is AB which is the distance between the points $A(0, 3)$ and $B(5, 0)$.

Distance between the points (x_1, y_1) and (x_2, y_2) is given as,

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Here, $x_1 = 0,\ y_1 = 3$, and $x_2 = 5,\ y_2 = 0$

Distance between the points $A(0, 3)$ and $B(5, 0)$

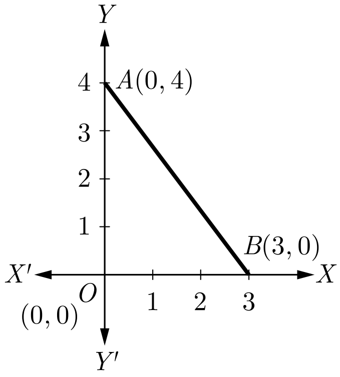
$$AB = \sqrt{(5-0)^2 + (0-3)^2}$$

$$= \sqrt{25+9} = \sqrt{34}$$

Hence, the required length of its diagonal is $\sqrt{34}$.

Thus (c) is correct option.

29. The perimeter of a triangle with vertices (0, 4), (0, 0) and (3, 0) is



g247

(a) 5 (b) 12

(c) 11 (d) $7 + \sqrt{5}$

Ans :

We have $OA = 4$

$OB = 3$

and $AB = \sqrt{3^2 + 4^2 = 5}$

Now, perimeter of ΔAOB is the sum of the length of all its sides.

$$p = OA + OB + AB = 4 + 3 + 5 = 12$$

Hence, the required perimeter of triangle is 12. However you can calculate perimeter direct from diagram.

Thus (b) is correct option.

30. The point which lies on the perpendicular bisector of the line segment joining the points $A(-2, -5)$ and $B(2, 5)$ is

(a) (0, 0) (b) (0, 2)

(c) (2, 0) (d) $(-2, 0)$

g249

Ans :

We know that, the perpendicular bisector of the any line segment divides the line segment into two equal parts i.e., the perpendicular bisector of the line segment always passes through the mid-point of the line segment.

Mid-point of the line segment joining the points $A(-2, -5)$ and $B(2, 5)$

$$= \left(\frac{-2+2}{2}, \frac{-5+5}{2}\right) = (0, 0)$$

Hence, (0, 0) is the required point lies on the perpendicular bisector of the lines segment.

Thus (a) is correct option.

31. If the point $P(2, 1)$ lies on the line segment joining points $A(4,2)$ and $B(8,4)$, then

(a) $AP = \frac{1}{3}AB$ (b) $AP = PB$

(c) $PB = \frac{1}{3}AB$ (d) $AP = \frac{1}{2}AB$

Ans :

Let, $AP : AB = m : n$

Using section formula, we have,

$$4 = \frac{8m+2n}{m+n}$$

and $$2 = \frac{4m+n}{m+n}$$

Solving these as linear equation, we get,

$$m = 1 \text{ and } n = 2$$

$$\frac{AP}{AB} = \frac{1}{2}$$

$$AP = \frac{1}{2}AB$$

Thus (d) is correct option.

32. If $P\left(\frac{a}{3}, 4\right)$ is the mid-point of the line segment joining the points $Q(-6,5)$ and $R(-2,3)$, then the value of a is

(a) -4 (b) -12

(c) 12 (d) -6

Ans :

Since $P\left(\frac{a}{3}, 4\right)$ is the mid-point of the points $Q(-6,5)$ and $R(-2,3)$,

$$\left(\frac{a}{3}, 4\right) = \left(\frac{-6-2}{2}, \frac{5+3}{2}\right)$$

$$\left(\frac{a}{3}, 4\right) = (-4, 4)$$

Now $$\frac{a}{3} = -4 \Rightarrow a = -12$$

Thus (b) is correct option.

33. The perpendicular bisector of the line segment joining the points $A(1,5)$ and $B(4,6)$ cuts the y-axis at

(a) (0, 13) (b) (0, – 13)

(c) (0, 12) (d) (13, 0)

Ans :

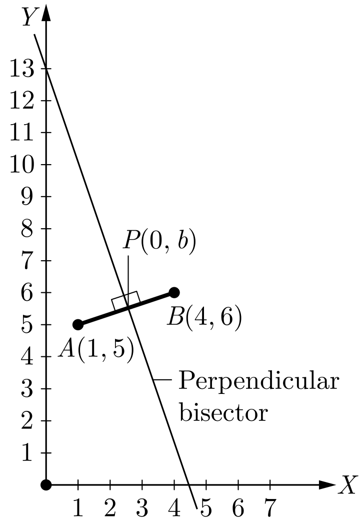
Let $P(0,b)$ be the required point. Since, any point on perpendicular bisector is equidistant from the end point of line segment.

i.e., $PA = PB$

$$\sqrt{(0-1)^2+(b-5)^2} = \sqrt{(0-4)^2+(b-6)^2}$$

$$1+b^2-10b+25 = 16+b^2-12b+36$$

$$2b = 26 \Rightarrow b = 13$$



Thus (a) is correct option.

34. If the distance between the points (4, p) and (1, 0) is 5, then the value of p is

(a) 4 only (b) ± 4

(c) -4 only (d) 0

Ans :

According to the question, the distance between the points (4, p) and (1, 0) is 5.

i.e., $$\sqrt{(1-4)^2+(0-p)^2} = 5$$

$$\sqrt{(-3)^2+p^2} = 5$$

$$\sqrt{9+p^2} = 5$$

Squaring both the sides, we get,

$$9+p^2 = 25$$

$$p^2 = 16 \Rightarrow p = \pm 4$$

Hence, the required value of p is ± 4.

Thus (b) is correct option.

35. Assertion : The value of y is 6, for which the distance between the points $P(2,-3)$ and $Q(10,y)$ is 10.

Reason : Distance between two given points $A(x_1, y_1)$ *and* $B(x_2, y_2)$ is given,

$$AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

$$PQ = 10$$
$$PQ^2 = 100$$
$$(10-2)^2 + (y+3)^2 = 100$$
$$(y+3)^2 = 100 - 64 = 36$$
$$y+3 = \pm 6$$
$$y = -3 \pm 6$$
$$y = 3, -9$$

g254

Assertion (A) is false but reason (R) is true. Thus (s) is correct option.

Fill in the Blank Questions

36. All the points equidistant from two given points A and B lie on the .... of the line segment AB.

Ans :

perpendicular bisector

g255

37. The distance of a point from the y-axis is called its ....

Ans :

abscissa

g256

38. The distance of a point from the x-axis is called its ....

Ans :

ordinate

g257

39. The value of the expression $\sqrt{x^2+y^2}$ is the distance of the point $P(x, y)$from the ....

Ans :

origin

g258

40. The distance of the point (p, q) from (a, b) is ....

Ans :

$\sqrt{(a-p)^2 + (b-q)^2}$

g259

41. If the area of the triangle formed by the vertices $A(x_1, y_1)$ $B(x_2, y_2)$ and $C(x_3, y_3)$ is zero, then the points A, B and C are ....

Ans :

g260

collinear

42. A point of the form $(b, 0)$ lies on ....

Ans :

x-axis

g261

43. The distance of the point (x_1, y_1) from the origin is ....

Ans :

$\sqrt{x_1^2 + y_1^2}$

g262

44. A point of the form $(0, a)$ lies on ....

Ans :

y-axis

g263

45. If the point $C(k,\ 4)$ divides the line segment joining two points $A(2,\ 6)$ and $B(5, 1)$ in ratio 2 : 3, the value of k is ....

Ans : [Board 2020 Delhi Basic]

We have $m : n = 2 : 3$

g264

By section formula,

$$\frac{mx_2 + nx_1}{m+n} = x$$

Now, $$\frac{2\times 5 + 3\times 2}{2+3} = k \Rightarrow k = \frac{16}{5}$$

46. If points $A(-3, 12)$, $B(7, 6)$ and $C(x, 9)$ are collinear, then the value of x is ....

Ans : [Board 2020 Delhi Basic]

If points are collinear, then area of triangle must be zero.

$$\frac{1}{2}[x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)] = 0$$

$$\frac{1}{2}[-3(6-9) + 7(9-12) + x(12-6)] = 0$$

g265

$$\frac{1}{2}(9-21+6x) = 0$$
$$\frac{1}{2}(-12+6x) = 0$$
$$6x = 12 \Rightarrow x = 2$$

47. The co-ordinate of the point dividing the line segment joining the points $A(1, 3)$ and $B(4, 6)$ in the ratio 2 : 1 is ....

Ans : [Board 2020 OD Basic]

Let point $P(x,\ y)$ divides the line segment joir points $A(1, 3)$ and $B(4, 6)$ in the ratio 2 : 1. Using section formula we have

g267

$$(x,\ y) = \left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$$

$$(x,\ y) = \left(\frac{2\times 4+1\times 1}{2+1}, \frac{2\times 6+1\times 3}{2+1}\right)$$

$$= \left(\frac{8+1}{3}, \frac{12+3}{3}\right) = \left(\frac{9}{3}, \frac{15}{3}\right) = (3,5)$$

VERY SHORT ANSWER QUESTIONS

48. Find the distance of a point $P(x,y)$ from the origin.

Ans : [Board 2018]

Distance between origin (0, 0) and point $P(x,y)$ is

$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$= \sqrt{(x-0)^2+(y-0)^2}$$

$$= \sqrt{x^2+y^2}$$

g268

Distance between P and origin is $\sqrt{x^2+y^2}$.

49. If the mid-point of the line segment joining the points $A(3,4)$ and $B(k,6)$ is $P(x,y)$ and $x+y-10=0$, find the value of k.

Ans : [Board 2020 OD Standard]

If $P(x,y)$ is mid point of $A(3,4)$ and $B(k,6)$, then we have

$$\frac{3+k}{2} = x \text{ and } y = \frac{4+6}{2} = \frac{10}{2} = 5$$

Substituting above value in $x+y-10=0$ we have

$$\frac{3+k}{2}+5-10 = 0$$

g269

$$\frac{3+k}{2} = 5$$

$$3+k = 10 \Rightarrow k = 10-3 = 7$$

50. Write the coordinates of a point P on x-axis which is equidistant from the points $A(-2,0)$ and $B(6,0)$.

Ans : [Board 2019 OD]

Since it is equidistant from the points $A(-2,0)$ and $B(6,0)$ then

$$AP = BP$$

270

$$AP^2 = BP^2$$

Using distance formula we have

$$[(x-(-2)]^2+(0-0)^2 = (x+6)^2+(0-0)^2$$

$$(x+2)^2 = (x+6)^2$$

$$x^2+4x+4 = x^2+12x+36$$

$$8x = -32$$

$$x = -4$$

Hence, required point P is $(-4,0)$.

Alternative :

You may easily observe that both point $A(-2,0)$ and $B(6,0)$ lies on x-axis because y ordinate is zero. Thus point P on x-axis equidistant from both point must be mid point of $A(-2,0)$ and $B(6,0)$.

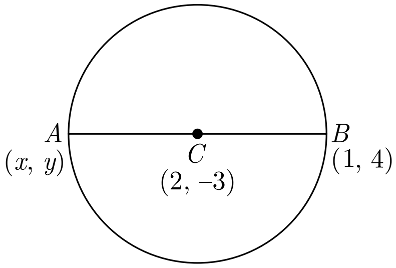
$$x = \frac{-2+6}{2} = 2$$

51. Find the coordinates of a point A, where AB is diameter of a circle whose centre is $(2,-3)$ and B is the point $(1,4)$.

Ans : [Board 2019 Delhi]

As per question we have shown the figure below. Since, AB is the diameter, centre C must be the mid point of the diameter of AB.

g271



Let the co-ordinates of point A be (x,y).

x-coordinate of C,

$$\frac{x+1}{2} = 2$$

$$x+1 = 4 \Rightarrow x = 3$$

and y-coordinate of C,

$$\frac{y+4}{2} = -3$$

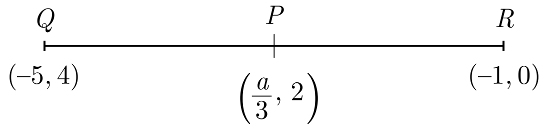
$$y+4 = -6 \Rightarrow y = -10$$

Hence, coordinates of point A are $(3,-10)$.

52. Find the value of a, for which point $P(\frac{a}{3},2)$ is the midpoint of the line segment joining the Points $Q(-5,4)$ and $R(-1,0)$.

Ans : [Board Term-2 SQP 2016]

As per question, line diagram is shown below.



Since P is mid-point of QR, we have

$$\frac{a}{3} = \frac{-5+(-1)}{2} = \frac{-6}{2} = -3$$

g101

Thus $a = -9$

53. The ordinate of a point A on y-axis is 5 and B has co-ordinates $(-3,1)$. Find the length of AB.

Ans : [Board Term-2 2014]

We have $A(0,5)$ and $B(-3,1)$.

g102

Distance between A and B,

$$AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(-3-0)^2+(1-5)^2}$$
$$= \sqrt{9+16}$$
$$= \sqrt{25} = 5$$

54. Find the perpendicular distance of $A(5,12)$ from the y-axis.

Ans : [Board Term-2 2011]

Perpendicular from point $A(5,12)$ on y-axis touch it at $(0,12)$.

Distance between $(5,12)$ and $(0,12)$ is,

$$d = \sqrt{(0-5)^2+(12-12)^2}$$
$$= \sqrt{25}$$
$$= 5 \text{ units.}$$

g103

55. If the centre and radius of circle is $(3,4)$ and 7 units respectively,, then what it the position of the point $A(5,8)$ with respect to circle?

Ans : [Board Term-2 2013]

Distance of the point, from the centre,

$$d = \sqrt{(5-3)^2+(8-4)^2}$$
$$= \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$$

g104

Since $2\sqrt{5}$ is less than 7, the point lies inside the circle.

56. Find the perimeter of a triangle with vertices $(0,4)$, $(0,0)$ and $(3,0)$.

Ans : [Board Term-2, 2011]

We have $A(0,4), B(0,0)$, and $C(3,0)$.

$$AB = \sqrt{(0-2)^2+(0-4)^2} = \sqrt{16} = 4$$
$$BC = \sqrt{(3-0)^2+(0-0)^2} = \sqrt{9} = 3$$
$$CA = \sqrt{(0-3)^2+(4-0)^2}$$
$$= \sqrt{9+16} = \sqrt{25} = 5$$

g105

Thus perimeter of triangle is $4+3+5 = 12$

57. Locate a point Q on line segment AB such that $BQ = \frac{5}{7} \times AB$. What is the ratio of line segment in which AB is divided?

Ans : [Board Term-2 2013]

We have $BQ = \frac{5}{7}AB$

$$\frac{BQ}{AB} = \frac{5}{7} \Rightarrow \frac{AB}{BQ} = \frac{7}{5}$$

g106

$$\frac{AB-BQ}{BQ} = \frac{7-5}{5}$$

$$\frac{AQ}{BQ} = \frac{2}{5}$$

Thus $AQ:BQ = 2:5$

58. Find the distance of the point $(-4,-7)$ from the y-axis.

Ans : [Board Term-2 2013]

Perpendicular from point $A(-4,-7)$ on y-axis touch it at $(0,-7)$.

Distance between $(-4,-7)$ and $(0,-7)$ is

$$d = \sqrt{(0+4)^2+(-7+7)^2}$$
$$= \sqrt{4^2+0} = \sqrt{16} = 4 \text{ units}$$

g107

59. If the distance between the points $(4,k)$ and $(1,0)$ is 5, then what can be the possible values of k.

Ans : [Board Term-2 2017]

Using distance formula we have

g108

$$\sqrt{(4-1)^2+(k-0)^2} = 5$$
$$3^2+k^2 = 25$$
$$k^2 = 25-9 = 16$$
$$k = \pm 4$$

60. Find the coordinates of the point on y-axis which is nearest to the point $(-2,5)$.

Ans : [Board Term-2 SQP 2017]

Point $(0,5)$ on y-axis is nearest to the point $(-2,5)$.

g109

61. In what ratio does the x-axis divide the line segment joining the points $(-4,-6)$ and $(-1,7)$? Find the coordinates of the point of division.

Ans : [Board Term-2 SQP 2017]

Let x-axis divides the line-segment joining $(-4,-6)$ and $(-1,7)$ at the point $P(x,y)$ in the ratio $1:k$.

Now, the coordinates of point of division P,

$$(x,y) = \frac{1(-1)+k(-4)}{k+1}, \frac{1(7)+k(-6)}{k+1}$$

g110

$$= \frac{-1-4k}{k+1}, \frac{7-6k}{k+1}$$

Since P lies on x axis, therefore $y=0$, which gives

$$\frac{7-6k}{k+1} = 0$$

$$7-6k = 0$$

$$k = \frac{7}{6}$$

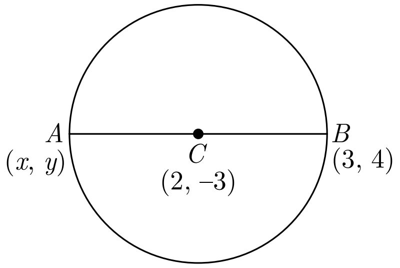
Hence, the ratio is $1:\frac{7}{6}$ or, $6:7$ and the coordinates of P are $\left(-\frac{34}{13}, 0\right)$.

TWO MARKS QUESTIONS

62. Find the coordinates of a point A, where AB is diameter of the circle whose centre is $(2,-3)$ and B is the point $(3,4)$.

Ans : [Board 2019 Delhi]

As per question we have shown the figure below. Since, AB is the diameter, centre C must be the mid point of the diameter of AB.



Let the co-ordinates of point A be (x,y).

x-coordinate of C,

g275

$$\frac{x+3}{2} = 2$$

$$x+3 = 4 \Rightarrow x = 1$$

and y-coordinate of C,

$$\frac{y+4}{2} = -3$$

$$y+4 = -6 \Rightarrow y = -10$$

Hence, coordinates of point A is $(1,-10)$.

63. Find a relation between x and y such that the point $P(x,y)$ is equidistant from the points $A(-5,3)$ and $B(7,2)$.

Ans : [Board Term-2 SQP 2016]

Let $P(x,y)$ is equidistant from $A(-5,3)$ and $B(7,2)$, then we have

$$AP = BP$$

$$\sqrt{(x+5)^2+(y-3)^2} = \sqrt{(x-7)^2+(y-2)^2}$$

$$(x+5)^2+(y-3)^2 = (x-7)^2+(y-2)^2$$

$$10x+25-6y+9 = -14x+49-4y+4$$

$$24x+34 = 2y+53$$

$$24x-2y = 19$$

g115

Thus $24x-2y-19 = 0$ is the required relation.

64. The x-coordinate of a point P is twice its y-coordinate. If P is equidistant from $Q(2,\ -5)$ and $R(-3,\ 6)$, find the co-ordinates of P.

Ans : [Board Term-2 2016]

Let the point P be $(2y,y)$. Since $PQ = PR$, we have

$$\sqrt{(2y-2)^2+(y+5)^2} = \sqrt{(2y+3)^2+(y-6)^2}$$

$$(2y-2)^2+(y+5)^2 = (2y+3)^2+(y-6)^2$$

$$-8y+4+10y+25 = 12y+9-12y+36$$

$$2y+29 = 45$$

$$y = 8$$

g116

Hence, coordinates of point P are $(16,8)$

65. Find the ratio in which y-axis divides the line segment joining the points $A(5,-6)$ and $B(-1,-4)$. Also find the co-ordinates of the point of division.

Ans : [Delhi Set I, II, III, 2016]

Let y -axis be divides the line-segment joining $A(5,-6)$ and $B(-1,-4)$ at the point $P(x,y)$ in the ratio $AP:PB = k:1$

Now, the coordinates of point of division P,

$$(x,y)=\left(\frac{k(-1)+1(5)}{k+1},\ \frac{k(-4)+1(-6)}{k+1}\right)$$

$$=\left(\frac{-k+5}{k+1},\ \frac{-4k-6}{k+1}\right)$$

Since P lies on y axis, therefore $x=0$, which gives

$$\frac{5-k}{k+1}=0 \Rightarrow k=5$$

Hence required ratio is 5:1,

g117

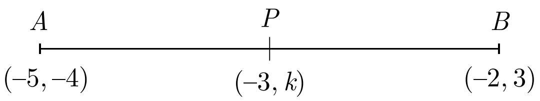
Now $$y=\frac{-4(5)-6}{6}=\frac{-13}{3}$$

Hence point on y-axis is $(0,-\frac{13}{3})$.

66. Find the ratio in which the point $(-3,k)$ divides the line segment joining the points $(-5,-4)$ and $(-2,3)$. Also find the value of k.

Ans : [Board Term-2 Foreign 2016]

As per question, line diagram is shown below.



Let AB be divides by P in ratio $n:1$.

x co-ordinate for section formula

g118

$$-3=\frac{(-2)n+1(-5)}{n+1}$$

$$-3(n+1)=-2n-5$$

$$-3n-3=-2n-5$$

$$5-3=3n-2n$$

$$2=n$$

Ratio $$\frac{n}{1}=\frac{2}{1} \text{ or } 2:1$$

Now, y co-ordinate,

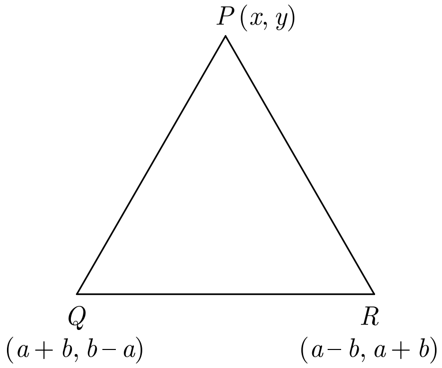
$$k=\frac{2(3)+1(-4)}{2+1}=\frac{6-4}{3}=\frac{2}{3}$$

67. If the point $P(x,y)$ is equidistant from the points $Q(a+b,b-a)$ and $R(a-b,a+b)$, then prove that $bx=ay$.

Ans : [Board Term-2 Delhi 2012, OD 2016]

We have $$|PQ|=|PR|$$

$$\sqrt{[x-(a+b)]^2+[y-(b-a)]^2}=\sqrt{[x-(a-b)]^2+[y-(b+a)]^2}$$



$$[x-(a+b)]^2+[y-(b-a)]^2=[x-(a-b)]^2+[y-(a+b)]^2$$

$$-2x(a+b)-2y(b-a)=-2x(a-b)-2y(a+b)$$

$$2x(a+b)+2y(b-a)=2x(a-b)+2y(a+b)$$

$$2x(a+b-a+b)+2y(b-a-a-b)=0$$

g119

$$2x(2b)+2y(-2a)=0$$

$$xb-ay=0$$

$$bx=ay$$ Hence Proved

68. Prove that the point $(3,0)$, $(6,4)$ and $(-1,3)$ are the vertices of a right angled isosceles triangle.

Ans : [Board Term-2 OD 2016]

We have $A(3,0)$, $B(6,4)$ and $C(-1,3)$

Now $$AB^2=(3-6)^2+(0-4)^2$$

g120

$$= 9+16 = 25$$

$$BC^2 = (6+1)^2+(4-3)^2$$

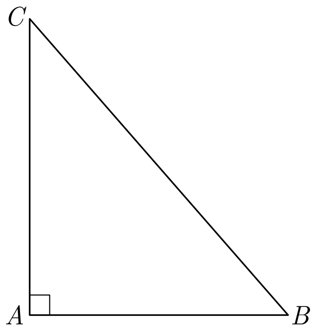
$$= 49+1 = 50$$

$$CA^2 = (-1-3)^2+(3-0)^2$$

$$= 16+9 = 25$$

$$AB^2 = CA^2 \text{ or, } AB = CA$$

Hence triangle is isosceles.



Also, $\quad 25+25 = 50$

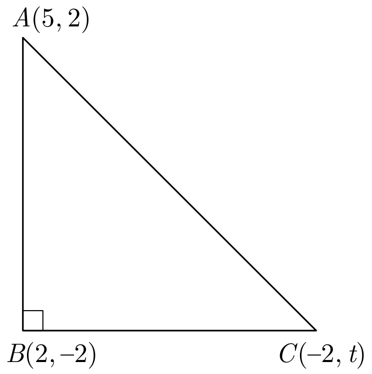
or, $\quad AB^2 + CA^2 = BC^2$

Since Pythagoras theorem is verified, therefore triangle is a right angled triangle.

69. If $A(5,2)$, $B(2,-2)$ and $C(-2,t)$ are the vertices of a right angled triangle with $\angle B = 90^o$, then find the value of t.

Ans : [Board Term-2 Delhi 2015]

As per question, triangle is shown below.



Now $\quad AB^2 = (2-5)^2+(-2-2)^2 = 9+16 = 25$

$$BC^2 = (-2-2)^2+(t+2)^2 = 16+(t+2)^2$$

$$AC^2 = (5+2)^2+(2-t)^2 = 49+(2-t^2)$$

Since $\Delta\,ABC$ is a right angled triangle

$$AC^2 = AB^2+BC^2$$

$$49+(2-t)^2 = 25+16+(t+2)^2$$

$$49+4-4t+t^2 = 41+t^2+4t+4$$

$$53-4t = 45+4t$$

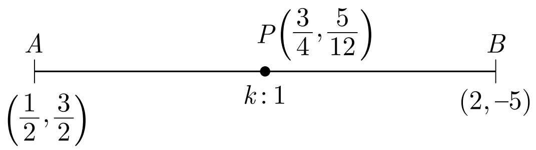
$$8t = 8$$

$$t = 1$$

70. Find the ratio in which the point $P(\frac{3}{4},\frac{5}{12})$ divides the line segment joining the point $A\ (\frac{1}{2},\frac{3}{2})$ and $(2,-5)$.

Ans : [Board Term-2 Delhi 2015]

Let P divides AB in the ratio $k:1$. Line diagram is shown below.



Now $\quad \dfrac{k(2)+1(\frac{1}{2})}{k+1} = \dfrac{3}{4}$

$$8k+2 = 3k+3$$

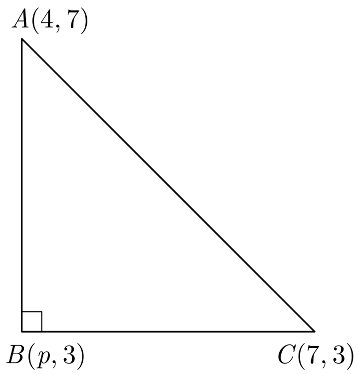
$$k = \frac{1}{5}$$

Thus required ratio is $\frac{1}{5}:1$ or 1:5.

71. The points $A(4,7)$, $B(p,3)$ and $C(7,3)$ are the vertices of a right triangle, right-angled at B. Find the value of p.

Ans : [Board Term-2 OD 2015]

As per question, triangle is shown below. Here $\Delta\,ABC$ is a right angle triangle,



$$AB^2 + BC^2 = AC^2$$

$$(p-4)^2 + (3-7)^2 + (7-p)^2 + (3-3)^2 = (7-4)^2 + (3-4)^2$$

$$(p-4)^2 + (-4)^2 + (7-p)^2 + 0 = (3)^2 + (-4)^2$$

$$p^2 - 8p + 16 + 16 + 49 + p^2 - 14p = 9 + 16$$

$$2p^2 - 22p + 81 = 25$$

$$2p^2 - 22p + 56 = 0$$

$$p^2 - 11p + 28 = 0$$

$$(p-4)(p-7) = 0$$

$$p = 7 \text{ or } 4$$

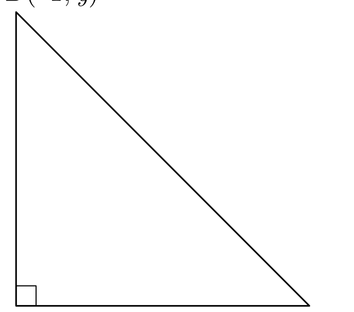
g123

72. If $A(4,3)$, $B(-1,y)$, and $C(3,4)$ are the vertices of a right triangle ABC, right angled at A, then find the value of y.

Ans : [Board Term-2 OD 2015]

As per question, triangle is shown below.

g124



Now $$AB^2 + AC^2 = BC^2$$

$$(4+1)^2 + (3-y)^2 + (4-3)^2 = (3+1)^2 + (4-y)^2$$

$$(5)^2 + (3-y)^2 + (-1)^2 + (1)^2 = (4)^2 + (4-y)^2$$

$$25 + 9 - 6y + y^2 + 1 + 1 = 16 + 16 - 8y + y^2$$

$$36 + 2y - 32 = 0$$

$$2y + 4 = 0$$

$$y = -2$$

73. Show that the points (a,a), $(-a,-a)$ and $(-\sqrt{3}\,a, \sqrt{3}\,a)$ are the vertices of an equilateral triangle.

Ans : [Board Term-2 Foreign 2015]

Let $A(a,a), B(-a,-a)$ and $C(-\sqrt{3}\,a, \sqrt{3}\,a)$.

Now $$AB = \sqrt{(a+a)^2 + (a+a)^2}$$

$$= \sqrt{4a^2 + 4a^2} = 2\sqrt{2}\,a$$

$$BC = \sqrt{(-a+\sqrt{3}\,a)^2 + (-a-\sqrt{3}\,a)^2}$$

$$= \sqrt{a^2 - 2\sqrt{3}\,a^2 + 3a^2 + a^2 + 2\sqrt{3}\,a^2 + 3a^2}$$

$$= 2\sqrt{2}\,a$$

$$AC = \sqrt{(a+\sqrt{3}\,a)^2 + (a-\sqrt{3}\,a)^2}$$

$$= \sqrt{a^2 + 2\sqrt{3}\,a^2 + 3a^2 + a^2 - 2\sqrt{3}\,a^2 + 3a^2}$$

$$= 2\sqrt{2}\,a$$

Since $AB = BC = AC$, therefore ABC is an equilateral triangle.

g125

74. If the mid-point of the line segment joining $A\left[\frac{x}{2}, \frac{y+1}{2}\right]$ and $B(x+1, y-3)$ is $C(5,-2)$, find x, y.

Ans : [Board Term-2 OD 2012, Delhi 2014]

If the mid-point of the line segment joining $A\left[\frac{x}{2}, \frac{y+1}{2}\right]$ and $B(x+1, y-3)$ is $C(5,-2)$, then at mid point,

$$\frac{\frac{x}{2} + x + 1}{2} = 5$$

$$\frac{3x}{2} + 1 = 10$$

$$3x = 18 \Rightarrow x = 6$$

also $$\frac{\frac{y+1}{2} + y - 3}{2} = -2$$

$$\frac{y+1}{2} + y - 3 = -4$$

$$y + 1 + 2y - 6 = -8 \Rightarrow y = -1$$

g126

75. Find the point on the x-axis which is equidistant from the points $(2,-5)$ and $(-2,9)$.

Ans : [Board Term-2 2012]

Let the point be $P(x,0)$ on the x-axis is equidistant from points $A(2,-5)$ and $B(-2,9)$.

g127

Now $$PA^2 = PB^2$$

$$(2-x)^2 + (-5-0)^2 = (-2-x)^2 + (9-0)^2$$

$$4 - 4x + x^2 + 25 = 4 + 4x + x^2 + 81$$

$$-8x = 56 \Rightarrow x = -7$$

Thus point is $(-7,0)$.

76. Show that $A(6,4)$, $B(5,-2)$ and $C(7,-2)$ are the vertices of an isosceles triangle.

Ans : [Board Term-2, 2012]

We have $A(6,4), B(5,-2), C(7,-2)$.

Now $AB = \sqrt{(6-5)^2+(4+2)^2}$

$= \sqrt{1^2+6^2} = \sqrt{37}$

$BC = \sqrt{(5-7)^2+(-2+2)^2}$

$= \sqrt{(-2)^2+0^2} = 2$

$CA = \sqrt{(7-6)^2+(-2-4)^2}$

$= \sqrt{1^2+6^2} = \sqrt{37}$

$AB = BC = \sqrt{37}$

g128

Since two sides of a triangle are equal in length, triangle is an isosceles triangle.

77. If $P(2,-1)$, $Q(3,4)$, $R(-2,3)$ and $S(-3,-2)$ be four points in a plane, show that $PQRS$ is a rhombus but not a square.

Ans : [Board Term-2 OD 2012]

We have $P(2,\ -1)$, $Q(3,\ 4)$, $R(-2,\ 3)$, $S(-3,\ -2)$

$PQ = \sqrt{1^2+5^2} = \sqrt{26}$

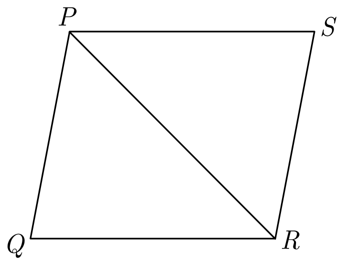
$QR = \sqrt{5^2+1^2} = \sqrt{26}$

$RS = \sqrt{1^2+5^2} = \sqrt{26}$

$PS = \sqrt{5^2+1^2} = \sqrt{26}$

g129

Since all the four sides are equal, $PQRS$ is a rhombus.



Now $PR = \sqrt{1^2+5^2} = \sqrt{26}$

$= \sqrt{4^2+4^2} = \sqrt{32}$

$PQ^2 + QR^2 = 2 \times 26 = 52 \neq (\sqrt{32})^2$

Since ΔPQR is not a right triangle, $PQRS$ is a rhombus but not a square.

78. Show that $A(-1,0)$, $B(3,1)$, $C(2,2)$ and $D(-2,1)$ are the vertices of a parallelogram $ABCD$.

Ans : [Board Term-2 2012]

Mid-point of AC,

$$\left(\frac{-1+2}{2}, \frac{0+2}{2}\right) = \left(\frac{1}{2}, 1\right)$$

g130

Mid-point of BD,

$$\left(\frac{3-2}{2}, \frac{1+1}{2}\right) = \left(\frac{1}{2}, 1\right)$$

Here Mid-point of AC = Mid-point of BD

Since diagonals of a quadrilateral bisect each other, $ABCD$ is a parallelogram.

79. If $(3,2)$ and $(-3,2)$ are two vertices of an equilateral triangle which contains the origin, find the third vertex.

Ans : [Board Term-2 OD 2012]

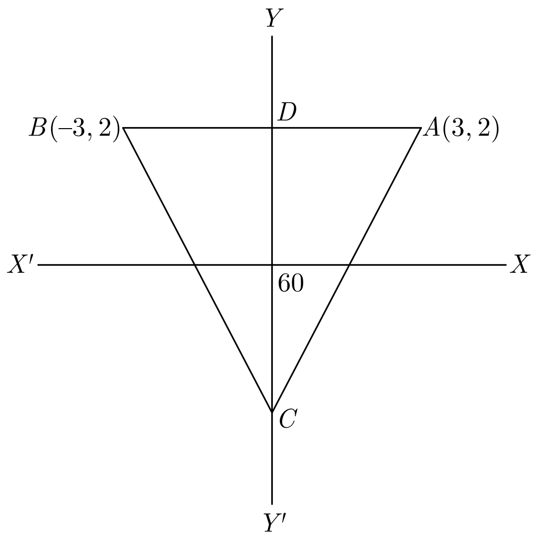
We have $A(3,2)$ and $B(-3,2)$.

It can be easily seen that mid-point of AB is lying on y-axis. Thus AB is equal distance from x-axis everywhere.

Also $OD \perp AB$

Hence 3^{rd} vertex of ΔABC is also lying on y-axis.

The digram of triangle should be as given below.



Let $C(x,y)$ be the coordinate of 3^{rd} vertex of ΔABC.

Now $AB^2 = (3+3)^2+(2-2)^2 = 36$

$BC^2 = (x+3)^2+(y-2)^2$

$AC^2 = (x-3)^2+(y-2)^2$

Since $AB^2 = AC^2 = BC^2$

$$(x+3)^2+(y-2)^2 = 36 \quad (1)$$

$$(x-3)^2+(y-2)^2 = 36 \quad (2)$$

Since $P(x,y)$ lie on y-axis, substituting $x=0$ in (1) we have

$$3^2 + (y-2)^2 = 36 - 9 = 27$$
$$(y-2)^2 = 36 - 9 = 27$$

Taking square root both side

$$y - 2 = \pm 3\sqrt{3}$$
$$y = 2 \pm 3\sqrt{3}$$

Since origin is inside the given triangle, coordinate of C below the origin,

$$y = 2 - 3\sqrt{3}$$

Hence Coordinate of C is $(0,\ 2-3\sqrt{3})$

80. Find a so that $(3, a)$ lies on the line represented by $2x - 3y - 5 = 0$. Also, find the co-ordinates of the point where the line cuts the x-axis.

Ans : [Board Term-2 2012]

Since $(3, a)$ lies on $2x - 3y - 5 = 0$, it must satisfy this equation. Therefore

$$2 \times 3 - 3a - 5 = 0$$
$$6 - 3a - 5 = 0$$
$$1 = 3a$$
$$a = \frac{1}{3}$$

Line $2x - 3y - 5 = 0$ will cut the x-axis at $(x, 0)$. and it must satisfy the equation of line.

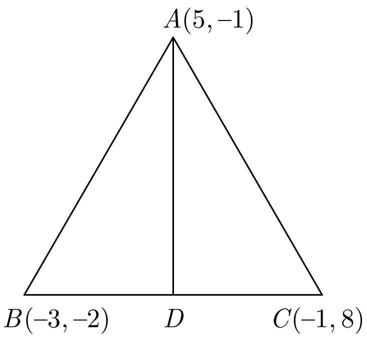
$$2x - 5 = 0 \Rightarrow x = \frac{5}{2}$$

Hence point is $\left(\frac{5}{2}, 0\right)$.

81. If the vertices of ΔABC are $A(5, -1)$, $B(-3, -2)$, $C(-1, 8)$, Find the length of median through A.

Ans : [Board Term-2 2012]

Let AD be the median. As per question, triangle is shown below.



Since D is mid-point of BC, co-ordinates of D,

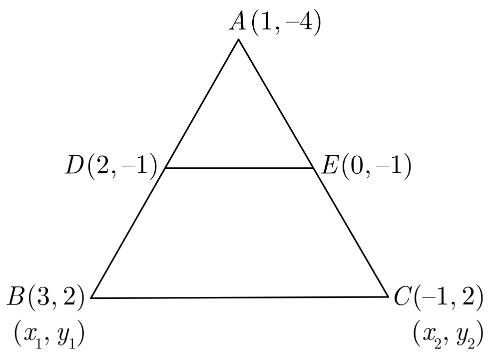
$$(x_1,\ y_2) = \left(\frac{-3-1}{2}, \frac{-2+8}{2}\right)$$
$$= (-2,\ 3)$$
$$AD = \sqrt{(5+2)^2 + (-1-3)^2}$$
$$= \sqrt{(7)^2 + (4)^2}$$
$$= \sqrt{49+16} = \sqrt{65} \text{ units}$$

Thus length of median is $\sqrt{65}$ units.

82. Find the mid-point of side BC of ΔABC, with $A(1, -4)$ and the mid-points of the sides through A being $(2, -1)$ and $(0, -1)$.

Ans : [Board Term-2 2012]

Assume co-ordinates of B and C are (x_1, y_1) and (x_2, y_2) respectively. As per question, triangle is shown below.



Now $$2 = \frac{1+x_1}{2} \Rightarrow x_1 = 3$$

and $$-1 = \frac{-4+y_1}{2} \Rightarrow y_1 = 2$$

$$0 = \frac{1+x_2}{2} \Rightarrow x = -1$$

$$-1 = \frac{-4+y_2}{2} \Rightarrow y_2 = 2$$

Thus $$B(x_1, y_1) = (3, 2),$$
$$C(x_2, y_2) = (-1, 2)$$

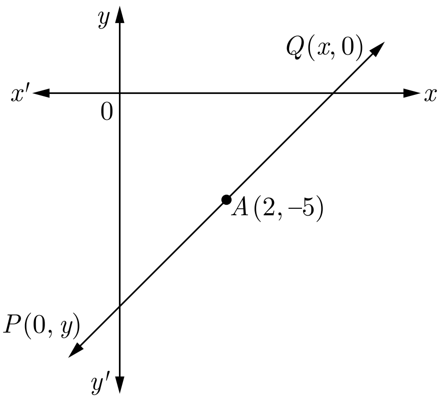
So, mid-point of BC is $\left(\frac{3-1}{2}, \frac{2+2}{2}\right) = (1, 2)$

83. A line intersects the y-axis and x-axis at the points P

and Q respectively. If $(2,-5)$ is the mid-point of PQ, then find the coordinates of P and Q.

Ans : [Board Term-2 OD 2017]

Let coordinates of P be $(0,y)$ and of Q be $(x,0)$.
$A(2,-5)$ is mid point of PQ.
As per question, line diagram is shown below.



Using section formula,

$$(2,-5) = \left(\frac{0+x}{2}+\frac{y+0}{2}\right)$$

$$2 = \frac{x}{2} \Rightarrow x = 4$$

and $$-5 = \frac{y}{2} \Rightarrow y = -10$$

Thus P is $(0,-10)$ and Q is $(4,0)$

84. If $(1,\frac{p}{3})$ is the mid point of the line segment joining the points $(2,0)$ and $(0,\frac{2}{9})$, then show that the line $5x+3y+2=0$ passes through the point $(-1,3p)$.

Ans :

Since $(1,\frac{p}{3})$ is the mid point of the line segment joining the points $(2,0)$ and $(0,\frac{2}{9})$, we have

$$\frac{p}{3} = \frac{0+\frac{2}{9}}{2} = \frac{1}{9}$$

$$p = \frac{1}{3}$$

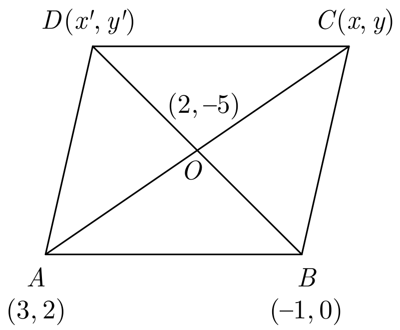
Now the point $(-1,3p)$ is $(-1,1)$.
The line $5x+3y+2=0$, passes through the point $(-1,1)$ as $5(-5)+3(1)+2=0$

85. If two adjacent vertices of a parallelogram are $(3,2)$ and $(-1,0)$ and the diagonals intersect at $(2,-5)$ then find the co-ordinates of the other two vertices.

Ans : [Board Term-2 Foreign 2017]

Let two other co-ordinates be (x,y) and (x',y') respectively using mid-point formula.

As per question parallelogram is shown below.



Now $$2 = \frac{x+3}{2} \Rightarrow x = 1$$

and $$-5 = \frac{2+y}{2} \Rightarrow y = -12$$

Again, $$\frac{-1+x'}{2} = 2 \Rightarrow x' = 5$$

and $$\frac{0+y'}{2} = -5 \Rightarrow y' = -10$$

Hence, coordinates of $C(1,-12)$ and $D(5,-10)$

86. In what ratio does the point $P(-4,6)$ divides the line segment joining the points $A(-6,10)$ and $B(3,-8)$?

Ans : [Board Term-2 Delhi Compt. 2017]

Let $$AP:PB = k:1$$

Now $$\frac{3k-6}{k+1} = -4$$

$$3k-6 = -4k-4$$

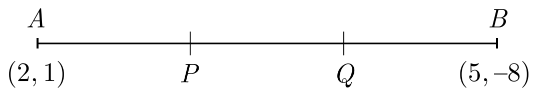
$$7k = 2 \Rightarrow k = \frac{2}{7}$$

Hence, $$AP:PB = 2:7$$

87. If the line segment joining the points $A(2,1)$ and $B(5,-8)$ is trisected at the points P and Q, find the coordinates P.

Ans : [Board Term-2 OD Compt. 2017]

As per question, line diagram is shown below.



Let $P(x,y)$ divides AB in the ratio $1:2$
Using section formula we get

$$x = \frac{1 \times 5 + 2 \times 2}{1+2} = 3$$

$$y = \frac{1 \times -8 + 2 \times 1}{1+2} = -2$$

Hence coordinates of P are $(3, -2)$.

88. Prove that the points $(2, -2), (-2, 1)$ and $(5, 2)$ are the vertices of a right angled triangle. Also find the area of this triangle.

Ans : [Board Term-2 Foreign 2016]

We have $A(2, -2), B(-2, 1)$ and $(5, 2)$
Now using distance formula we get

$$AB^2 = (2+2)^2 + (-2-1)^2$$

$$= 16 + 9 = 25$$

$$AB^2 = 25 \Rightarrow AB = 5.$$

Thus $AB = 5$.

Similarly $\quad BC^2 = (-2-5)^2 + (1-2)^2$

$$= 49 + 1 = 50$$

$$BC^2 = 50 \Rightarrow BC = 5\sqrt{2}$$

$$AC^2 = (2-5)^2 + (-2-2)^2$$

$$= 9 + 16 = 25$$

$$AC^2 = 25 \Rightarrow AC = 5$$

Clearly $\quad AB^2 + AC^2 = BC^2$

$$25 + 25 = 50$$

Hence the triangle is right angled,

$$\text{Area of } \Delta ABC = \frac{1}{2} \times \text{Base} \times \text{Height}$$

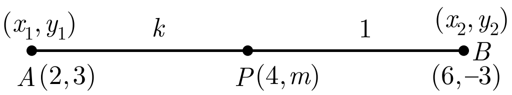
$$= \frac{1}{2} \times 5 \times 5 = \frac{25}{2} \text{ sq unit.}$$

THREE MARKS QUESTIONS

89. Find the ratio in which $P(4, m)$ divides the segment joining the points $A(2, 3)$ and $B(6, -3)$. Hence find m.

Ans : [Board 2018]

Let $P(x, y)$ be the point which divide AB in $k:1$ ratio.



Now $\quad x = \dfrac{m_1x_2 + m_2x_1}{m_1 + m_2}$

$$4 = \frac{k(6) + 1(2)}{k+1}$$

$$4k + 4 = 6k + 2$$

$$6k - 4k = 4 - 2$$

$$2k = 2 \Rightarrow k = 1$$

Thus point P divides the line segment AB in $1:1$ ratio.

Now $\quad y = \dfrac{m_1y_2 + m_2y_1}{m_1 + m_2}$

$$m = \frac{1 \times (-3) + 1(3)}{1+1}$$

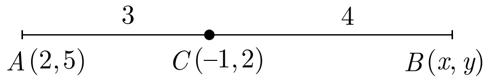
$$= \frac{-3+3}{2} = 0$$

Thus $m = 0$.

90. If the point $C(-1, 2)$ divides internally the line segment joining $A(2, 5)$ and $B(x, y)$ in the ratio 3 :4 find the coordinates of B.

Ans : [Board 2020 Delhi Standard]

From the given information we have drawn the figure as below.



Using section formula,

$$-1 = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}$$

$$-1 = \frac{3 \times x + 4 \times 2}{3+4} = \frac{3x+8}{7}$$

$$3x + 8 = -7$$

$$3x = -15 \Rightarrow x = -5$$

and $\quad 2 = \dfrac{m_1y_2 + m_2y_1}{m_1 + m_2}$

$$2 = \frac{3y + 4 \times 5}{3+4} = \frac{3y+20}{7}$$

$$3y + 20 = 14$$

$$3y = 14 - 20 = -6$$

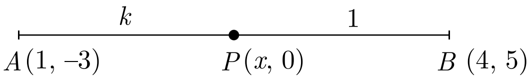
$$y = -2$$

Hence, the coordinates of $B(x, y)$ is $(-5, -2)$.

91. Find the ratio in which the segment joining the points $(1, -3)$ and $(4, 5)$ is divided by x-axis? Also find the coordinates of this point on x-axis.

Ans : [Board 2019 Delhi]

Let the required ratio be $k:1$ and the point on x-axis be $(x, 0)$.



Here, $(x_1, y_1) = (1, -3)$

and $(x_2, y_2) = (4, 5)$

Using section formula y coordinate, we obtain,

$$y = \frac{my_2 + ny_1}{m+n}$$

$$0 = \frac{k \times 5 + 1 \times 1(-3)}{k+1}$$

$$0 = 5k - 3$$

$$5k = 3 \Rightarrow k = \frac{3}{5}$$

Hence, the required ratio is $\frac{3}{5}$ i.e $3:5$.

Now, again using section formula for x, we obtain

$$x = \frac{mx_2 + nx_1}{m+n}$$

$$x = \frac{k \times (4) + 1 \times 1}{k+1}$$

$$= \frac{\frac{3}{5}(4) + 1}{\frac{3}{5} + 1} = \frac{12+5}{3+5} = \frac{17}{8}$$

Co-ordinate of P is $\left(\frac{17}{8}, 0\right)$.

92. Find the point on y-axis which is equidistant from the points $(5, -2)$ and $(-3, 2)$.

Ans : [Board 2019 Delhi]

We have point $A = (5, -2)$ and $B = (-3, 2)$

Let $C(0, a)$ be point on y-axis.

According to question, point C is equidistant from A and B.

Thus $AC = BC$

Using distance formula we have

$$\sqrt{(0-5)^2 + (a+2)^2} = \sqrt{(0+3)^2 + (a-2)^2}$$

$$\sqrt{25 + a^2 + 4 + 4a} = \sqrt{9 + a^2 + 4 - 4a}$$

$$25 + a^2 + 4 + 4a = 9 + a^2 + 4 - 4a$$

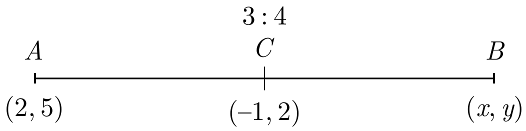
$$8a = -16 \Rightarrow a = -2$$

Hence, point on y-axis is $(0 - 2)$.

93. If the point $C(-1, 2)$ divides internally the line segment joining the points $A(2, 5)$ and $B(x, y)$ in the ratio $3:4$, find the value of $x^2 + y^2$.

Ans : [Board Term-2 Foreign 2016]

As per question, line diagram is shown below.



We have $\frac{AC}{BC} = \frac{3}{4}$

Applying section formula for x co-ordinate,

$$-1 = \frac{3x + 4(2)}{3+4}$$

$$-7 = 3x + 8 \Rightarrow x = -5$$

Similarly applying section formula for y co-ordinate,

$$2 = \frac{3y + 4(5)}{3+4}$$

$$14 = 3y + 20 \Rightarrow y = -2$$

Thus (x, y) is $(-5, -2)$.

Now $x^2 + y^2 = (-5)^2 + (-2)^2$

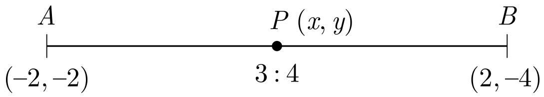
$$= 25 + 4 = 29$$

94. If the co-ordinates of points A and B are $(-2, -2)$ and $(2, -4)$ respectively, find the co-ordinates of P such that $AP = \frac{3}{7}AB$, where P lies on the line segment AB.

Ans : [Board Term-2 OD 2017]

We have $AP = \frac{3}{7}AB \Rightarrow AP:PB = 3:4$

As per question, line diagram is shown below.



Section formula :

$$x = \frac{mx_2 + nx_1}{m+n} \text{ and } y = \frac{my_2 + ny_1}{m+n}$$

Applying section formula we get

$$x = \frac{3 \times 2 + 4 \times (-2)}{3+4} = -\frac{2}{7}$$

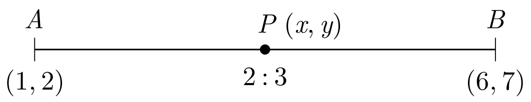
$$y = \frac{3 \times (-4) + 4 \times (-2)}{3+4} = -\frac{20}{7}$$

Hence P is $\left(-\frac{2}{7}, -\frac{20}{7}\right)$.

95. Find the co-ordinate of a point P on the line segment joining $A(1,2)$ and $B(6,7)$ such that $AP = \frac{2}{5}AB$.

Ans : [Board Term-2 OD 2015]

As per question, line diagram is shown below.



We have $AP = \frac{2}{5}AB \Rightarrow AP:PB = 2:3$

Section formula :

$$x = \frac{mx_2 + nx_1}{m+n} \text{ and } y = \frac{my_2 + nx_1}{m+n}$$

Applying section formula we get

$$x = \frac{2 \times 6 + 3 \times 1}{2+3} = \frac{12+3}{5} = 3$$

and $$y = \frac{2 \times 7 + 3 \times 2}{2+3} = \frac{14+6}{5} = 4$$

Thus $P(x,y) = (3,4)$

96. If the distance of $P(x,y)$ from $A(6,2)$ and $B(-2,6)$ are equal, prove that $y = 2x$.

Ans : [Board Term-2, 2015]

We have $P(x,y), A(6,2), B(-2,6)$

Now $PA = PB$

$$PA^2 = PB^2$$

$$(x-6)^2 + (y-2)^2 = (x+2)^2 + (y-6)^2$$

$$-12x + 36 - 4y + 4 = 4x + 4 - 12y + 36$$

$$-12x - 4y = 4x - 12y$$

$$12y - 4y = 4x + 12x$$

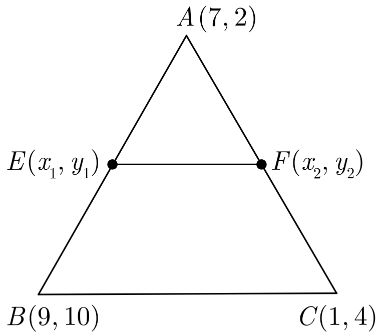
$$8y = 16x$$

$$y = 2x$$ Hence Proved

97. The co-ordinates of the vertices of ΔABC are $A(7,2)$, $B(9,10)$ and $C(1,4)$. If E and F are the mid-points of AB and AC respectively, prove that $EF = \frac{1}{2}BC$.

Ans : [Board Term-2 2015]

Let the mid-points of AB and AC be $E(x_1, y_1)$ and $F(x_2, y_2)$. As per question, triangle is shown below.



Co-ordinates of point E,

$$(x_1, y_1) = \left(\frac{9+7}{2}, \frac{10+2}{2}\right) = (8,6)$$

Co-ordinates of point F,

$$(x_2, y_2) = \left(\frac{7+1}{2}, \frac{2+4}{2}\right) = (4,3)$$

Length, $$EF = \sqrt{(8-4)^2 + (6-3)^2}$$

$$= \sqrt{4^2 + 3^2}$$

$$= 5 \text{ units} \quad ...(1)$$

Length $$BC = \sqrt{(9-1)^2 + (10-4)^2}$$

$$= \sqrt{8^2 + 6^2}$$

$$= 10 \text{ units} \quad ...(2)$$

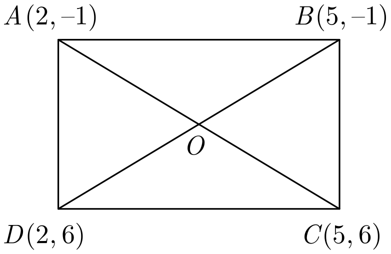
From equation (1) and (2) we get

$$EF = \frac{1}{2}BC$$ Hence proved.

98. Prove that the diagonals of a rectangle $ABCD$, with vertices $A(2,-1), B(5,-1), C(5,6)$ and $D(2,6)$ are equal and bisect each other.

Ans : [Board Term-2 2014]

As per question, rectangle $ABCD$, is shown below.



$$\text{Now} \quad AC = \sqrt{(5-2)^2 + (6+1)^2} = \sqrt{3^2 + 7^2} = \sqrt{9+49} = \sqrt{58}$$

g149

$$BD = \sqrt{(5-2)^2 + (-1-6)^2} = \sqrt{3^2 + 7^2} = \sqrt{9+49} = \sqrt{58}$$

Since $AC = BD = \sqrt{58}$ the diagonals of rectangle $ABCD$ are equal.

Mid-point of AC,

$$= \left(\frac{2+5}{2}, \frac{-1+6}{2}\right) = \left(\frac{7}{2}, \frac{5}{2}\right)$$

Mid-point of BD,

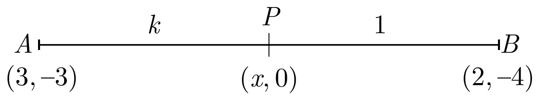
$$= \left(\frac{2+5}{2}, \frac{6-1}{2}\right) = \left(\frac{7}{2}, \frac{5}{2}\right)$$

Since the mid-point of diagonal AC and mid-point of diagonal BD is same and equal to $\left(\frac{7}{5}, \frac{5}{2}\right)$. Hence they bisect each other.

99. Find the ratio in which the line segment joining the points $A(3, -3)$ and $B(-2, 7)$ is divided by x-axis. Also find the co-ordinates of point of division.

Ans : [Board Term-2 Delhi 2014]

We know that y co-ordinate of any point on the $x-$axis will be zero. Let $(x, 0)$ be point on x axis which cut the line. As per question, line diagram is shown below.



Let the ratio be $k:1$. Using section formula for y co-ordinate we have

$$0 = \frac{1(-3) + k(7)}{1+k}$$

g150

$$k = \frac{3}{7}$$

Using section formula for x co-ordinate we have

$$x = \frac{1(3) + k(-2)}{1+k} = \frac{3 - 2 \times \frac{3}{7}}{1 + \frac{3}{7}} = \frac{3}{2}$$

Thus co-ordinates of point are $\left(\frac{3}{2}, 0\right)$.

100. Find the ratio in which $(11, 15)$ divides the line segment joining the points $(15, 5)$ and $(9, 20)$.

Ans : [Board Term-2 2014]

Let the two points $(15, 5)$ and $(9, 20)$ are divided in the ratio $k:1$ by point $P(11, 15)$.
Using Section formula, we get

$$x = \frac{m_2 x_1 + m_1 x_2}{m_2 + m_1}$$

g151

$$11 = \frac{1(15) + k(9)}{1+k}$$

$$11 + 11k = 15 + 9k$$

$$k = 2$$

Thus ratio is 2 : 1.

101. Find the point on y-axis which is equidistant from the points $(5, -2)$ and $(-3, 2)$.

Ans : [Board Term-2 2014, Delhi 2012]

Let point be $(0, y)$.

$$5^2 + (y+2)^2 = (3)^2 + (y-2)^2$$

g152

$$\text{or,} \quad y^2 + 25 + 4y + 4 = 9 - 4y + 4$$

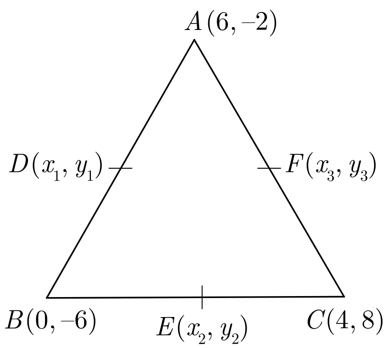
$$8y = -16 \text{ or, } y = -2$$

or, Point $(0, -2)$

102. The vertices of ΔABC are $A(6, -2)$, $B(0, -6)$ and $C(4, 8)$. Find the co-ordinates of mid-points of AB, BC and AC.

Ans : [Board Term-2, 2014]

Let mid-point of AB, BC and AC be $D(x_1, y_1)$, $E(x_2, y_2)$ and $F(x_2, y_3)$. As per question, triangle is shown below.



Using section formula, the co-ordinates of the points D, E, F are

For D, $x_1 = \frac{6+0}{2} = 3$

$y_1 = \frac{-2-6}{2} = -4$

For E, $x_2 = \frac{0+4}{2} = 2$

$y_2 = \frac{-6+8}{2} = 1$

For F, $x_3 = \frac{4+6}{2} = 5$

$y_3 = \frac{-2+8}{2} = 3$

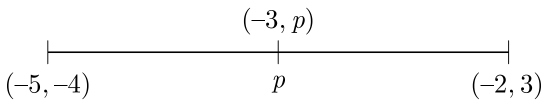
g153

The co-ordinates of the mid-points of AB, BC and AC are $D(3,-4)$, $E(2,1)$ and $F(5,3)$ respectively.

103. Find the ratio in which the point $(-3, p)$ divides the line segment joining the points $(-5,-4)$ and $(-2,3)$. Hence find the value of p.

Ans : [Board Term-2, 2012]

As per question, line diagram is shown below.



Let $X(-3, p)$ divides the line joining of $A(-5,-4)$ and $B(-2,3)$ in the ratio $k:1$.

The co-ordinates of p are $\left[\frac{-2k-5}{k+1}, \frac{3k-4}{k+1}\right]$

But co-ordinates of P are $(-3, p)$. Therefore we get

$$\frac{-2k-5}{k+1} = -3 \Rightarrow k = 2$$

g154

and $$\frac{3k-4}{k+1} = p$$

Substituting $k = 2$ gives

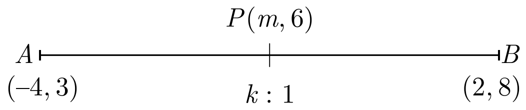
$$p = \frac{2}{3}$$

Hence ratio of division is 2:1 and $p = \frac{2}{3}$

104. Find the ratio in which the point $p(m, 6)$ divides the line segment joining the points $A(-4,3)$ and $B(2,8)$. Also find the value of m.

Ans : [Board Term-2, 2012]

As per question, line diagram is shown below.



Let the ratio be $k:1$.

Using section formula, we have

$$m = \frac{2k+(-4)}{k+1} \quad (1)$$

$$6 = \frac{8k+3}{k+1} \quad (2)$$

$$8k+3 = 6k+6$$

$$2k = 3 \Rightarrow k = \frac{3}{2}$$

g155

Thus ratio is $\frac{3}{2}:1$ or 3:2.

Substituting value of k in (1) we have

$$m = \frac{2\left(\frac{3}{2}\right)+(-4)}{\frac{3}{2}+1} = \frac{3-4}{\frac{5}{2}} = \frac{-1}{\frac{5}{2}} = \frac{-2}{5}$$

105. If $A(4,-1), B(5,3), C(2,y)$ and $D(1,1)$ are the vertices of a parallelogram $ABCD$, find y.

Ans : [Board Term-2, 2012]

Diagonals of a parallelogram bisect each other. Mid-points of AC and BD are same.

Thus $$\left(3, \frac{-1+y}{2}\right) = (3,2)$$

g156

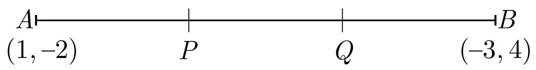
$$\frac{-1+y}{2} = 2 \Rightarrow y = 5$$

106. Find the co-ordinates of the points of trisection of the line segment joining the points $A(1,-2)$ and $B(-3,4)$.

Ans : [Board Term-2, 2012]

Let $P(x_1, y_1), Q(x_2, y_2)$ divides AB into 3 equal parts. Thus P divides AB in the ratio of 1:2.

As per question, line diagram is shown below.



Now $$x_1 = \frac{1(-3)+2(1)}{1+3} = \frac{-3+2}{3} = \frac{-1}{3}$$

$$y_1 = \frac{1(4)+2(-2)}{1+2} = \frac{4-4}{3} = 0$$

g157

Co-ordinates of P is $\left(-\frac{1}{3}, 0\right)$.

Here Q is mid-point of PB.

Thus $$x_2 = \frac{-\frac{1}{3}+(-3)}{2} = \frac{-10}{6} = \frac{-5}{3}$$

$$y_2 = \frac{0+4}{2} = 2$$

Thus co-ordinates of Q is $\left(-\frac{5}{2}, 2\right)$.

107. If (a, b) is the mid-point of the segment joining the points $A(10,-6)$ and $B(k,4)$ and $a-2b=18$, find the value of k and the distance AB.

Ans : [Board Term-2, 2012]

We have $A(10,-6)$ and $B(k,4)$.
If $P(a,b)$ is mid-point of AB, then we have

$$(a,b) = \left(\frac{k+10}{2}, \frac{-6+4}{2}\right)$$

g158

$$a = \frac{k+10}{2} \text{ and } b = -1$$

From given condition we have

$$a-2b = 18$$

Substituting value $b=-1$ we obtain

$$a+2 = 18 \Rightarrow a = 16$$

$$a = \frac{k+10}{2} = 16 \Rightarrow k = 22$$

$$P(a,b) = (16,1)$$

$$AB = \sqrt{(22-10)^2+(4+6)^2}$$

$$= 2\sqrt{61} \text{ units}$$

108. Find the ratio in which the line $2x+3y-5=0$ divides the line segment joining the points $(8,-9)$ and $(2,1)$. Also find the co-ordinates of the point of division.

Ans : [Board Term-2, 2012]

Let a point $P(x,y)$ on line $2x+3y-5=0$ divides AB in the ratio $k:1$.

Now $$x = \frac{2k+8}{k+1}$$

and $$y = \frac{k-9}{k+1}$$

Substituting above value in line $2x+3y-5=0$ we have

$$2\left(\frac{2k+8}{k+1}\right)+3\left(\frac{k-9}{k+1}\right)-5 = 0$$

g159

$$4k+16+3k-27-5k-5 = 0$$

$$2k-16 = 0$$

$$k = 8$$

Thus ratio is 8 : 1.

Substituting the value $k=8$ we get

$$x = \left(\frac{2\times 8+8}{8+1}\right) = \frac{8}{3}$$

$$y = \left(\frac{8-9}{8+1}\right) = -\frac{1}{9}$$

Thus $$P(x,y) = \left(\frac{8}{3}, -\frac{1}{9}\right)$$

109. Find the area of the rhombus of vertices $(3,0),(4,5)$, $(-1,4)$ and $(-2,-1)$ taken in order.

Ans : [Board Term-2, 2012]

We have $A(3,0)$, $B(4,5)$, $C(-1,4)$, $D(-2,-1)$

Diagonal AC, $$d_1 = \sqrt{(3+1)^2+(0-4)^2}$$

$$= \sqrt{16+16} = \sqrt{32}$$

g160

$$= \sqrt{16\times 2} = 4\sqrt{2}$$

Diagonal BD, $$d_2 = \sqrt{(4+2)^2+(5+1)^2}$$

$$= \sqrt{36+36} = \sqrt{72}$$

$$= \sqrt{36\times 2} = 6\sqrt{2}$$

Area of rhombus $$= \frac{1}{2}\times d_1 \times d_2$$

$$= \frac{1}{2}4\sqrt{2}\times 6\sqrt{2}$$

$$= 24 \text{ sq. unit.}$$

110. Find the ratio in which the line joining points $(a+b, b+a)$ and $(a-b, b-a)$ is divided by the point (a,b).

Ans : [Board Term-2, 2013]

Let $A(a+b,b+a), B(a-b,b-a)$ and $P(a,b)$ and P divides AB in $k:1$, then we have

$$a = \frac{k(a-b)+1(a+b)}{k+1}$$

g161

$$a(k+1) = k(a-b)+a+b$$

$$ak+a = ak-bk+a+b$$

$$bk = b$$

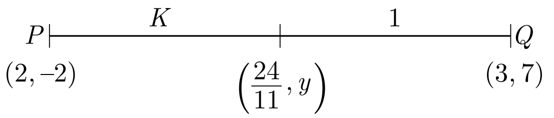
$$k = 1$$

Thus (a, b) divides $A(a+b, b+a)$ and $B(a-b, b-a)$ in 1:1 internally.

111. In what ratio does the point $(\frac{24}{11}, y)$ divides the line segment joining the points $P(2, -2)$ and $Q(3, 7)$? Also find the value of y.

Ans : [Board Term-2 SQP 2012]

As per question, line diagram is shown below.



Let $P(\frac{24}{11}, y)$ divides the segment joining the points $P(2, -2)$ and $Q(3, 7)$ in ratio $k:1$.

Using intersection formula $x = \frac{mx_2 + nx_1}{m+n}$ we have

$$\frac{3k+2}{k+1} = \frac{24}{11}$$

$$33k + 22 = 24k + 24$$

$$9k = 2 \Rightarrow k = \frac{2}{9}$$

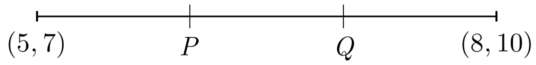
Hence, $$y = \frac{-18+14}{11} = -\frac{4}{11}$$

112. Find the co-ordinates of the points which divide the line segment joining the points $(5, 7)$ and $(8, 10)$ in 3 equal parts.

Ans : [Board Term-2 OD Compt. 2017]

Let $P(x_1, y_2)$ and $Q(x_2, y_2)$ trisect AB. Thus P divides AB in the ratio 1:2

As per question, line diagram is shown below.



Using section formula we have,

Now $$x = \frac{1(8)+2(5)}{3} = 6$$

$$y = \frac{1(10)+2(7)}{3} = 8$$

Thus $P(x_1, y_1)$ is $P(6, 8)$. Since Q is the mid point of PB, we have

$$x_1 = \frac{6+8}{2} = 7$$

$$y_1 = \frac{8+10}{2} = 9$$

Thus $Q(x_2, y_2)$ is $Q(7, 9)$

113. Find the co-ordinates of a point on the $x-$axis which is equidistant from the points $A(2, -5)$ and $B(-2, 9)$.

Ans : [Board Term-2 Delhi Compt. 2017]

Let the point P on the x axis be $(x, 0)$. Since it is equidistant from the given points $A(2, -5)$ and $B(-2, 9)$

$$PA = PB$$

$$PA^2 = PB^2$$

$$(x-2)^2 + [0-(-5)]^2 = (x-(-2))^2 + (0-9)^2$$

$$x^2 - 4x + 4 + 25 = x^2 + 4x + 4 + 81$$

$$-4x + 29 = 4x + 85$$

$$x = -\frac{56}{8} = -7$$

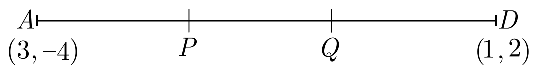
Hence the point on x axis is $(-7, 0)$

114. The line segment joining the points $A(3, -4)$ and $B(1, 2)$ is trisected at the points P and Q. Find the coordinate of the PQ.

Ans : [Delhi Compt. Set-II, 2017]

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ trisect AB. Thus P divides AB in the ratio 1:2.

As per question, line diagram is shown below.



Using intersection formula

$$x = \frac{1 \times 1 + 2 \times 3}{1+2} = \frac{7}{3}$$

$$y = \frac{1 \times 2 + 2 \times -4}{1+2} = -2$$

Hence point P is $\left(\frac{7}{3}, -2\right)$

115. Show that ΔABC with vertices $A(-2, 0), B(0, 2)$ and $C(2, 0)$ is similar to ΔDEF with vertices

$D(-4,0)$, $F(4,0)$ and $E(0,4)$.

Ans : [Board Term-2 Delhi 2017, Foreign 2017]

Using distance formula

$$AB = \sqrt{(0+2)^2+(2-0)^2} = \sqrt{4+4}$$
$$= 2\sqrt{2} \text{ units}$$
$$BC = \sqrt{(2-0)^2+(0-2)^2} = \sqrt{4+4}$$
$$= 2\sqrt{2} \text{ units}$$
$$CA = \sqrt{(-2-2)^2+(0-0)^2} = \sqrt{16}$$
$$= 4 \text{ units}$$

and
$$DE = \sqrt{(0+4)^2+(4-0)^2} = \sqrt{32}$$
$$= 4\sqrt{2} \text{ units}$$
$$EF = \sqrt{(4-0)^2+(0-4)^2} = \sqrt{32}$$
$$= 4\sqrt{2} \text{ units}$$
$$FD = \sqrt{(-4-4)^2+(0-0)^2} = \sqrt{64}$$
$$= 8 \text{ units}$$
$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$
$$\frac{2\sqrt{2}}{4\sqrt{2}} = \frac{2\sqrt{2}}{4\sqrt{2}} = \frac{4}{8} = \frac{1}{2}$$

Since ratio of the corresponding sides of two similar Δs is equal, we have

$$\Delta ABC \sim \Delta DEF$$ Hence Proved.

116. Find the co-ordinates of the point on the $y-$ axis which is equidistant from the points $A(5,3)$ and $B(1,-5)$

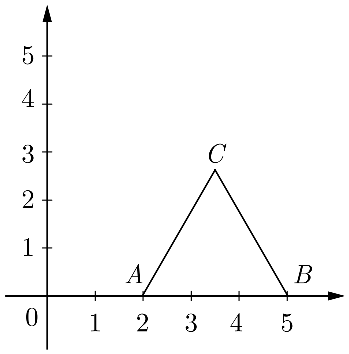
Ans : [Board Term-2 OD Compt. 2017]

Let the points on y-axis be $P(0,y)$

Now
$$PA = PB$$
$$PA^2 = PB^2$$
$$(0-5)^2+(y-3)^2 = (0-1)^2+(y+5)^2$$
$$5^2+y^2-6y+9 = 1+y^2+10y+25$$
$$16y = 8 \Rightarrow y = \frac{1}{2}$$

Hence point on y-axis is $(0,\frac{1}{2})$.

117. In the given figure ΔABC is an equilateral triangle of side 3 units. Find the co-ordinates of the other two vertices.



Ans : [Board Term-2 Foreign 2017]

The co-ordinates of B will be $(2+3,0)$ or $.(5,0)$

Let co-ordinates of C be (x,y). Since triangle is equilateral, we have

$$AC^2 = BC^2$$
$$(x-2)^2(y-0)^2 = (x-5)^2+(y-0)^2$$
$$x^2+4-4x+y^2 = x^2+25-10x+y^2$$
$$6x = 21$$
$$x = \frac{7}{2}$$

and
$$(x-2)^2+(y-0)^2 = 9$$
$$\left(\frac{7}{2}-2\right)^2+y^2 = 9$$
$$\frac{9}{4}+y^2 = 9 \text{ or, } y^2 = 9-\frac{9}{4}$$
$$y^2 = \frac{27}{4} = \frac{3\sqrt{3}}{2}$$

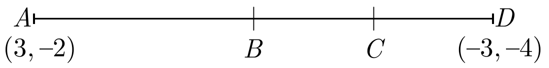
Hence C is $\left(\frac{7}{2}, \frac{3\sqrt{3}}{2}\right)$.

118. Find the co-ordinates of the points of trisection of the line segment joining the points $(3,-2)$ and $(-3,-4)$.

Ans : [Board Term-2 Foreign 2017]

Let $P(x_1,y_1)$ and $Q(x_2,y_2)$ trisect the line joining $A(3,-2)$ and $B(-3,-4)$.

As per question, line diagram is shown below.



Thus P divides AB in the ratio 1:2.

Using intersection formula $x = \frac{mx_2+nx_1}{m+n}$ and $y = \frac{my_2+ny_1}{m+n}$

$$x_1 = \frac{1(-3)+2(3)}{1+2} = 1$$

and $$y_1 = \frac{1(-4)+2(-2)}{1+2} = -\frac{8}{3}$$

Thus we have $x = 1$ and $y = -\frac{8}{3}$

Since Q is at the mid-point of PB, using mid-point formula

$$x_2 = \frac{1-3}{2} = -1$$

and $$y_2 = \frac{-\frac{8}{3}+(-4)}{2} = -\frac{10}{3}$$

Hence the co-ordinates of P and Q are $(1, -\frac{8}{3})$ and $(-1, -\frac{10}{3})$

119. If the distances of $P(x, y)$ from $A(5, 1)$ and $B(-1, 5)$ are equal, then prove that $3x = 2y$.

Ans : [Board Term-2 OD 2016]

Since $P(x, y)$ is equidistant from the given points $A(5, 1)$ and $B(-1, 5)$,

$$PA = PB$$

$$PA^2 = PB^2$$

Using distance formula,

$$(5-x)^2 + (1-y)^2 = (-1-x)^2 + (5-y)^2$$

$$(5-x)^2 + (1-y)^2 = (1+x)^2 + (5-y)^2$$

$$25 - 10x + 1 - 2y = 1 + 2x + 25 - 10y$$

$$-10x - 2y = 2x - 10y$$

$$8y = 12x$$

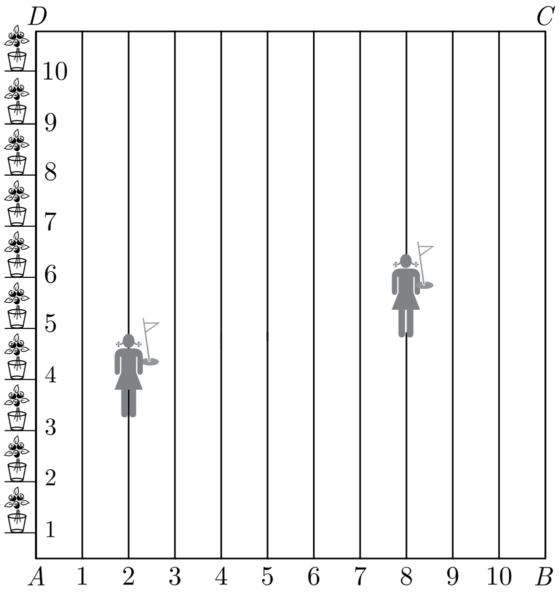
$$3x = 2y$$ Hence proved.

FOUR MARKS QUESTIONS

120. To conduct Sports Day activities, in your rectangular school ground $ABCD$, lines have been drawn with chalk powder at a distance of 1 m each. 100 flower pots have been placed at a distance of 1 m from each other along AD, as shown in Figure. Niharika runs ¼th the distance AD on the 2nd line and posts a green flag. Preet runs $\frac{1}{5}$th distance AD on the eighth line and posts a red flag.

(i) What is the distance between the two flags?

(ii) If Rashmi has to post a blue flag exactly half way between the line segment joining the two flags, where should she post the blue flag?



Ans : [Board 2020 Delhi Basic]

We assume A as origin (0, 0), AB as x-axis and AD as y-axis.

Niharika runs in the 2nd line with green flag and distance covered (parallel to AD),

$$= \frac{1}{4} \times 100 = 25 \text{ m}$$

Thus co-ordinates of green flag are (2, 25) and we label it as P i.e., $P(2, 25)$.

Similarly, Preet runs in the eighth line with red flag and distance covered (parallel to AD),

$$= \frac{1}{5} \times 100 = 20 \text{ m}$$

Co-ordinates of red flag are (8, 20) and we label it as Q i.e., $Q(8, 20)$

(i) Now, using distance formula, distance between green flag and red flag,

$$PQ = \sqrt{(8-2)^2 + (20-25)^2}$$

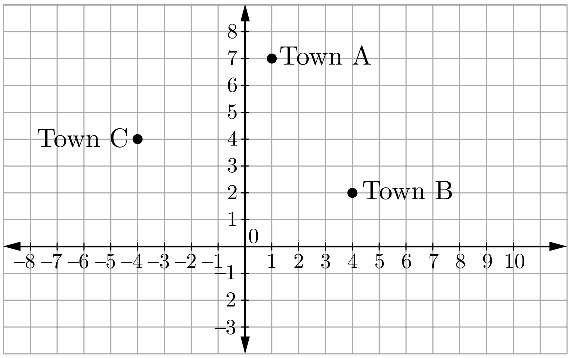
$$= \sqrt{6^2 + (-5)^2} = \sqrt{36+25}$$

$$= \sqrt{61} \text{ m}$$

(ii) Also, Rashmi has to post a blue flag the mid-point of PQ, therefore by using mid-point formula, we obtain $\left(\frac{2+8}{2}, \frac{25+20}{2}\right)$ i.e. $\left(5, \frac{45}{2}\right)$

Hence, the blue flag is in the fifth line, at a distance of $\frac{45}{2}$ i.e., 22.5 m along the direction parallel to AD.

121. Two friends Seema and Aditya work in the same office at Delhi. In the Christmas vacations, both decided to go to their hometown represented by Town A and Town B respectively in the figure given below. Town A and Town B are connected by trains from the same station C (in the given figure) in Delhi. Based on the given situation answer the following questions:



(i) Who will travel more distance, Seema or Aditya, to reach to their hometown?

(ii) Seema and Aditya planned to meet at a location D situated at a point D represented by the mid-point of the line joining the points represented by Town A and Town B. Find the coordinates of the point represented by the point D.

(iii) Find the area of the triangle formed by joining the points represented by A, B and C.

Ans : [Board 2020 SQP Standard]

From the given figure, the coordinates of points A, B and C are $(1,7),(4,2)$ and $(-4,4)$ respectively.

(i) Distance travelled by seema

$$CA = \sqrt{(-4-1)^2+(4-7)^2}$$
$$= \sqrt{(-5)^2+(-3)^2}$$
$$= \sqrt{25+9} \qquad = \sqrt{34}$$

g282

units

Thus distance travelled by seema is $\sqrt{34}$ units.

Similarly, distance travelled by Aditya

$$CB = \sqrt{(4+4)^2+(4-2)^2}$$
$$= \sqrt{8^2+2^2} = \sqrt{64+4}$$
$$= \sqrt{68} \text{ units}$$

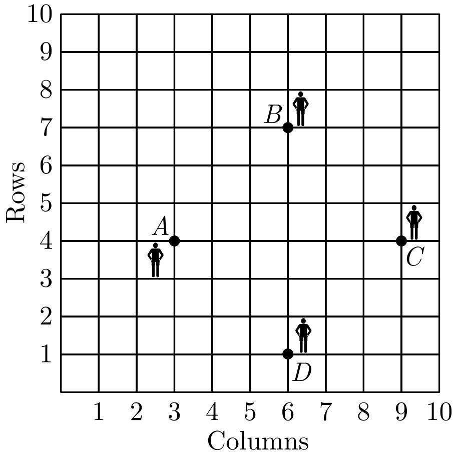
Distance travelled by Aditya is $\sqrt{68}$ units and Aditya travels more distance.

(ii) Since, D is mid-point of town A and town B

$$D = \left(\frac{1+4}{2}, \frac{7+2}{2}\right) = \left(\frac{5}{2}, \frac{9}{2}\right)$$

(iii) Removed from syllabus

122. In a classroom, 4 friends are seated at the points A, B, C, and D as shown in Figure. Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli, Don't you think $ABCD$ is a square? Chameli disagrees. Using distance formula, find which of them is correct.



Ans : [Board 2020 Delhi Basic]

Coordinates of points A, B, C, D are $A(3,4)$, $B(6,7)$, $C(9,4)$ and $D(6,1)$.

Distance formula, $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Now $$AB = \sqrt{(3-6)^2+(4-7)^2}$$
$$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2} \text{ units}$$

g283

$$BC = \sqrt{(6-9)^2+(7-4)^2}$$
$$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2} \text{ units}$$
$$CD = \sqrt{(9-6)^2+(4-1)^2}$$
$$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2} \text{ units}$$
$$DA = \sqrt{(6-3)^2+(1-4)^2}$$
$$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2} \text{ units}$$

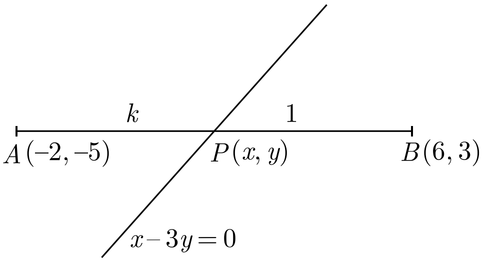
Now $$AC = \sqrt{(3-9)^2+(4-4)^2}$$
$$= \sqrt{36+0} = 6 \text{ units}$$
$$DB = \sqrt{(6-6)^2+(1-7)^2}$$
$$= \sqrt{0+36} = 6 \text{ units}$$

Since, $AB = BC = CD = DA$ and $AC = DB$, $ABCD$ is a square and Champa is right.

123. Find the ratio in which the line $x-3y=0$ divides the line segment joining the points $(-2,-5)$ and $(6,3)$. Find the coordinates of the point of intersection.

Ans : [Board 2019 OD]

Let $k:1$ be the ratio in which line $x-3y=0$ divides the line segment at $p(x, y)$.



g285

Using section formula, we get

$$x = \frac{mx_2 + nx_1}{m+n} = \frac{k \times 6 + 1 \times (-2)}{k+1}$$

$$x = \frac{6k-2}{k+1} \quad ...(1)$$

and $$y = \frac{my_2 + ny_1}{m+n} = \frac{k \times 3 + 1 \times (-5)}{k+1}$$

$$y = \frac{3k-5}{k+1} \quad ...(2)$$

The point $P(x, y)$ lies on the line, hence it satisfies the equation of the given line.

$$\frac{6k-2}{k+1} - 3\left(\frac{3k-5}{k+1}\right) = 0$$

$$6k - 2 - 3(3k-5) = 0$$

$$6k - 2 - 9k + 15 = 0$$

$$-3k + 13 = 0 \Rightarrow k = \frac{13}{3}$$

Hence, the required ratio is 13 : 3.

Now, substituting value of k in x and y, we get

$$x = \frac{6 \times \frac{13}{3} - 2}{\frac{13}{3} + 1} = \frac{78-6}{16} = \frac{72}{16} = \frac{9}{2}$$

$$y = \frac{3 \times \frac{13}{3} - 5}{\frac{13}{3} + 1} = \frac{8 \times 3}{16} = \frac{24}{16} = \frac{3}{2}$$

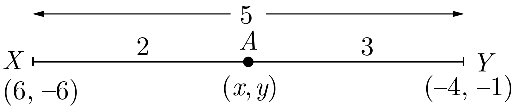
Hence, the co-ordinates of point of intersection

$$P(x, y) = \left(\frac{9}{2}, \frac{3}{2}\right)$$

124. Point A lies on the line segment XY joining $X(6, -6)$ and $Y(-4, -1)$ in such a way that $\frac{XA}{XY} = \frac{2}{5}$. If point A also lies on the line $3x + k(y+1) = 0$, find the value of k.

Ans : [Board 2019 OD]

As per given information in question we have drawn the figure given below.



We use section formula for point $A(x, y)$.

Here, $m_1 = 2$, $m_2 = 3$, $x_1 = 6$, $x_2 = -4$, $y_1 = -6$ and $y_2 = -1$

Now $$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} = \frac{2 \times (-4) + 3(6)}{2+3}$$

$$= \frac{-8+18}{5} = \frac{10}{5} = 2$$

and $$y = \frac{m_1y_2 + m_2y_2}{m_1 + m_2} = \frac{2 \times (-1) + 3(-6)}{2+3}$$

$$= \frac{-2-18}{5} = \frac{-20}{5} = -4$$

Hence, coordinates of point A is $(2, -4)$.

Since point A also lies on the line $3x + k(y+1) = 0$, its coordinates must satisfies this line.

Thus $$3(2) + k(-4+1) = 0$$

$$6 + (-3k) = 0$$

$$3k = 6 \Rightarrow k = 2$$

g286

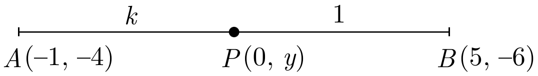
Hence, value of k is 2.

125. Find the ratio in which the y-axis divides the line segment joining the points $(-1, -4)$ and $(5, -6)$. Also find the coordinates of the point of intersection.

Ans : [Board 2019 OD]

Let points $P(0, y)$ divides the line joining the point $A(-1, -4)$ and $B(5, -6)$ in ratios $k:1$.

As per given information in question we have drawn figure below.



Section formula is given by

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} \quad ...(1)$$

and $$y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2} \quad ...(2)$$

Here, $m_1 = k$ and $m_2 = 1$,

$x_1 = -1$ and $x_2 = 5$

$y_1 = -4$ and $y_2 = -6$

g287

Now $$0 = \frac{k \times 5 + 1 \times (-1)}{k+1}$$

$$5k - 1 = 0 \Rightarrow k = \frac{1}{5}$$

Substitute value of k in eq (2), we get

$$y = \frac{k(-6) + 1(-4)}{k+1}$$

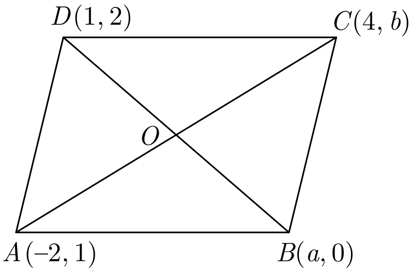
$$= \frac{\frac{1}{5}(-6) + 1(-4)}{\frac{1}{5} + 1} = \frac{-26}{6} = \frac{-13}{3}$$

Hence, value of k is $\frac{1}{5}$ and required point is $\left(0, -\frac{13}{3}\right)$

126. If $A(-2,1)$, $B(a,0)$, $C(4,b)$ and $D(1,2)$ are the vertices of a parallelogram $ABCD$, find the values of a and b. Hence find the lengths of its sides.

Ans : [Board 2018]

As per information given in question we have drawn the figure below.



Here $ABCD$ is a parallelogram and diagonals AC and BD bisect each other. Therefore mid point of BD is same as mid point of AC.

$$\left(\frac{a+1}{2}, \frac{2}{2}\right) = \left(\frac{-2+4}{2}, \frac{b+1}{2}\right)$$

g288

$$\frac{a+1}{2} = 1 \Rightarrow a = 1$$

and $$\frac{b+1}{2} = 1 \Rightarrow b = 1$$

Now $$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(1+2)^2 + (0-1)^2}$$

$$= \sqrt{9+1} = \sqrt{10} \text{ unit}$$

$$BC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(4-1)^2 + (1-0)^2}$$

$$= \sqrt{9+1} = \sqrt{10} \text{ unit}$$

Since $ABCD$ is a parallelogram,

$$AB = CD = \sqrt{10} \text{ unit}$$

$$BC = AD = \sqrt{10} \text{ unit}$$

Therefore length of sides are $\sqrt{10}$ units each.

127. If $P(9a-2, -b)$ divides the line segment joining $A(3a+1, -3)$ and $B(8a, 5)$ in the ratio 3:1. find the values of a and b.

Ans : [Board Term-2 SQP 2016]

Using section formula we have

$$9a - 2 = \frac{3(8a) + 1 + (3a+1)}{3+1} \quad ...(1)$$

$$-b = \frac{3(5) + 1(-3)}{3+1} \quad ...(2)$$

Form (2) $$-b = \frac{15-3}{4} = 3 \Rightarrow b = -3$$

From (1), $$9a - 2 = \frac{24a + 3a + 1}{4}$$

$$4(9a-2) = 27a + 1$$

g180

$$36a - 8 = 27a + 1$$

$$9a = 9 \Rightarrow a = 1$$

128. Find the coordinates of the point which divide the line segment joining $A(2,-3)$ and $B(-4,-6)$ into three equal parts.

Ans : [Board Term-2 SQP 2016]

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ trisect the line joining $A(3,-2)$ and $B(-3,-4)$.

As per question, line diagram is shown below.

P divides AB in the ratio of 1:2 and Q divides AB in the ratio 2:1.

g181

By section formula

$$x_1 = \frac{mx_2 + nx_1}{1+2} \text{ and } y = \frac{my_2 + ny_1}{m+n}$$

$$P(x_1, y_1) = \left(\frac{1(-4) + 2(2)}{2+1}, \frac{2(-6) + 1(-3)}{2+1}\right)$$

$$= \left(\frac{-4+4}{3}, \frac{-6-(-6)}{3}\right) = (0, -4)$$

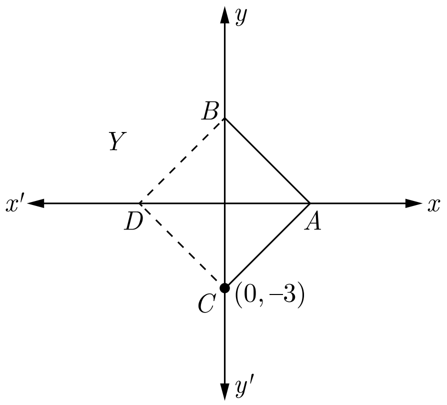
$$Q(x_2, y_2) = \left(\frac{2(-4) + 1(2)}{2+1}, \frac{2(-6) + 1(-3)}{2+1}\right)$$

$$= \left(\frac{-8+2}{3}, -\frac{12+(-3)}{3}\right) = (-2, -5)$$

129. The base BC of an equilateral triangle ABC lies on y-axis. The co-ordinates of point C are $(0,3)$. The origin is the mid-point of the base. Find the co-ordinates of the point A and B. Also find the co-ordinates of another point D such that $BACD$ is a rhombus.

Ans : [Board Term-2 Foreign 2015]

As per question, diagram of rhombus is shown below.



Co-ordinates of point B are $(0,3)$.

Thus $BC = 6$ unit

Let the co-ordinates of point A be $(x,0)$

Now $AB = \sqrt{x^2+9}$

Since $AB = BC$, thus we have

$$x^2+9 = 36$$

$$x^2 = 27 \Rightarrow x = \pm 3\sqrt{3}$$

Co-ordinates of point A is $(3\sqrt{3},0)$.

Since $ABCD$ is a rhombus,

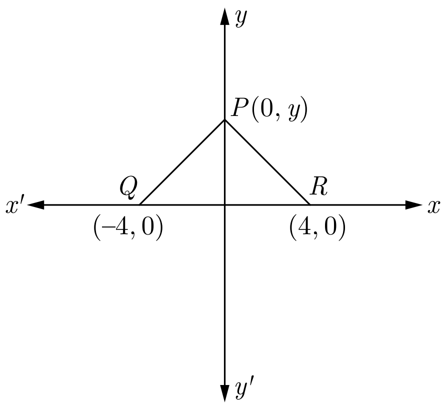
$$AB = AC = CD = DB$$

Thus co-ordinate of point D is $(-3\sqrt{3},0)$.

130. The base QR of an equilateral triangle PQR lies on x-axis. The co-ordinates of point Q are $(-4,0)$ and the origin is the mid-point of the base. find the co-ordinates of the point P and R.

Ans : [Board Term-2 Delhi 2017, Foreign 2015]

As per question, line diagram is shown below.



Co-ordinates of point R is $(4,0)$.

Thus $QR = 8$ units

Let the co-ordinates of point P be $(0,y)$

Since $PQ = QR$

$$(-4-0)^2+(0-y)^2 = 64$$

$$16+y^2 = 64$$

$$y = \pm 4\sqrt{3}$$

Coordinates of P are $(0,4\sqrt{3})$ or $(0,-4\sqrt{3})$

131. The vertices of quadrilateral $ABCD$ are $A(5,-1)$, $B(8,3)$, $C(4,0)$ and $D(1,-4)$. Prove that $ABCD$ is a rhombus.

Ans : [Board Term-2 Delhi 2015]

The vertices of the quadrilateral $ABCD$ are $A(5,-1)$, $B(8,3)$, $C(4,0)\,D(1,-4)$.

Now $AB = \sqrt{(8-5)^2+(3+1)^2}$

$= \sqrt{3^2+4^2} = 5$ units

$BC = \sqrt{(8-4)^2+(3-0)^2}$

$= \sqrt{4^2+3^2} = 5$ units

$CD = \sqrt{(4-1)^2+(0+4)^2}$

$= \sqrt{(3)^2+(4)^2} = 5$ units

$AD = \sqrt{(5-1)^2+(-1+4)^2}$

$= \sqrt{(4)^2+(3)^2} = 5$ units

Diagonal, $AC = \sqrt{(5-4)^2+(-1-0)^2}$

$= \sqrt{1^2+1^2} = \sqrt{2}$ units

Diagonal $BD = \sqrt{(8-1)^2+(3+4)^2}$

$= \sqrt{(7)^2+(7)^2} = 7\sqrt{2}$ units

As the length of all the sides are equal but the length of the diagonals are not equal. Thus $ABCD$ is not square but a rhombus.

132. The co-ordinates of vertices of ΔABC are $A(0,0)$, $B(0,2)$ and $C(2,0)$. Prove that ΔABC is an isosceles triangle. Also find its area.

Ans : [Board Term-2 Delhi 2014]

Using distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ we have

$$AB = \sqrt{(0-0)^2 + (0-2)^2} = \sqrt{4} = 2$$
$$AC = \sqrt{(0-2)^2 + (0-0)^2} = \sqrt{4} = 2$$
$$BC = \sqrt{(0-2)^2 + (2-0)^2} = \sqrt{4+4} = 2\sqrt{2}$$

Clearly, $AB = AC \neq BC$

Thus ΔABC is an isosceles triangle.

Now, $AB^2 + AC^2 = 2^2 + 2^2 = 4 + 4 = 8$

also, $BC^2 = (2\sqrt{2})^2 = 8$

$$AB^2 + AC^2 = BC^2$$

Thus ΔABC is an isosceles right angled triangle.

Now, area of ΔABC

$$\Delta_{ABC} = \frac{1}{2}\text{base} \times \text{height}$$

g193

$$= \frac{1}{2} \times 2 \times 2$$
$$= 2 \text{ sq. units.}$$
$$= \frac{1}{2}[3 \times (-1) + 7 \times 2 + 5 \times (-1)]$$
$$= \frac{1}{2}[-3 + 14 - 5]$$
$$= 3 \text{ units}$$

Area $\square_{ABCD} = \frac{5}{2} + 3 = \frac{11}{2}$ sq. units.

133. Find the ratio is which the line segment joining the points $A(3,-3)$ and $B(-2,7)$ is divided by x-axis. Also find the co-ordinates of the point of division.

Ans : [Board Term-2 OD 2014]

We have $A(3,-3)$ and $B(-2,7)$.
At any point on x-axis y-coordinate is always zero.
So, let the point be $(x,0)$ that divides line segment AB in ratio $k:1$.

Now $$(x,\ 0) = \left(\frac{-2k+3}{k+1}, \frac{7k-3}{k+1}\right)$$

g200

$$\frac{7k-3}{k+1} = 0$$
$$7k - 3 = 0 \Rightarrow k = \frac{3}{7}$$

The line is divided in the ratio of 3 : 7.

Now $$\frac{-2k+3}{k+1} = x$$

$$\frac{-2 \times \frac{3}{7} + 3}{\frac{3}{7} + 1} = x$$
$$\frac{-6+21}{3+7} = x$$
$$\frac{15}{10} = x \Rightarrow x = \frac{3}{2}$$

The coordinates of the point is $\left(\frac{3}{2}, 0\right)$.

134. Determine the ratio in which the straight line $x - y - 2 = 0$ divides the line segment joining $(3,-1)$ and $(8,9)$.

Ans : [Board Term-2, 2012]

Let co-ordinates of P be (x_1, y_1) and it divides line AB in the ratio $k:1$.

Now $$x_1 = \frac{8k+3}{k+1}$$

g201

$$y_1 = \frac{9k-1}{k+1}$$

Since point $P(x_1, y_1)$ lies on line $x - y - 2 = 0$, so co-ordinates of P must satisfy the equation of line.

Thus $$\frac{8k+3}{k+1} - \frac{9k-1}{k+1} - 2 = 0$$
$$8k + 3 - 9k + 1 - 2k - 2 = 0$$
$$-3k + 2 = 0 \Rightarrow k = \frac{2}{3}$$

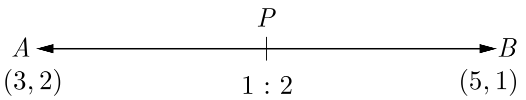
So, line $x - y - 2 = 0$ divides AB in the ratio 2:3

135. The line segment joining the points $A(3,2)$ and $B(5,1)$ is divided at the point P in the ratio 1:2 and P lies on the line $3x - 18y + k = 0$. Find the value of k.

Ans : [Board Term-2 Delhi 2012]

Let co-ordinates of P be (x_1, y_1) and it divides line AB in the ratio $1:2$.

g202



$$x_1 = \frac{mx_2 + nx_1}{m+n} = \frac{1 \times 5 + 2 \times 3}{1+2} = \frac{11}{3}$$
$$y_2 = \frac{my_2 + ny_1}{m+n} = \frac{1 \times 2 + 2 \times 2}{1+2} = \frac{5}{3}$$

Since point $P(x_1, y_1)$ lies on line ,$3x - 18y + k = 0$, so co-ordinates of P must satisfy the equation of line.

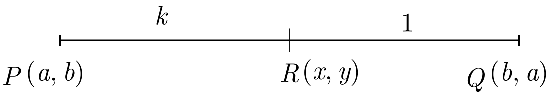
$$3 \times \frac{11}{3} - 18 \times \frac{5}{3} + k = 0$$
$$k = 19$$

136. If $R(x, y)$ is a point on the line segment joining the points $P(a, b)$ and $Q(b, a)$, then prove that $x + y = a + b$.

Ans : [Board Term-2, 2012 Set (28)]

As per question line is shown below.

g203



Let point $R(x, y)$ divides the line joining P and Q in the ratio $k:1$, then we have

$$x = \frac{kb + a}{k + 1}$$

and $$y = \frac{ka + b}{k + 1}$$

Adding, $$x + y = \frac{kb + a + ka + b}{k + 1}$$

$$= \frac{k(a + b) + (a + b)}{k + 1}$$

$$= \frac{(k + 1)(a + b)}{k + 1} = a + b$$

$$x + y = a + b$$ Hence Proved

137. (i) Derive section formula.

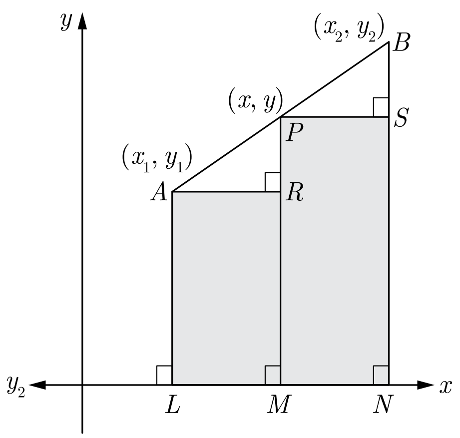
(ii) In what ratio does $(-4, 6)$ divides the line segment joining the point $A(-6, 4)$ and $B(3, -8)$

Ans : [Board Term-2 Delhi 2014]

(i) Section Formula : Let $A(x_1, y_1)$ and $B(x_2, y_2)$ are two points. Let $P(x, y)$ be a point on line, joining A and B, such that P divides it in the ratio $m_1 : m_2$.

Now $$(x, y) = \left(\frac{m_2 x_1 + m_1 x_2}{m_1 + m_2}, \frac{m_2 y_1 + m_1 y_2}{m_1 + m_2}\right)$$

g204



Proof : Let AB be a line segment joining the points. $A(x_1, y_1)$, $B(x_2, y_2)$.

Let P divides AB in the ratio $m_1 : m_2$. Let P have co-ordinates (x, y).

Draw $AL, PM, PN, \perp$ to x-axis

It is clear form figure, that

$$AR = LM = OM - OL = x - x_1$$

$$PR = PM - RM = y - y_1.$$

also, $$PS = ON - OM = x_2 - x$$

$$BS = BN - SN = y_2 - y$$

Now $$\Delta APR \sim \Delta PBS \quad [AAA]$$

Thus $$\frac{AR}{PS} = \frac{PR}{BS} = \frac{AP}{PB}$$

and $$\frac{AR}{PS} = \frac{AP}{PB}$$

$$\frac{x - x_1}{x_2 - x} = \frac{m_1}{m_2}$$

$$m_2 x - m_2 x_1 = m_1 x_2 - m_1 x$$

$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$$

Now $$\frac{PR}{BS} = \frac{AP}{PB}$$

$$\frac{y - y_2}{y_2 - y} = \frac{m_1}{m_2}$$

$$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

Thus co-ordinates of P are $\left(\frac{m_2 x_1 + m_1 x_2}{m_1 + m_2}, \frac{m_2 y_1 + m_1 y_2}{m_1 + m_2}\right)$

(ii) Assume that $(-4, 6)$ divides the line segment joining the point $A(-6, 4)$ and $B(3, -8)$ in ratio $k:1$

Using section formula for x co-ordinate we have

$$-4 = \frac{k(3) - 6}{k + 1}$$

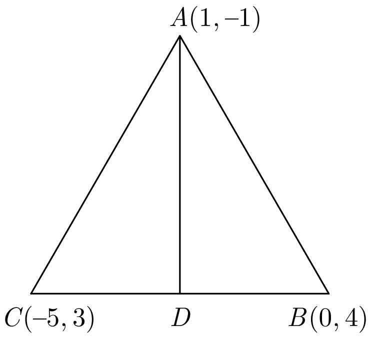
$$-4k - 4 = 3k - 6 \Rightarrow k = \tfrac{2}{7}$$

138. $(1, -1)$, $(0, 4)$ and $(-5, 3)$ are vertices of a triangle. Check whether it is a scalene triangle, isosceles triangle or an equilateral triangle. Also, find the length of its median joining the vertex $(1, -1)$ the mid-point of the opposite side.

Ans : [Board Term-2, 2015]

Let the vertices of ΔABC be $A(1, -1)$, $B(0, 4)$ and $C(-5, 3)$. Let $D(x, y)$ be mid point of BC. N triangle is shown below.

g210



Using distance formula, we get

$$AB = \sqrt{(1-0)^2+(-1-4)^2} = \sqrt{1+5^2} = \sqrt{26}$$

$$BC = \sqrt{(-5-0)^2+(3-4)^2} = \sqrt{25+1} = \sqrt{26}$$

$$AC = \sqrt{(-5-1)^2+(3+1)^2} = \sqrt{36+16} = 2\sqrt{13}$$

Since $AB = BC \neq AC$, triangle ΔABC is isosceles.

Now, using mid-section formula, the co-ordinates of mid-point of BC are

$$x = \frac{-5+0}{2} = -\frac{5}{2}$$

$$y = \frac{3+4}{2} = \frac{7}{2}$$

$$D(x,y) = \left(-\frac{5}{2},\frac{7}{2}\right)$$

Length of median AD,

$$AD = \sqrt{\left(\frac{-5}{2}-1\right)^2+\left(\frac{7}{2}+1\right)^2}$$

$$= \sqrt{\left(\frac{-7}{2}\right)^2+\left(\frac{9}{2}\right)^2}$$

$$= \sqrt{\frac{130}{4}} = \frac{\sqrt{130}}{2} \text{ square unit}$$

Thus length of median AD is $\frac{\sqrt{130}}{2}$ units.

139. Point $(-1,y)$ and $B(5,7)$ lie on a circle with centre $O(2,-3y)$. Find the values of y. Hence find the radius of the circle.

Ans : [Board Term-2 Delhi 2014]

Since, $A(-1,y)$ and $B(5,7)$ lie on a circle with centre $O(2,-3y)$, OA and OB are the radius of circle and are equal. Thus

g214

$$OA = OB$$

$$\sqrt{(-1-2)^2+(y+3y)^2} = \sqrt{(5-2)^2+(7+3y)^2}$$

$$9+16y^2 = 9y^2+42y+58$$

$$y^2-6y-7 = 0$$

$$(y+1)(y-7) = 0$$

$$y = -1,7$$

When $y=-1$, centre is $O(2,-3y) = (2,3)$ and radius

$$OB = \left|\sqrt{(5-2)^2+(7-3)^2}\right|$$

$$= \sqrt{9+16} = 5 \text{ unit}$$

When $y=7$, centre is $O(2,-3y) = (2,-21)$ and radius

$$OB = \left|\sqrt{(2-5)^2+(-21-7)^2}\right|$$

$$= \left|\sqrt{9+784}\right| = \sqrt{793} \text{ unit}$$

CHAPTER 8

INTRODUCTION OF TRIGONOMETRY

ONE MARK QUESTIONS

Multiple Choice Questions

1. Given that $\sin\alpha = \frac{\sqrt{3}}{2}$ and $\cos\beta = 0$, then the value of $\beta - \alpha$ is
   (a) 0° (b) 90°
   (c) 60° (d) 30°

h215

Ans : [Board 2020 SQP Standard]

We have $\sin\alpha = \frac{\sqrt{3}}{2}$

$$\sin\alpha = \sin 60^\circ \Rightarrow \alpha = 60^\circ \quad ...(1)$$

and $\cos\beta = 0$

$$\cos\beta = \cos 90^\circ \Rightarrow \beta = 90^\circ \quad ...(2)$$

Now, $\beta - \alpha = 90^\circ - 60^\circ = 30^\circ$

Thus (d) is correct option.

2. If ΔABC is right angled at C, then the value of $\sec(A+B)$ is
   (a) 0 (b) 1
   (c) $\frac{2}{\sqrt{3}}$ (d) not defined

Ans : [Board 2020 SQP Standard]

h216

We have $\angle C = 90^\circ$

Since, $\angle A + \angle B + \angle C = 180^\circ$

$$\angle A + \angle B = 180^\circ - \angle C$$

$$= 180^\circ - 90^\circ = 90^\circ$$

Now, $\sec(A+B) = \sec 90^\circ$ not defined

Thus (d) is correct option.

3. If $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$, $(\theta \neq 90^\circ)$ then the value of $\tan\theta$ is
   (a) $\sqrt{2}-1$ (b) $\sqrt{2}+1$
   (c) $\sqrt{2}$ (d) $-\sqrt{2}$

Ans : [Board 2020 SQP Standard]

h217

We have $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$

Dividing both sides by $\cos\theta$, we get

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta} = \sqrt{2}\frac{\cos\theta}{\cos\theta}$$

$$\tan\theta + 1 = \sqrt{2}$$

$$\tan\theta = \sqrt{2} - 1$$

Thus (a) is correct option.

4. If $\cos A = \frac{4}{5}$, then the value of $\tan A$ is
   (a) $\frac{3}{5}$ (b) $\frac{3}{4}$
   (c) $\frac{4}{3}$ (d) $\frac{5}{3}$

h218

Ans :

We have $\cos A = \frac{4}{5}$

We know that, $\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{4}{5}$

$$\text{Perpendicular} = \sqrt{5^2 - 4^2} = \sqrt{25 - 16} = 3$$

Now, $\tan A = \frac{\text{Perpendicular}}{\text{Base}} = \frac{3}{4}$

Thus (b) is correct option.

5. If $\sin A = \frac{1}{2}$, then the value of $\cot A$ is
   (a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$
   (c) $\frac{\sqrt{3}}{2}$ (d) 1

h219

Ans :

We have $\sin A = \frac{1}{2}$

$$\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{1}{2}$$

Now, $\text{Base} = \sqrt{2^2 - 1^2} = \sqrt{3}$

So, $\cot A = \frac{\text{Base}}{\text{Perpendicular}} = \frac{\sqrt{3}}{1} = \sqrt{3}$

Hence, the required value of $\cot A$ is $\sqrt{3}$.
Thus (a) is correct option.

6. If $\sin\theta = \frac{a}{b}$, then $\cos\theta$ is equal to

(a) $\frac{b}{\sqrt{b^2 - a^2}}$ (b) $\frac{b}{a}$

(c) $\frac{\sqrt{b^2 - a^2}}{b}$ (d) $\frac{a}{\sqrt{b^2 - a^2}}$

h221

Ans :

We have $\sin\theta = \frac{a}{b} = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$

$$\text{Base} = \sqrt{b^2 - a^2}$$

So, $\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{\sqrt{b^2 - a^2}}{b}$

Thus (c) is correct option.

7. If $\cos(\alpha + \beta) = 0$, then $\sin(\alpha - \beta)$ can be reduced to

(a) $\cos\beta$ (b) $\cos 2\beta$

(c) $\sin\alpha$ (d) $\sin 2\alpha$

Ans :

Given, $\cos(\alpha + \beta) = 0 = \cos 90^\circ$ $[\cos 90^\circ = 0]$

$$\alpha + \beta = 90^\circ$$

$$\alpha = 90^\circ - \beta$$

h222

Now, $\sin(\alpha - \beta) = \sin(90^\circ - \beta - \beta)$

$$= \sin(90^\circ - 2\beta)$$

$= \cos 2\beta$

Thus (b) is correct option.

8. If $\cos 9\alpha = \sin\alpha$ and $9\alpha < 90^\circ$, then the value of $\tan 5\alpha$ is

(a) $\frac{1}{\sqrt{3}}$ (b) $\sqrt{3}$

(c) 1 (d) 0

h223

Ans :

We have $\cos 9\alpha = \sin\alpha$ where $9\alpha < 90^\circ$

$$\sin(90^\circ - 9\alpha) = \sin\alpha$$

$$90^\circ - 9\alpha = \alpha$$

$$10\alpha = 90^\circ \Rightarrow \alpha = 9^\circ$$

$$\tan 5\alpha = \tan(5 \times 9^\circ)$$

$$= \tan 45^\circ = 1 \qquad [\tan 45^\circ = 1]$$

Thus (c) is correct option.

9. If ΔABC is right angled at C, then the value of $\cos(A + B)$ is

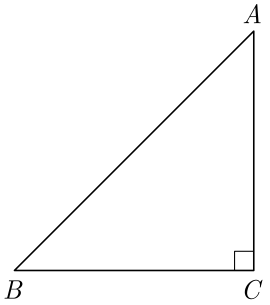
(a) 0 (b) 1

(c) $\frac{1}{2}$ (d) $\frac{\sqrt{3}}{2}$

h224

Ans :

We know that in ΔABC,



$$\angle A + \angle B + \angle C = 180^\circ$$

But right angled at C i.e., $\angle C = 90^\circ$, thus

$$\angle A + \angle B + 90^\circ = 180^\circ$$

$$A + B = 90^\circ$$

$$\cos(A + B) = \cos 90^\circ = 0$$

Thus (a) is correct option.

10. If $\sin\alpha = \frac{1}{2}$ and $\cos\beta = \frac{1}{2}$, then the value of $(\alpha + \beta)$ is

(a) 0° (b) 30°

(c) 60° (d) 90°

h225

Ans :

Given, $\sin\alpha = \frac{1}{2} = \sin 30^\circ \Rightarrow \alpha = 30^\circ$

and $\cos\beta = \frac{1}{2} = \cos 60^\circ \Rightarrow \beta = 60^\circ$

$$\alpha + \beta = 30^\circ + 60^\circ = 90^\circ$$

Thus (d) is correct option.

11. If $4\tan\theta = 3$, then $\left(\frac{4\sin\theta - \cos\theta}{4\sin\theta + \cos\theta}\right)$ is equal to

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$

(c) $\frac{1}{2}$ (d) $\frac{3}{4}$

h226

Ans :

Given, $4\tan\theta = 3$

$$\tan\theta = \frac{3}{4} \qquad ...(i)$$

$$\frac{4\sin\theta - \cos\theta}{4\sin\theta + \cos\theta} = \frac{4\frac{\sin\theta}{\cos\theta} - 1}{4\frac{\sin\theta}{\cos\theta} + 1} = \frac{4\tan\theta - 1}{4\tan\theta + 1}$$

$$= \frac{4\left(\frac{3}{4}\right) - 1}{4\left(\frac{3}{4}\right) + 1} = \frac{3-1}{3+1} = \frac{2}{4} = \frac{1}{2}$$

Thus (c) is correct option.

12. If $\sin\theta - \cos\theta = 0$, then the value of $(\sin^4\theta + \cos^4\theta)$ is

(a) 1 (b) $\frac{3}{4}$

(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

h227

Ans :

Given, $\sin\theta - \cos\theta = 0$

$$\sin\theta = \cos\theta$$

$$\sin\theta = \sin(90^\circ - \theta)$$

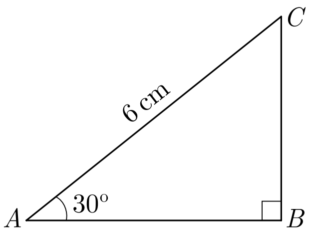
$$\theta = 90^\circ - \theta \Rightarrow \theta = 45^\circ$$

Now, $\sin^4\theta + \cos^4\theta = \sin^4 45^\circ + \cos^4 45^\circ$

$$= \left(\frac{1}{\sqrt{2}}\right)^4 + \left(\frac{1}{\sqrt{2}}\right)^4 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

Thus (c) is correct option.

13. In the adjoining figure, the length of BC is



h228

(a) $2\sqrt{3}$ cm (b) $3\sqrt{3}$ cm

(c) $4\sqrt{3}$ cm (d) 3 cm

Ans :

In ΔABC, $\sin 30^\circ = \frac{BC}{AC}$

$$\frac{1}{2} = \frac{BC}{6}$$

$$BC = 3\,\text{cm}$$

Thus (d) is correct option.

14. If $x = p\sec\theta$ and $y = q\tan\theta$, then

(a) $x^2 - y^2 = p^2q^2$ (b) $x^2q^2 - y^2p^2 = pq$

(c) $x^2q^2 - y^2p^2 = \frac{1}{p^2q^2}$ (d) $x^2q^2 - y^2p^2 = p^2q^2$

Ans :

We know, $\sec^2\theta - \tan^2\theta = 1$

Substituting $\sec\theta = \frac{x}{p}$ and $\tan\theta = \frac{y}{q}$ in above equation we have

$$\left(\frac{x}{p}\right)^2 - \left(\frac{y}{q}\right)^2 = 1$$

h229

$$x^2q^2 - y^2p^2 = p^2q^2$$

Thus (d) is correct option.

15. If $b\tan\theta = a$, the value of $\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta}$ is

(a) $\frac{a-b}{a^2+b^2}$ (b) $\frac{a+b}{a^2+b^2}$

(c) $\frac{a^2+b^2}{a^2-b^2}$ (d) $\frac{a^2-b^2}{a^2+b^2}$

Ans :

h230

We have $\tan\theta = \frac{a}{b}$

$$\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \frac{a\frac{\sin\theta}{\cos\theta} - b}{a\frac{\sin\theta}{\cos\theta} + b} = \frac{a\tan\theta - b}{a\tan\theta + b}$$

$$= \frac{a^2 - b^2}{a^2 + b^2}$$

Thus (d) is correct option.

16. $(\cos^4 A - \sin^4 A)$ is equal to

(a) $1 - 2\cos^2 A$ (b) $2\sin^2 A - 1$

(c) $\sin^2 A - \cos^2 A$ (d) $2\cos^2 A - 1$

Ans :

$$\cos^4 A - \sin^4 A = (\cos^2 A)^2 - (\sin^2 A)^2$$

$$= (\cos^2 A - \sin^2 A)(\cos^2 A + \sin^2 A)$$

$$= (\cos^2 A - \sin^2 A)(1)$$

$$= \cos^2 A - (1 - \cos^2 A)$$

h231

$$= 2\cos^2 A - 1$$

Thus (d) is correct option.

17. If $\sec 5A = \operatorname{cosec}(A + 30^\circ)$, where $5A$ is an acute angle, then the value of A is

(a) 15° (b) 5°

(c) 20° (d) 10°

h232

Ans :

We have, $\sec 5A = \operatorname{cosec}(A+30^\circ)$

$$\sec 5A = \sec[90^\circ - (A-30^\circ)]$$

$$\sec 5A = \sec(60^\circ - A)$$

$$5A = 60^\circ - A$$

$$6A = 60^\circ \Rightarrow A = 10^\circ$$

Thus (d) is correct option.

18. If $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ and $x\sin\theta = y\cos\theta$, than x^2+y^2 is equal to

(a) 0 (b) 1/2

(c) 1 (d) 3/2

h233

Ans :

We have, $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$

$$(x\sin\theta)\sin^2\theta + (y\cos\theta)\cos^2\theta = \sin\theta\cos\theta$$

$$x\sin\theta(\sin^2\theta) + (x\sin\theta)\cos^2\theta = \sin\theta\cos\theta$$

$$x\sin\theta(\sin^2\theta+\cos^2\theta) = \sin\theta\cos\theta$$

$$x\sin\theta = \sin\theta\cos\theta \Rightarrow x = \cos\theta$$

Now, $x\sin\theta = y\cos\theta$

$$\cos\theta\sin\theta = y\cos\theta$$

$$y = \sin\theta$$

Hence, $x^2+y^2 = \cos^2\theta + \sin^2\theta = 1$

Thus (c) is correct option.

19. If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, then $m^2 - n^2$ is equal to

(a) $\sqrt{mn}$ (b) $\sqrt{\frac{m}{n}}$

(c) $4\sqrt{mn}$ (d) None of these

h235

Ans :

Given, $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$

$$m^2 - n^2 = (\tan\theta+\sin\theta)^2 - (\tan\theta-\sin\theta)^2$$

$$= 4\tan\theta\sin\theta$$

$$= 4\sqrt{\tan^2\theta\sin^2\theta}$$

$$= 4\sqrt{\sin^2\theta\frac{\sin^2\theta}{\cos^2\theta}}$$

$$= 4\sqrt{\sin^2\theta\frac{(1-\cos^2\theta)}{\cos^2\theta}}$$

$$= 4\sqrt{\frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta}$$

$$= 4\sqrt{\tan^2\theta - \sin^2\theta}$$

$$= 4\sqrt{(\tan\theta+\sin\theta)(\tan\theta-\sin\theta)}$$

$$= 4\sqrt{mn}$$

Thus (c) is correct option.

20. If $0 < \theta < \frac{\pi}{4}$, then the simplest form of $\sqrt{1-2\sin\theta\cos\theta}$ is

(a) $\sin\theta - \cos\theta$ (b) $\cos\theta - \sin\theta$

(c) $\cos\theta + \sin\theta$ (d) $\sin\theta\cos\theta$

h236

Ans :

$$\sqrt{1-2\sin\theta\cos\theta} = \sqrt{\sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta}$$

$$= \sqrt{(\cos\theta - \sin\theta)^2}$$

$$= \cos\theta - \sin\theta$$

For $0^\circ < \theta < 45^\circ$

	0	$\pi/6$	$\pi/4$
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$
$\sin\theta$	0	$1/2$	$1/\sqrt{2}$

Here, we see that $\cos\theta > \sin\theta$, when $0 < \theta < \frac{\pi}{4}$, that's why we take $(\cos\theta - \sin\theta)^2$ instead of taking $(\sin\theta - \cos\theta)^2$.

Thus (b) is correct option.

21. If $f(x) = \cos^2 x + \sec^2 x$, then $f(x)$

(a) ≥ 1 (b) ≤ 1

(c) ≥ 2 (d) ≤ 2

h237

Ans : (c) ≥ 2

Given, $f(x) = \cos^2 x + \sec^2 x$

$$= \cos^2 x + \sec^2 x - 2 + 2$$

$$= \cos^2 x + \sec^2 x - 2\cos x \cdot \sec x + 2$$

$$= (\cos x - \sec x)^2 + 2$$

We know that, square of any expression is always greater than equal to zero.

$f(x) \geq 2$ Hence proved.

Thus (c) is correct option.

22. Assertion : The value of $\sin\theta = \frac{4}{3}$ is not possible.

Reason : Hypotenuse is the largest side in any right angled triangle.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

$$\sin\theta = \frac{P}{H} = \frac{4}{3}$$

h238

Here, perpendicular is greater than the hypotenuse which is not possible in any right triangle.

Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Thus (a) is correct option.

23. Assertion : $\sin^2 67^\circ + \cos^2 67^\circ = 1$

Reason : For any value of $\theta, \sin^2\theta + \cos^2\theta = 1$

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

We have $\sin^2\theta + \cos^2\theta = 1$

h239

$$\sin^2 67^\circ + \cos^2 67^\circ = 1$$

Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Thus (a) is correct option.

1. FILL IN THE BLANK

1. Maximum value for sine of any angle is ....

Ans :

1

h241

2. Triangle in which we study trigonometric ratios is called ....

Ans :

Right Triangle

h242

3. Cosine of 90° is ....

Ans :

Zero

h243

4. Sum of .... of sine and cosine of angle is one.

Ans :

Square

h244

5. Reciprocal of $\sin\theta$ is ....

Ans :

$\operatorname{cosec}\theta$

h245

6. The value of $\sin A$ or $\cos A$ never exceeds ....

Ans :

1

h246

7. sine of $(90^\circ - \theta)$ is ....

Ans :

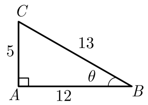
$\cos\theta$

h247

8. If $\sin\theta = \frac{5}{13}$, then the value of $\tan\theta$ is ....

Ans : [Board 2020 OD Basic]

From $\sin\theta = \frac{5}{13}$ we can draw the figure as given below.



h248

Now, $\tan\theta = \frac{AC}{BC} = \frac{5}{12}$

9. The value of the $(\tan^2 60^\circ + \sin^2 45^\circ)$ is ....

Ans : [Board 2020 OD Basic]

$$\tan^2 60^\circ + \sin^2 45^\circ = (\sqrt{3})^2 + \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= 3 + \frac{1}{2} = \frac{7}{2}$$

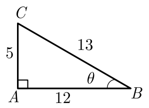
h250

10. If $\cot\theta = \frac{12}{5}$, then the value of $\sin\theta$ is ....

Ans : [Board 2020 Delhi Basic]

Given, $\cot\theta = \frac{12}{5} \Rightarrow \tan\theta = \frac{5}{12}$

From $\tan\theta = \frac{5}{12}$ we can draw the figure as given below.



So, $\sin\theta = \frac{AC}{CB} = \frac{5}{13}$

11. If $\tan(A+B) = \sqrt{3}$ and $\tan(A-B) = \frac{1}{\sqrt{3}}$, $A > B$, then the value of A is ....

Ans : [Board 2020 Delhi Basic]

We have $\tan(A+B) = \sqrt{3}$

$= \tan 60^\circ$

Hence, $A + B = 60^\circ$

...(1)

Again, $\tan(A-B) = \frac{1}{\sqrt{3}}$

$= \tan 30^\circ$

$A - B = 30^\circ$ ...(2)

Adding equation (1) and (2) we get

$2A = 90^\circ \Rightarrow A = 45^\circ$

12. The value of $\left(\sin^2\theta + \frac{1}{1+\tan^2\theta}\right) =$ ....

Ans : [Board 2020 Delhi Standard]

$$\sin^2\theta + \frac{1}{1+\tan^2\theta} = \sin^2\theta + \frac{1}{\sec^2\theta}$$

$$= \sin^2\theta + \cos^2\theta = 1$$

13. The value of $(1+\tan^2\theta)(1-\sin\theta)(1+\sin\theta) =$ ....

Ans : [Board 2020 Delhi Standard]

$(1+\tan^2\theta)(1-\sin\theta)(1+\sin\theta)$

$$= \sec^2\theta(1-\sin^2\theta)$$

$$= \sec^2\theta \times \cos^2\theta$$

$$= \frac{1}{\cos^2\theta} \times \cos^2\theta = 1$$

Very Short Answer Questions

14. Prove that

$(1+\tan A - \sec A) \times (1 + \tan A + \sec A) = 2\tan A$

Ans : [Board 2020 Delhi Basic]

LHS $= (1+\tan A - \sec A) \times (1+\tan A + \sec A)$

$$= (1+\tan A)^2 - \sec^2 A$$

$$= 1 + \tan^2 A + 2\tan A - \sec^2 A$$

$$= \sec^2 A + 2\tan A - \sec^2 A$$

$$= 2\tan A = \text{RHS}$$

15. If $\tan A = \cot B$, then find the value of $(A+B)$.

Ans : [Board 2020 OD Standard]

We have $\tan A = \cot B$

$\tan A = \tan(90^\circ - B)$

$A = 90^\circ - B$

Thus $A + B = 90^\circ$

16. If $x = 3\sin\theta + 4\cos\theta$ and $y = 3\cos\theta - 4\sin\theta$ then prove that $x^2 + y^2 = 25$.

Ans : [Board 2020 OD Basic]

We have $x = 3\sin\theta + 4\cos\theta$

and $y = 3\cos\theta - 4\sin\theta$

$x^2 + y^2$

$= (3\sin\theta + 4\cos\theta)^2 + (3\cos\theta - 4\sin\theta)^2$

$$= (9\sin^2\theta + 16\cos^2\theta + 24\sin\theta\cos\theta) + (9\cos^2\theta + 16\sin^2\theta - 24\sin\theta\cos\theta)$$

$$= 9(\sin^2\theta + \cos^2\theta) + 16(\sin^2\theta + \cos^2\theta)$$

$$= 9 + 16 = 25$$

17. Evaluate $\sin^2 60^\circ - 2\tan 45^\circ - \cos^2 30^\circ$

Ans : [Board 2019 OD]

$$\sin^2 60^\circ - 2\tan 45^\circ - \cos^2 30^\circ = \left(\frac{\sqrt{3}}{2}\right)^2 - 2(1) - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= \frac{3}{4} - 2 - \frac{3}{4} = -2$$

18. If $\sin\theta + \sin^2\theta = 1$ then prove that $\cos^2\theta + \cos^4\theta = 1$.

Ans : [Board 2020 OD Basic]

We have
$$\sin\theta + \sin^2\theta = 1$$
$$\sin\theta + (1 - \cos^2\theta) = 1$$
$$\sin\theta - \cos^2\theta = 0$$
$$\sin\theta = \cos^2\theta$$

h272

Squaring both sides, we get
$$\sin^2\theta = \cos^4\theta$$
$$1 - \cos^2\theta = \cos^4\theta$$
$$\cos^4\theta + \cos^2\theta = 1$$
Hence Proved

19. In a triangle ABC, write $\cos\left(\frac{B+C}{2}\right)$ in terms of angle A.

Ans : [Board Term-1 2016]

h101

In a triangle $A + B + C = 180^\circ$
$$B + C = 180^\circ - A$$
Thus
$$\cos\left(\frac{B+C}{2}\right) = \cos\left[\frac{180^\circ - A}{2}\right]$$
$$= \cos\left[90 - \frac{A}{2}\right]$$
$$= \sin\frac{A}{2}$$

20. If $\sec\theta \cdot \sin\theta = 0$, then find the value of θ.

Ans : [Board Term-1 2016]

We have $\sec\theta \cdot \sin\theta = 0$
$$\frac{1}{\cos\theta} \cdot \sin\theta = 0$$
$$\frac{\sin\theta}{\cos\theta} = 0$$
$$\tan\theta = 0 = \tan 0^\circ$$

h102

Thus $\theta = 0^\circ$

21. If $\tan 2A = \cot(A + 60^\circ)$, find the value of A where $2A$ is an acute angle.

Ans : [Board Term-1 2016]

We have
$$\tan 2A = \cot(A + 60^\circ)$$
$$\cot(90^\circ - 2A) = \cot(A + 60^\circ)$$
$$90^\circ - 2A = A + 60^\circ$$
$$3A = 30^\circ \Rightarrow A = 10^\circ$$

h104

22. If $\tan(3x + 30^\circ) = 1$ then find the value of x.

Ans : [Board Te

h107

We have
$$\tan(3x + 30^\circ) = 1 = \tan 45^\circ$$
$$3x + 30^\circ = 45^\circ$$
$$x = 5^\circ$$

23. What happens to value of $\cos\theta$ when θ increases from 0° to 90°.

Ans : [Board Term-1 2015]

$\cos\theta$ decreases from 1 to θ.

h108

24. If A and B are acute angles and $\sin A = \cos B$, then find the value of $A + B$.

Ans : [Board Term-1 2016]

We have
$$\sin A = \cos B$$
$$\sin A = \sin(90^\circ - B)$$
$$A = 90^\circ - B$$
$$A + B = 90^\circ$$

h110

25. If $\cos A = \frac{2}{5}$, find the value of $4 + 4\tan^2 A$.

Ans : [Board SQP 2018]

h113

$$4 + 4\tan^2 A = 4(1 + \tan^2 A)$$
$$4\sec^2 A = \frac{4}{\cos^2 A} = \frac{4}{\left(\frac{2}{5}\right)^2} = 4 \times \frac{25}{4} = 25$$

26. If $k + 1 = \sec^2\theta(1 + \sin\theta)(1 - \sin\theta)$, then find the value of k.

Ans : [Board Term-1 2015]

We have
$$k + 1 = \sec^2\theta(1 + \sin\theta)(1 - \sin\theta)$$
$$= \sec^2\theta(1 - \sin^2\theta)$$
$$= \sec^2\theta.\cos^2\theta$$
$$= \sec^2\theta \times \frac{1}{\sec^2\theta}$$
$$k + 1 = 1 \Rightarrow k = 1 - 1 = 0$$

h154

Thus $k = 0$

27. Find the value of $\sin^2 41^\circ + \sin^2 49^\circ$

Ans : [Board Term-1 2012, NCERT]

We have
$$\sin^2 41 + \sin^2 49 = \sin^2(90^\circ - 49^\circ) + \sin^2 49^\circ$$

$$= \cos^2 49 + \sin^2 49^\circ$$

$$= 1$$

h155

TWO MARKS QUESTIONS

28. Prove that $1 + \dfrac{\cot^2\alpha}{1 + \operatorname{cosec}\alpha} = \operatorname{cosec}\alpha$

Ans : [Board 2020 OD Standard]

h273

$$1 + \frac{\cot^2\alpha}{1 + \operatorname{cosec}\alpha} = 1 + \frac{\operatorname{cosec}^2\alpha - 1}{1 + \operatorname{cosec}\alpha}$$

$$= 1 + \frac{(1 + \operatorname{cosec}\alpha)(\operatorname{cosec}\alpha - 1)}{1 + \operatorname{cosec}\alpha}$$

$$= 1 + \operatorname{cosec}\alpha - 1$$

$$= \operatorname{cosec}\alpha$$ Hence Proved

29. Prove that : $\dfrac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \tan A$.

Ans : [Board 2018]

h275

$$\frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \frac{\sin A(1 - 2\sin^2 A)}{\cos A(2\cos^2 A - 1)}$$

$$= \frac{\sin A(1 - 2\sin^2 A)}{\cos A(2\cos^2 A - 1)}$$

$$= \tan A \frac{[1 - 2(1 - \cos^2 A)]}{(2\cos^2 A - 1)}$$

$$= \tan A \frac{[1 - 2 + 2\cos^2 A)]}{(2\cos^2 A - 1)}$$

$$= \tan A \frac{(2\cos^2 A - 1)}{(2\cos^2 A - 1)}$$

$$= \tan A$$ Hence Proved

30. Show that $\tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta$

Ans : [Board 2020 OD Standard]

h276

$$\tan^4\theta + \tan^2\theta = \tan^2\theta(1 + \tan^2\theta)$$

$$= \tan^2\theta \times \sec^2\theta$$

$$= (\sec^2\theta - 1)\sec^2\theta$$

$$= \sec^4\theta - \sec^2\theta$$ Hence Proved

31. Prove that $\sqrt{\dfrac{1 - \sin\theta}{1 + \sin\theta}} = \sec\theta - \tan\theta$.

Ans :

h278

$$\text{LHS} = \sqrt{\frac{1 - \sin\theta}{1 + \sin\theta}} = \sqrt{\frac{(1 - \sin\theta)(1 - \sin\theta)}{(1 + \sin\theta)(1 - \sin\theta)}}$$

$$= \sqrt{\frac{(1 - \sin\theta)^2}{1 - \sin^2\theta}} = \sqrt{\frac{(1 - \sin\theta)^2}{\cos^2\theta}}$$

$$= \frac{1 - \sin\theta}{\cos\theta} = \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$$

$$= \sec\theta - \tan\theta = \text{RHS}$$ Hence Proved

32. Prove that : $\dfrac{1 - \tan^2\theta}{1 + \tan^2\theta} = \cos^2\theta - \sin^2\theta$

Ans : [Board 2020 OD Basic]

h279

$$\frac{1 - \tan^2\theta}{1 + \tan^2\theta} = \frac{1 - \tan^2\theta}{\sec^2\theta}$$

$$= \frac{1}{\sec^2\theta} - \frac{\tan^2\theta}{\sec^2\theta}$$

$$= \cos^2\theta - \frac{\sin^2\theta}{\cos^2\theta} \times \cos^2\theta$$

$$= \cos^2\theta - \sin^2\theta$$ Hence Proved

33. Prove that $\dfrac{\tan^2\theta}{1 + \tan^2\theta} + \dfrac{\cot^2\theta}{1 + \cot^2\theta} = 1$.

Ans : [Board 2020 Delhi Basic]

h280

$$\text{LHS} = \frac{\tan^2\theta}{1 + \tan^2\theta} + \frac{\cot^2\theta}{1 + \cot^2\theta}$$

$$= \frac{\tan^2\theta}{\sec^2\theta} + \frac{\cot^2\theta}{\operatorname{cosec}^2\theta}$$

$$= \frac{\frac{\sin^2\theta}{\cos^2\theta}}{\frac{1}{\cos^2\theta}} + \frac{\frac{\cos^2\theta}{\sin^2\theta}}{\frac{1}{\sin^2\theta}}$$

$$= \sin^2\theta + \cos^2\theta = 1 = \text{RHS}$$

34. Prove that : $\dfrac{1}{1 + \sin\theta} + \dfrac{1}{1 - \sin\theta} = 2\sec^2\theta$

Ans : [Board 2020 Delhi Basic]

h281

$$\text{LHS} = \frac{1}{1 + \sin\theta} + \frac{1}{1 - \sin\theta}$$

$$= \frac{(1 - \sin\theta) + (1 + \sin\theta)}{(1 + \sin\theta)(1 - \sin\theta)}$$

$$= \frac{2}{1 - \sin^2\theta} = 2\sec^2\theta = \text{RHS}$$

35. Prove that $\dfrac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta - 1} + \dfrac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta + 1} = 2\sec^2\theta$.

Ans : [Board 2020 Delhi Basic]

$$\text{LHS} = \frac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta - 1} + \frac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta + 1}$$

$$= \operatorname{cosec}\theta\left[\frac{1}{\operatorname{cosec}\theta - 1} + \frac{1}{\operatorname{cosec}\theta + 1}\right]$$

$$= \operatorname{cosec}\theta\left[\frac{\operatorname{cosec}\theta + 1 + \operatorname{cosec}\theta - 1}{(\operatorname{cosec}\theta - 1)(\operatorname{cosec}\theta + 1)}\right]$$

$$= \operatorname{cosec}\theta\left(\frac{2\operatorname{cosec}\theta}{\operatorname{cosec}^2\theta - 1}\right)$$

h283

$$= \frac{2\operatorname{cosec}^2\theta}{\operatorname{cosec}^2\theta - 1} = \frac{2\operatorname{cosec}^2\theta}{\cot^2\theta}$$

$$= \frac{2 \times \frac{1}{\sin^2\theta}}{\frac{\cos^2\theta}{\sin^2\theta}} = \frac{2}{\cos^2\theta}$$

$$= 2\sec^2\theta = \text{RHS}$$ Hence Proved

36. If $5\tan\theta = 3$, then what is the value of $\left(\frac{5\sin\theta - 3\cos\theta}{4\sin\theta + 3\cos\theta}\right)$?

Ans : [Board 2020 Delhi Basic]

We have $5\tan\theta = 3 \Rightarrow \tan\theta = \frac{3}{5}$

h285

Dividing numerator and denominator by $\cos\theta$ we have

$$\frac{5\sin\theta - 3\cos\theta}{4\sin\theta + 3\cos\theta} = \frac{5\frac{\sin\theta}{\cos\theta} - 3}{4\frac{\sin\theta}{\cos\theta} + 3} = \frac{5\tan\theta - 3}{4\tan\theta + 3}$$

$$= \frac{5 \times \frac{3}{5} - 3}{4 \times \frac{3}{5} + 3} = \frac{3 - 3}{\frac{12}{5} + 3} = 0$$

37. Evaluate :

$$\frac{3\tan^2 30^\circ + \tan^2 60^\circ + \operatorname{cosec} 30^\circ - \tan 45^\circ}{\cot^2 45^\circ}$$

Ans : [Board Term-1 2016]

$$\frac{3\tan^2 30^\circ + \tan^2 60^\circ + \operatorname{cosec} 30^\circ - \tan 45^\circ}{\cot^2 45^\circ}$$

$$= \frac{3 \times \left(\frac{1}{\sqrt{3}}\right)^2 + (\sqrt{3})^2 + 2 - 1}{(1)^2}$$

$$= \frac{3 \times \frac{1}{3} + 3 + 2 - 1}{1}$$

h114

$$= 1 + 3 + 2 - 1 = 5$$

38. If $\sin(A+B) = 1$ and $\sin(A-B) = \frac{1}{2}$, $0 \le A+B < 90^0$ and $A > B$, then find A and B.

Ans : [Board Term-1 2016]

h115

We have $\sin(A+B) = 1 = \sin 90^\circ$

$$A + B = 90^\circ \quad \text{...(1)}$$

and $\sin(A - B) = \frac{1}{2} = \sin 30^\circ$

$$A - B = 30^\circ \quad \text{...(2)}$$

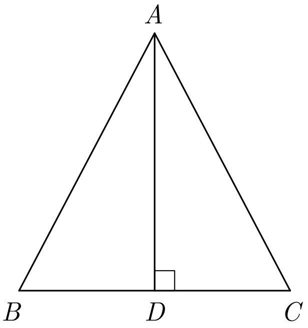
Solving eq. (1) and (2), we obtain

$$A = 60^\circ \text{ and } B = 30^\circ$$

39. Find $\operatorname{cosec} 30^\circ$ and $\cos 60^\circ$ geometrically.

Ans : [Board Term-1 2015]

Let a triangle ABC with each side equal to $2a$ as shown below.



h116

In ΔABC, $\angle A = \angle B = \angle C = 60^\circ$

Now we draw AD perpendicular to BC, then

$$\Delta BDA \cong \Delta CDA$$

$$BD = CD$$

$$\angle BAD = CAD = 30^\circ \quad \text{by } CPCT$$

$$AD = \sqrt{3}a$$

In ΔBDA, $\operatorname{cosec} 30^\circ = \frac{AB}{BD} = \frac{2a}{a} = 2$

and $\cos 60^\circ = \frac{BD}{AB} = \frac{a}{2a} = \frac{1}{2}$

40. Evaluate : $\frac{\sin 90^\circ}{\cos 45^\circ} + \frac{1}{\operatorname{cosec} 30^\circ}$

Ans : [Board Term-1 2013]

h118

We have $\frac{\sin 90^\circ}{\cos 45^\circ} + \frac{1}{\operatorname{cosec} 30^\circ} = \frac{1}{\frac{1}{\sqrt{2}}} + \frac{1}{2}$

$$= \sqrt{2} + \frac{1}{2} = \frac{2\sqrt{2}+1}{2}$$

41. If $\sqrt{2}\sin\theta = 1$, find the value of $\sec^2\theta - \text{cosec}^2\theta$.

Ans : [Board Term-1 2012]

We have $\sqrt{2}\sin\theta = 1$

$$\sin\theta = \frac{1}{\sqrt{2}} = \sin 45^o$$

h120

Thus $\theta = 45^o$

Now $\sec^2\theta - \text{cosec}^2\theta = \sec^2 45^o - \text{cosec}^2 45^o$

$$= (\sqrt{2})^2 - (\sqrt{2})^2$$

$$= 0$$

42. If $4\cos\theta = 11\sin\theta$, find the value of $\frac{11\cos\theta - 7\sin\theta}{11\cos\theta + 7\sin\theta}$.

Ans : [Board Term-1 2012]

We have $4\cos\theta = 11\sin\theta$

or, $\cos\theta = \frac{11}{4}\sin\theta$

h121

Now $\frac{11\cos\theta - 7\sin\theta}{11\cos\theta + 7\sin\theta} = \frac{11\times\frac{11}{4}\sin\theta - 7\sin\theta}{11\times\frac{11}{4}\sin\theta + 7\sin\theta}$

$$= \frac{\sin\theta(\frac{121}{4} - 7)}{\sin\theta(\frac{121}{4} + 7)}$$

$$= \frac{121-28}{121+28} = \frac{93}{149}$$

43. If $\tan(A+B) = \sqrt{3}$, $\tan(A-B) = \frac{1}{\sqrt{3}}$ $0^\circ < A+B \leq 90^\circ$, then find A and B.

Ans : [Board Term-1 2012]

We have $\tan(A+B) = \sqrt{3} = \tan 60^o$

$$A + B = 60^o \quad ...(1)$$

Also $\tan(A-B) = \frac{1}{\sqrt{3}} = \tan 30^o$

$$A - B = 30^o \quad ...(2)$$

Adding equations (1) and (2), we obtain,

$$2A = 90^o$$

$$A = \frac{90^o}{2} = 45^o$$

h122

Substituting this value of A in equation (1), we get

$$B = 60^o - A = 60^o - 45^o = 15^o$$

Hence, $A = 45^o$ and $B = 15^o$

44. If $\cos(A-B) = \frac{\sqrt{3}}{2}$ and $\sin(A+B) = \frac{\sqrt{3}}{2}$, find $\sin A$ and B, where $(A+B)$ and $(A-B)$ are acute angles.

Ans : [Board Term-1 2012]

We have $\cos(A-B) = \frac{\sqrt{3}}{2} = \cos 30^o$

$$A - B = 30^o \quad ...(1)$$

Also $\sin(A+B) = \frac{\sqrt{3}}{2} = \sin 60^o$

$$A + B = 60^o \quad ...(2)$$

Adding equations (1) and (2), we obtain,

$$2A = 90^o$$

$$A = 45^o$$

h123

Substituting this value of A in equation (1), we get $B = 15^o$

45. Find the value of $\cos 2\theta$, if $2\sin 2\theta = \sqrt{3}$.

Ans : [Board Term-1 2012, Set-25]

We have $2\sin 2\theta = \sqrt{3}$

$$\sin 2\theta = \frac{\sqrt{3}}{2} = \sin 60^o$$

h125

$$2\theta = 60^o$$

Hence, $\cos 2\theta = \cos 60^o = \frac{1}{2}$.

46. Find the value of $\sin 30^o \cos 60^o + \cos 30^o \sin 60^o$ is it equal to $\sin 90^o$ or $\cos 90^o$?

Ans : [Board Term-1 2016]

$$\sin 30^o \cos 60^o + \cos 30^o \sin 60^o = \frac{1}{2}\times\frac{1}{2} + \frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2}$$

$$= \frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$$

It is equal to $\sin 90^o = 1$ but not equal to $\cos 90^o$ as $\cos 90^o = 0$.

h126

47. If $\sqrt{3}\sin\theta - \cos\theta = 0$ and $0^o < \theta < 90^o$, find the value of θ.

Ans : [Boar Term-1, 2012]

We have

h128

$$\sqrt{3}\sin\theta - \cos\theta = 0 \text{ and } 0^o < \theta < 90^o$$

$$\sqrt{3}\sin\theta = \cos\theta$$

$$\frac{\sin\theta}{\cos\theta} = \frac{1}{\sqrt{3}}$$

$$\tan\theta = \frac{1}{\sqrt{3}} = \tan 30^o \quad \left[\tan\theta = \frac{\sin\theta}{\cos\theta}\right]$$

$$\theta = 30^\circ$$

48. Evaluate : $\dfrac{\cos 45^\circ}{\sec 30^\circ} + \dfrac{1}{\sec 60^\circ}$

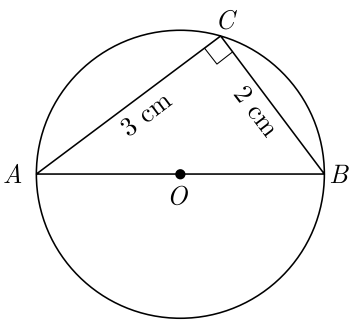
Ans : [Board Term-1 2012]

We have $\dfrac{\cos 45^\circ}{\sec 30^\circ} + \dfrac{1}{\sec 60^\circ} = \dfrac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}}} + \dfrac{1}{2}$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{2}$$

$$= \frac{\sqrt{6}}{4} + \frac{1}{2} = \frac{\sqrt{6}+2}{4}$$

49. In the given figure, AOB is a diameter of a circle with centre O, find $\tan A \tan B$.



Ans : [Board Term-1 2012]

In ΔABC, $\angle C$ is a angle in a semi-circle, thus

$$\angle C = 90^\circ$$

$$\tan A = \frac{BC}{AC} = \frac{2}{3}$$

and $$\tan B = \frac{AC}{BC} = \frac{3}{2}$$

$$\tan A \tan B = \frac{2}{3} \times \frac{3}{2} = 1$$

50. If $\sin\phi = \frac{1}{2}$, show that $3\cos\phi - 4\cos^3\phi = 0$.

Ans :

We have $$\sin\phi = \frac{1}{2}$$

$$\phi = 30^\circ$$

Now substituting this value of θ in LHS we have

$$3\cos\phi - 4\cos^3\phi = 3\cos 30^\circ - 4\cos^3 30^\circ$$

$$= 3\left(\frac{\sqrt{3}}{2}\right) - 4\left(\frac{\sqrt{3}}{2}\right)^3$$

$$= \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2}$$

$$= 0$$ Hence Proved

51. Express the trigonometric ratio of $\sec A$ and $\tan A$ in terms of $\sin A$.

Ans : [Board Term-1 2015]

We have $$\sec A = \frac{1}{\cos A} = \frac{1}{\sqrt{1-\sin^2 A}}$$

and $$\tan A = \frac{\sin A}{\cos A} = \frac{\sin A}{\sqrt{1-\sin^2 A}}$$

52. Prove that : $\dfrac{(\sin^4\theta + \cos^4\theta)}{1 - 2\sin^2\theta\cos^2\theta} = 1$

Ans : [Board Term-1 2015]

$$\frac{(\sin^4\theta + \cos^4\theta)}{1 - 2\sin^2\theta\cos^2\theta} = \frac{(\sin^2\theta)^2 + (\cos^2\theta)^2}{1 - 2\sin^2\theta\cos^2\theta}$$

$$= \frac{(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta}{1 - 2\sin^2\theta\cos^2\theta}$$

$$= \frac{1 - 2\sin^2\theta\cos^2\theta}{1 - 2\sin^2\cos^2\theta}$$

$$= 1$$

53. Prove that : $\sec^4\theta - \sec^2\theta = \tan^4\theta + \tan^2\theta$

Ans : [Board Term-1 2015]

We have

$$\sec^4\theta - \sec^2\theta = \sec^2\theta(\sec^2\theta - 1)$$

$$[[1 + \tan^2\theta = \sec^2\theta]]$$

$$= \sec^2\theta(\tan^2\theta)$$

$$= (1 + \tan^2\theta)\tan^2\theta$$

$$= \tan^2\theta + \tan^4\theta$$

Hence Proved.

54. Find the value of θ, if,
$\dfrac{\cos\theta}{1-\sin\theta} + \dfrac{\cos\theta}{1+\sin\theta} = 4; \theta \le 90^\circ$

Ans : [Board Term-1 2015]

We have $$\frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta} = 4$$

$$\frac{\cos\theta(1+\sin\theta)+\cos\theta(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)} = 4$$

$$\frac{\cos\theta[1+\sin\theta+1-\sin\theta]}{1-\sin^2\theta} = 4$$

$$\frac{\cos\theta(2)}{\cos^2\theta} = 4$$

$$\frac{1}{\cos\theta} = 2$$

$$\cos\theta = \frac{1}{2}$$

$$\cos\theta = \cos 60^\circ$$

Thus $\theta = 60^\circ$.

55. Prove that : $-1+\dfrac{\sin A\sin(90^\circ - A)}{\cot(90^\circ - A)} = -\sin^2 A$

Ans : [Board Term-1 2012]

$$-1+\frac{\sin A\sin(90^\circ-A)}{\cot(90^\circ-A)} = -\sin^2 A$$

$$\frac{\sin A\sin(90^\circ-A)}{\cot(90^\circ-A)} = 1-\sin^2 A$$

h160

$$\frac{\sin A\cos A}{\tan A} = \cos^2 A$$

$$\frac{\sin A\cos A}{\frac{\sin A}{\cos A}} = \cos^2 A$$

$$\frac{\cos A}{\sin A}\sin A\cos A = \cos^2 A$$

$\cos^2 A = \cos^2 A$ Hence Proved.

56. Prove that : $\sqrt{\dfrac{1-\cos A}{1+\cos A}} = \operatorname{cosec} A - \cot A$

Ans : [Board Term-1 2012]

$$\sqrt{\frac{1-\cos A}{1+\cos A}} = \sqrt{\frac{1-\cos A}{1+\cos A}\times\frac{1-\cos A}{1-\cos A}}$$

$$= \sqrt{\frac{(1-\cos A)^2}{(1-\cos^2 A)}}$$

h161

$$= \sqrt{\frac{(1-\cos A)^2}{\sin^2 A}}$$

$$= \frac{1-\cos A}{\sin A} = \frac{1}{\sin A} - \frac{\cos A}{\sin A}$$

$= \operatorname{cosec} A - \cot A$ Hence Proved.

57. If $\sin\theta - \cos\theta = \frac{1}{2}$, then find the value of $\sin\theta + \cos\theta$.

Ans : [Board Term-1 2013]

We have $\sin\theta - \cos\theta = \frac{1}{2}$

Squaring both sides, we get

h162

$$(\sin\theta-\cos\theta)^2 = \left(\frac{1}{2}\right)^2$$

$$\sin^2\theta+\cos^2\theta-2\sin\theta\cos\theta = \frac{1}{4}$$

$$1-2\sin\theta\cos\theta = \frac{1}{4}$$

$$2\sin\theta\cos\theta = 1-\frac{1}{4} = \frac{3}{4}$$

Again, $(\sin\theta+\cos\theta)^2 = \sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta$

$$= 1+2\sin\theta\cos\theta$$

$$= 1+\frac{3}{4} = \frac{7}{4}$$

Thus $\sin\theta+\cos\theta = \sqrt{\frac{7}{4}} = \frac{\sqrt{7}}{2}$

58. If θ be an acute angle and $5\operatorname{cosec}\theta = 7$, then evaluate $\sin\theta + \cos^2\theta - 1$.

Ans : [Board Term-1 2012]

We have $5\operatorname{cosec}\theta = 7$

h163

$$\operatorname{cosec}\theta = \frac{7}{5}$$

$$\sin\theta = \frac{5}{7} \qquad \left[\operatorname{cosec}\theta = \tfrac{1}{\sin\theta}\right]$$

$$\sin\theta+\cos^2\theta-1 = \sin\theta-(1-\cos^2\theta)$$

$$= \sin\theta-\sin^2\theta \quad [\sin^2\theta+\cos^2\theta=1]$$

$$= \frac{5}{7}-\left(\frac{5}{7}\right)^2 = \frac{35-25}{49} = \frac{10}{49}$$

59. If $\sin A = \frac{\sqrt{3}}{2}$, find the value of $2\cot^2 A - 1$.

Ans : [Board Term-1 2012]

Using $\cot^2\theta = -1+\operatorname{cosec}^2\theta$ we have

$$2\cot^2 A - 1 = 2(\operatorname{cosec}^2 A - 1) - 1$$

h164

$$= \frac{2}{\sin^2 A} - 3$$

$$= \frac{2}{\left(\frac{\sqrt{3}}{2}\right)^2} - 3 = \frac{8}{3} - 3 = \frac{-1}{3}$$

Thus $2\cot^2 A - 1 = \frac{-1}{3}$

THREE MARKS QUESTIONS

60. Show that : $\dfrac{\cos^2(45^\circ+\theta)+\cos^2(45^\circ-\theta)}{\tan(60^\circ+\theta)\tan(30^\circ-\theta)}=1$

Ans : [Board 2020 OD Standard]

$$\text{LHS} = \frac{\cos^2(45^\circ+\theta)+\cos^2(45^\circ-\theta)}{\tan(60^\circ+\theta)\tan(30^\circ-\theta)}$$

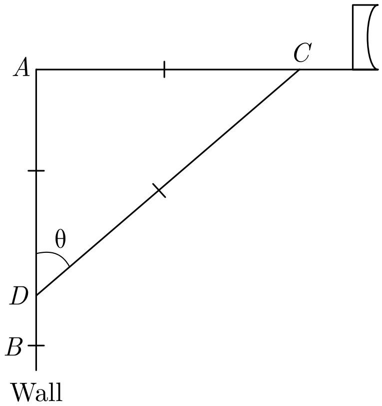
$$= \frac{\cos^2(45^\circ+\theta)+\sin^2(90^\circ-45^\circ+\theta)}{\tan(60^\circ+\theta)\cot(90^\circ-30^\circ+\theta)}$$

$$= \frac{\cos^2(45^\circ+\theta)+\sin^2(45^\circ+\theta)}{\tan(60^\circ+\theta)\cot(60^\circ+\theta)}$$

$$= \frac{1}{1} = 1 = \text{RHS}$$

h286

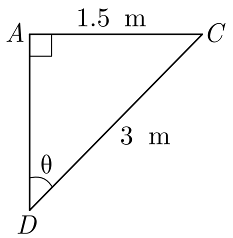
61. The rod of TV disc antenna is fixed at right angles to wall AB and a rod CD is supporting the disc as shown in Figure. If $AC = 1.5$ m long and $CD = 3$ m, find (i) $\tan\theta$ (ii) $\sec\theta + \operatorname{cosec}\theta$.



h287

Ans : [Board 2020 Delhi Standard]

From the given information we draw the figure as below



In right angle triangle $\Delta\, CAD$, applying Pythagoras theorem,

$$AD^2 + AC^2 = DC^2$$

$$AD^2 + (1.5)^2 = (3)^2$$

$$AD^2 = 9 - 2.25 = 6.75$$

$$AD = \sqrt{6.75} = 2.6 \text{ m (Approx)}$$

(i) $$\tan\theta = \frac{AC}{AD} = \frac{1.5}{2.6} = \frac{15}{26}$$

(ii) $$\sec\theta + \operatorname{cosec}\theta = \frac{CD}{AD} + \frac{CD}{AC} = \frac{3}{2.6} + \frac{3}{1.5} = \frac{41}{13}$$

62. Prove that : $\dfrac{\cot\theta + \operatorname{cosec}\theta - 1}{\cot\theta - \operatorname{cosec}\theta + 1} = \dfrac{1+\cot\theta}{\sin\theta}$

Ans : [Board 2020 Delhi Standard]

h288

$$\text{LHS} = \frac{\cot\theta + \operatorname{cosec}\theta - 1}{\cot\theta - \operatorname{cosec}\theta + 1}$$

$$= \frac{\frac{\cos\theta}{\sin\theta} + \frac{1}{\sin\theta} - 1}{\frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta} + 1}$$

$$= \frac{\sin\theta(\cos\theta + 1 - \sin\theta)}{\sin\theta(\cos\theta - 1 + \sin\theta)}$$

$$= \frac{\sin\theta\cos\theta + \sin\theta - \sin^2\theta}{\sin\theta(\cos\theta + \sin\theta - 1)}$$

$$= \frac{\sin\theta\cos\theta + \sin\theta - (1-\cos^2\theta)}{\sin\theta(\cos\theta + \sin\theta - 1)}$$

$$= \frac{\sin\theta(\cos\theta + 1) - (1-\cos^2\theta)}{\sin\theta(\cos\theta + \sin\theta - 1)}$$

$$= \frac{(1+\cos\theta)(\sin\theta - 1 + \cos\theta)}{\sin\theta(\cos\theta + \sin\theta - 1)}$$

$$= \frac{1+\cos\theta}{\sin\theta} = \text{RHS}$$

63. If $\sin\theta + \cos\theta = \sqrt{2}$ prove that $\tan\theta + \cot\theta = 2$

Ans : [Board 2020 OD Standard]

We have $$\sin\theta + \cos\theta = \sqrt{2}$$

Squaring both the sides, we get

$$(\sin\theta + \cos\theta)^2 = (\sqrt{2})^2$$

$$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 2$$

$$1 + 2\sin\theta\cos\theta = 2$$

$$2\sin\theta\cos\theta = 1$$

$$\sin\theta\cos\theta = \frac{1}{2} \quad ...(1)$$

h289

Now $$\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}$$

$$= \frac{1}{\sin\theta\cos\theta} = \frac{1}{\frac{1}{2}} = 2 = \text{RHS}$$

64. If $\sin\theta + \cos\theta = \sqrt{3}$, then prove that $\tan\theta + \cot\theta = 1$.

Ans : [Board 2020 SQP Standard]

Given, $\sin\theta + \cos\theta = \sqrt{3}$

Squaring above equation, we have

$$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 3$$
$$1 + 2\sin\theta\cos\theta = 3$$
$$2\sin\theta\cos\theta = 3 - 1 = 2$$
$$\sin\theta\cos\theta = 1$$

Now, $\tan\theta + \cot\theta = \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$$
$$= \frac{1}{\sin\theta\cos\theta}$$

Substituting value of $\sin\theta\cos\theta$ we have

$$\tan\theta + \cot\theta = \frac{1}{\sin\theta\cos\theta} = \frac{1}{1} = 1$$

65. If $1 + \sin^2\theta = 3\sin\theta\cos\theta$, prove that $\tan\theta = 1$ or ½.

Ans : [Board 2020 OD Standard]

We have, $1 + \sin^2\theta = 3\sin\theta\cos\theta$

Dividing by $\sin^2\theta$ on both sides, we get

$$\frac{1}{\sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta} = \frac{3\sin\theta\cos\theta}{\sin^2\theta}$$
$$\frac{1}{\sin^2\theta} + 1 = 3\cot\theta$$
$$\text{cosec}^2\theta + 1 = 3\cot\theta$$
$$1 + \cot^2\theta + 1 = 3\cot\theta$$
$$\cot^2\theta - 3\cot\theta + 2 = 0$$
$$\cot^2\theta - 2\cot\theta - \cot\theta + 2 = 0$$
$$\cot\theta(\cot\theta - 2) - 1(\cot\theta - 2) = 0$$
$$(\cot\theta - 2)(\cot\theta - 1) = 0$$
$$\cot\theta = 1 \text{ or } 2$$
$$\tan\theta = 1 \text{ or } ½.$$

66. Prove that $(\sin\theta + \text{cosec}\,\theta)^2 + (\cos\theta + \sec\theta)^2 = 7 + \tan^2\theta + \cot^2\theta$

Ans : [Board 2019 Delhi Standard]

$$\text{LHS} = (\sin\theta + \text{cosec}\,\theta)^2 + (\cos\theta + \sec\theta)^2$$
$$= (\sin^2\theta + \text{cosec}^2\theta + 2\sin\theta\,\text{cosec}\,\theta) +$$
$$+(\cos^2\theta + \sec^2\theta + 2\cos\theta\sec\theta)$$
$$= (\sin^2\theta + \cos^2\theta) + (\text{cosec}^2\theta + \sec^2\theta)$$
$$+ 2\sin\theta \times \frac{1}{\sin\theta} + 2\cos\theta \times \frac{1}{\cos\theta}$$
$$= 1 + (1 + \cot^2\theta) + (1 + \tan^2\theta) + 2 + 2$$
$$= 7 + \tan^2\theta + \cot^2\theta$$
$$= \text{RHS}$$

67. Prove that $(1 + \cot A - \text{cosec}\,A)(1 + \tan A + \sec A) = 2$

Ans : [Board 2019 Delhi]

$$\text{LHS} = (1 + \cot A - \text{cosec}\,A)(1 + \tan A + \sec A)$$
$$= \left(1 + \frac{\cos A}{\sin A} - \frac{1}{\sin A}\right)\left(1 + \frac{\sin A}{\cos A} + \frac{1}{\cos A}\right)$$
$$= \left(\frac{\sin A + \cos A - 1}{\sin A}\right)\left(\frac{\cos A + \sin A + 1}{\cos A}\right)$$
$$= \frac{(\sin A + \cos A - 1)(\cos A + \sin A + 1)}{\sin A\cos A}$$
$$= \frac{(\sin A + \cos A)^2 - (1)^2}{\sin A\cos A}$$
$$= \frac{\sin^2 A + \cos^2 A + 2\sin A\cos A - 1}{\sin A\cos A}$$
$$= \frac{1 + 2\sin A\cos A - 1}{\sin A\cos A}$$
$$= 2 = \text{RHS}$$

68. Prove that $\dfrac{\sin A - \cos A - 1}{\sin A + \cos A - 1} = \dfrac{1}{\sec A - \tan A}$

Ans : [Board 2019 Delhi]

$$\text{LHS} = \frac{\sin A - \cos A + 1}{\sin A + \cos A - 1}$$
$$= \frac{\sin A - \cos A + 1}{\sin A + \cos A - 1} \times \frac{1 + \sin A}{1 + \sin A}$$
$$= \frac{(\sin A - \cos A + 1)(1 + \sin A)}{\sin A + \cos A - 1 + \sin^2 A + \cos A\sin A - \sin A}$$
$$= \frac{(\sin A - \cos A + 1)(1 + \sin A)}{-1 + \cos A + (1 - \cos^2 A) + \sin A\cos A}$$
$$= \frac{(\sin A - \cos A + 1)(1 + \sin A)}{\cos A(1 - \cos A + \sin A)}$$
$$= \frac{1 + \sin A}{\cos A} = \frac{1}{\cos A} + \frac{\sin A}{\cos A}$$
$$= \sec A + \tan A$$
$$= \frac{(\sec A + \tan A)}{(\sec A - \tan A)} \times (\sec A - \tan A)$$

$$= \frac{\sec^2 A - \tan^2 A}{\sec A - \tan A}$$

$$= \frac{1}{\sec A - \tan A} = \text{RHS}$$

69. Prove that : $2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1 = 0$

Ans : [Board 2020 Delhi Standard]

$$\text{LHS} = 2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1$$

$$= 2[(\sin^2\theta)^3 + (\cos^2\theta)^3] - 3(\sin^4\theta + \cos^4\theta) + 1$$

$$= 2[(\sin^2\theta + \cos^2\theta)(\sin^4\theta - \sin^2\theta\cos^2\theta + \cos^4\theta] + -3(\sin^4\theta + \cos^4\theta) + 1$$

$$= 2(\sin^4\theta - \sin^2\theta\cos^2\theta + \cos^4\theta) - 3(\sin^4\theta + \cos^4\theta) + 1$$

$$= 2(\sin^4\theta + \cos^4\theta - \sin^2\theta\cos^2\theta) - 3(\sin^4\theta + \cos^4\theta) + 1$$

$$= -\sin^4\theta - \cos^4\theta - 2\sin^2\theta\cos^2\theta + 1$$

$$= -(\sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta) + 1$$

$$= -(\sin^2\theta + \cos^2\theta)^2 + 1$$

$$= -1 + 1 = 0 = \text{RHS}$$

70. Prove that $\frac{\tan^2 A}{\tan^2 A - 1} + \frac{\text{cosec}^2 A}{\sec^2 A - \text{cosec}^2 A} = \frac{1}{1 - 2\cos^2 A}$

Ans : [Board 2019 Delhi]

$$\text{LHS} = \frac{\tan^2 A}{\tan^2 A - 1} + \frac{\text{cosec}^2 A}{\sec^2 A - \text{cosec}^2 A}$$

$$= \frac{\frac{\sin^2 A}{\cos^2 A}}{\frac{\sin^2 A}{\cos^2 A} - 1} + \frac{\frac{1}{\sin^2 A}}{\frac{1}{\cos^2 A} - \frac{1}{\sin^2 A}}$$

$$= \frac{\frac{\sin^2 A}{\cos^2 A}}{\frac{\sin^2 A - \cos^2 A}{\cos^2 A}} + \frac{\frac{1}{\sin^2 A}}{\frac{\sin^2 A - \cos^2 A}{\cos^2 A \sin^2 A}}$$

$$= \frac{\sin^2 A}{\sin^2 A - \cos^2 A} + \frac{\cos^2 A}{\sin^2 A - \cos^2 A}$$

$$= \frac{1}{1 - \cos^2 A - \cos^2 A}$$

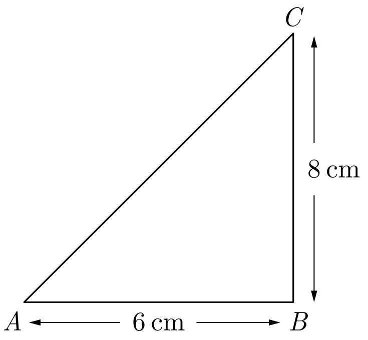
$$= \frac{1}{1 - 2\cos^2 A}$$

$$= \text{RHS}$$

71. If in a triangle ABC right angled at B, $AB = 6$ units and $BC = 8$ units, then find the value of $\sin A \cos C + \cos A \sin C$.

Ans : [Board Term-1 2016]

As per question statement figure is shown below.



We have $AC^2 = 8^2 + 6^2 = 100$

$AC = 10$ cm

Now $\sin A = \frac{BC}{AC} = \frac{8}{10}$;

$\cos A = \frac{AB}{AC} = \frac{6}{10}$

and $\sin C = \frac{AB}{AC} = \frac{6}{10}$;

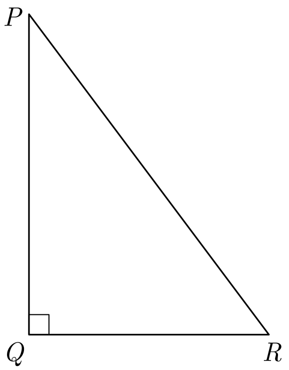
$\cos C = \frac{BC}{AC} = \frac{8}{10}$

Thus $\sin A \cos C + \cos A \sin C = \frac{8}{10} \times \frac{8}{10} + \frac{6}{10} \times \frac{6}{10}$

$$= \frac{64}{100} + \frac{36}{100}$$

$$= \frac{100}{100} = 1$$

72. In the given $\angle PQR$, right-angled at Q, $QR = 9$ cm and $PR - PQ = 1$ cm. Determine the value of $\sin R + \cos R$.



Ans : [Board Term-1 2015]

Using Pythagoras theorem we have

$$PQ^2 + QR^2 = PR^2$$

$$PQ^2 + 9^2 = (PQ+1)^2$$

h133

$$PQ^2 + 81 = (PQ+1)^2$$

$$PQ^2 + 81 = PQ^2 + 1 + 2PQ$$

$$PQ = 40$$

Since $PR - PQ = 1$, thus,

$$PR = 1 + 40 = 41$$

$$\sin R + \cos R = \frac{40}{41} + \frac{9}{41} = \frac{49}{41}$$

73. If $\cos(40^\circ + x) = \sin 30^\circ$, find the value of x.

Ans : [Board Term-1 2015]

We have

$$\cos(40^\circ - x) = \sin 30^\circ$$

$$\cos(40^\circ + x) = \sin(90^\circ - 60^\circ)$$

$$\cos(40^\circ + x) = \cos 60^\circ$$

$$40^\circ + x = 60^\circ$$

$$x = 60^\circ - 40^\circ = 20^\circ$$

h135

Thus $x = 20^\circ$.

74. Evaluate : $\dfrac{5\cos^2 60^\circ + 4\cos^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 60^\circ}$

Ans : [Board Term-1 2013]

$$\frac{5\cos^2 60^\circ + 4\cos^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 60^\circ}$$

$$= \frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{\sqrt{3}}{2}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$$

h136

$$= \frac{\frac{5}{4} + 3 - 1}{\frac{1}{4} + \frac{1}{4}}$$

$$= \frac{\frac{5}{4} + 2}{\frac{1}{2}} = \frac{\frac{13}{4}}{\frac{1}{2}} = \frac{13}{2}$$

75. Verify : $\sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}} = \dfrac{\sin\theta}{1+\cos\theta}$, for $\theta = 60^\circ$

Ans :

$$\text{LHS} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \sqrt{\frac{1-\cos 60^\circ}{1+\cos 60^\circ}}$$

$$= \sqrt{\frac{1-\frac{1}{2}}{1+\frac{1}{2}}} = \sqrt{\frac{\frac{1}{2}}{\frac{3}{2}}} = \frac{1}{\sqrt{3}} \quad \left(\cos 60^\circ = \frac{1}{2}\right)$$

$$\text{RHS} = \frac{\sin\theta}{1+\cos\theta} = \frac{\sin 60^\circ}{1+\cos 60^\circ}$$

$$= \frac{\frac{\sqrt{3}}{2}}{1+\frac{1}{2}} = \frac{\frac{\sqrt{3}}{2}}{\frac{3}{2}} = \frac{1}{\sqrt{3}}$$

h139

$$\text{RHS} = \text{LHS}$$

Hence, relation is verified for $\theta = 60^\circ$.

76. If $\tan A + \cot A = 2$, then find the value of $\tan^2 A + \cot^2 A$.

Ans : [Board Term-1 2015]

We have $\tan A + \cot A = 2$

Squaring both sides, we have

$$(\tan A + \cot A)^2 = (2)^2$$

$$\tan^2 A + \cot^2 A + 2\tan A \cot A = 4$$

h140

$$\tan^2 A + \cot^2 A + 2\tan A \times \frac{1}{\tan A} = 4$$

$$\tan^2 A + \cot^2 A + 2 = 4$$

$$\tan^2 A + \cot^2 A = 4 - 2$$

$$\tan^2 A + \cot^2 A = 2$$

77. If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, show that $\cos\theta - \sin\theta = \sqrt{2}\cos\theta$.

Ans : [Board Term-1 2011]

We have $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$

We have $\sin\theta = \sqrt{2}\cos\theta - \cos\theta$

h141

$$= (\sqrt{2} - 1)\cos\theta$$

$$= \frac{(\sqrt{2}-1)(\sqrt{2}+1)}{(\sqrt{2}+1)}\cos\theta$$

Thus $\sin\theta = \dfrac{1}{\sqrt{2}+1}\cos\theta$

$$(\sqrt{2}+1)\sin\theta = \cos\theta$$

$$\sqrt{2}\sin\theta + \sin\theta = \cos\theta$$

$$\cos\theta - \sin\theta = \sqrt{2}\sin\theta$$

Hence proved.

78. Prove that : $\dfrac{\cos A}{1-\tan A} + \dfrac{\sin A}{1-\cot A} = \sin A + \cos A$.

Ans : [Board Term-1 2013, 2011]

$$\text{LHS} = \frac{\cos A}{1-\tan A} + \frac{\sin A}{1-\cot A}$$

h142

$$= \frac{\cos A}{1-\left(\frac{\sin A}{\cos A}\right)} + \frac{\sin A}{1-\left(\frac{\cos A}{\sin A}\right)}$$

$$= \frac{\cos^2 A}{\cos A - \sin A} + \frac{\sin^2 A}{\sin A - \cos A}$$

$$= \frac{\cos^2 A}{\cos A - \sin A} - \frac{\sin^2 A}{\cos A - \sin A}$$

$$= \frac{\cos^2 A - \sin^2 A}{\cos A - \sin A}$$

$$= \frac{(\cos A - \sin A)(\cos A + \sin A)}{(\cos A - \sin A)}$$

$$= \cos A + \sin A$$

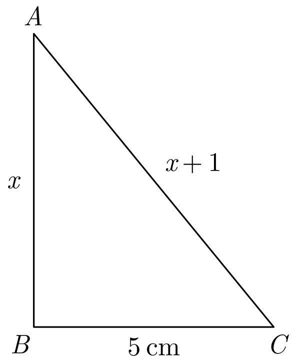
$$= \sin A + \cos A$$

$= \text{RHS}$ Hence proved.

79. In ΔABC, $\angle B = 90°$, $BC = 5$ cm, $AC - AB = 1$, Evaluate : $\frac{1+\sin C}{1+\cos C}$.

Ans : [Board Term-1 2011]

As per question we have drawn the figure given below.



We have $AC - AB = 1$

Let $AB = x$, then we have

$$AC = x+1$$

Now $AC^2 = AB^2 + BC^2$

$$(x+1)^2 = x^2 + 5^2$$

$$x^2 + 2x + 1 = x^2 + 25$$

$$2x = 24$$

$$x = \frac{24}{2} = 12 \text{ cm}$$

h143

Hence, $AB = 12$ cm and $AC = 13$ cm

Now $\sin C = \frac{AB}{AC} = \frac{12}{13}$

$$\cos C = \frac{BC}{AC} = \frac{5}{13}$$

Now $\frac{1+\sin C}{1+\cos C} = \frac{1+\frac{12}{13}}{1+\frac{5}{13}} = \frac{\frac{25}{13}}{\frac{18}{13}} = \frac{25}{18}$

80. Prove that : $\frac{\cos A}{1+\tan A} - \frac{\sin A}{1+\cot A} = \cos A - \sin A$

Ans : [Board Term-1 2016]

$$\frac{\cos A}{1+\tan A} - \frac{\sin A}{1+\cot A}$$

$$= \frac{\cos A}{1+\frac{\sin A}{\cos A}} - \frac{\sin A}{1+\frac{\cos A}{\sin A}}$$

h165

$$= \frac{\cos^2 A}{\cos A + \sin A} - \frac{\sin^2 A}{\sin A + \cos A}$$

$$= \frac{\cos^2 A - \sin^2 A}{(\sin A + \cos A)}$$

$$= \frac{(\cos A + \sin A)(\cos A - \sin A)}{\sin A + \cos A}$$

$= \cos A - \sin A$ Hence Proved.

81. If $b\cos\theta = a$, then prove that $\text{cosec}\,\theta + \cot\theta = \sqrt{\frac{b+a}{b-a}}$.

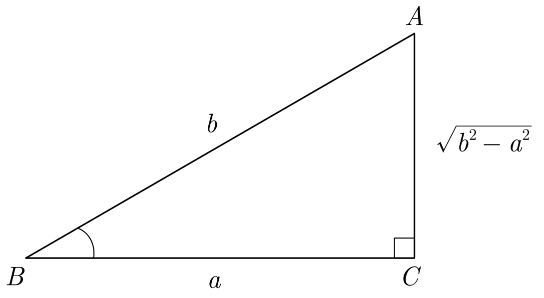
Ans : [Board Term-1 2015]

We have $b\cos\theta = a$

or, $\cos\theta = \frac{a}{b}$

h166

Now consider the triangle shown below.



$$AC^2 = AB^2 - BC^2$$

or, $$\cos\theta = \frac{a}{b}$$

$$AC = \sqrt{b^2-a^2}$$

Now $$\operatorname{cosec}\theta = \frac{b}{\sqrt{b^2-a^2}}, \cot\theta = \frac{a}{\sqrt{b^2-a^2}}$$

$$\operatorname{cosec}\theta + \cot\theta = \frac{b+a}{\sqrt{b^2-a^2}} = \sqrt{\frac{b+a}{b-a}}$$

82. Prove that : $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3 - \cos\theta} = \tan\theta$

Ans : [Bard Term-1 2015]

$$\frac{\sin\theta - 2\sin^3\theta}{2\cos^3 - \cos\theta} = \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2\cos^2\theta - 1)}$$

$$= \frac{\sin\theta(\sin^2\theta + \cos^2\theta - 2\sin^2\theta)}{\cos\theta(2\cos^2\theta - \sin^2\theta - \cos^2\theta)}$$

$$= \frac{\tan\theta(\cos^2\theta - \sin^2\theta)}{(\cos^2\theta - \sin^2\theta)}$$

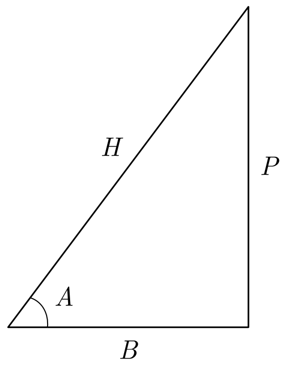
$$= \tan\theta$$

h167

83. When is an equation called 'an identity'. Prove the trigonometric identity $1 + \tan^2 A = \sec^2 A$.

Ans : [Board Term-1 2015, NCERT]

Equations that are true no matter what value is plugged in for the variable. On simplifying an identity equation, one always get a true statement. Consider the triangle shown below.



Let $\tan A = \dfrac{P}{B}$ and $\sec A = \dfrac{H}{B}$

h168

$$H^2 = P^2 + B^2$$

Now $$1 + \tan^2 A = 1 + \left(\frac{P}{B}\right)^2 = 1 + \frac{P^2}{B^2}$$

$$= \frac{B^2 + P^2}{B^2} = \frac{H^2}{B^2}$$

$$= \left(\frac{H}{B}\right)^2$$

$$= \sec^2 A$$ Hence Proved.

84. Prove that : $(\cot\theta - \operatorname{cosec}\theta)^2 = \dfrac{1-\cos\theta}{1+\cos\theta}$

Ans : [Board Term-1 2015]

$$\cot\theta - \operatorname{cosec}\theta = \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}$$

$$(\cot\theta - \operatorname{cosec}\theta)^2 = \left(\frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)^2$$

h169

$$= \left(\frac{\cos\theta - 1}{\sin\theta}\right)^2$$

$$= \frac{(1-\cos\theta)^2}{\sin^2\theta} \quad [[\sin^2\theta + \cos^2\theta = 1]]$$

$$= \frac{(1-\cos\theta)^2}{(1-\cos^2\theta)}$$

$$= \frac{(1-\cos\theta)(1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$$

$$= \frac{1-\cos\theta}{1+\cos\theta}$$ Hence Proved.

85. Prove that :

$(\operatorname{cosec}\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta) = 1$

Ans : [Board Term-1 2015]

$$\text{LHS} = (\operatorname{cosec}\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta)$$

$$= \left(\frac{1}{\sin\theta} - \sin\theta\right)\left(\frac{1}{\cos\theta} - \cos\theta\right)\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$$

$$= \left(\frac{1-\sin^2\theta}{\sin\theta}\right)\left(\frac{1-\cos^2\theta}{\cos\theta}\right)\left(\frac{\sin^2\theta+\cos^2\theta}{\sin\theta.\cos\theta}\right)$$

$$= \frac{\cos^2\theta}{\sin\theta} \times \frac{\sin^2\theta}{\cos\theta} \times \left(\frac{1}{\sin\theta\cos\theta}\right) \quad [\sin^2\theta + \cos^2\theta = 1]$$

$$= \cos\theta\sin\theta \times \frac{1}{\sin\theta\cos\theta} = 1$$

h170

86. Show that :
$\operatorname{cosec}^2\theta - \tan^2(90^\circ - \theta) = \sin^2\theta + \sin(90^\circ - \theta)$

Ans : [Board Term-1 2013]

$$\operatorname{cosec}^2\theta - \tan^2(90^\circ - \theta)$$

$$= \operatorname{cosec}^2\theta - \cot^2\theta$$

h171

$$= \frac{1}{\sin^2\theta} - \frac{\cos^2\theta}{\sin^2\theta}$$

$$= \frac{1-\cos^2\theta}{\sin^2\theta} = \frac{\sin^2\theta}{\sin^2\theta}$$

$$= 1$$

$$= \sin^2\theta + \cos^2\theta$$

$$= \sin^2\theta + \sin^2(90^\circ - \theta)$$

Hence Proved

87. Prove that : $\frac{\operatorname{cosec}^2\theta}{\operatorname{cosec}\theta - 1} - \frac{\operatorname{cosec}^2\theta}{\operatorname{cosec}\theta + 1} = 2\sec^2\theta$

Ans : [Board Term-1 2013]

We have

$$\frac{\operatorname{cosec}^2\theta}{\operatorname{cosec}\theta - 1} - \frac{\operatorname{cosec}^2\theta}{\operatorname{cosec}\theta + 1} = \operatorname{cosec}^2\theta\left[\frac{1}{\frac{1}{\sin\theta} - 1} - \frac{1}{\frac{1}{\sin\theta} + 1}\right]$$

$$= \operatorname{cosec}^2\theta\left[\frac{\sin\theta}{1-\sin\theta} - \frac{\sin\theta}{1+\sin\theta}\right]$$

$$= \frac{1}{\sin^2\theta}\sin\theta\left[\frac{(1+\sin\theta)-(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}\right]$$

h172

$$= \frac{1}{\sin\theta}\left[\frac{2\sin\theta}{1-\sin^2\theta}\right]$$

$$= \frac{2}{\cos^2\theta} = 2\sec^2\theta$$

Hence Proved

88. Prove that :

$$\frac{1}{\operatorname{cosec}A - \cot A} - \frac{1}{\sin A} = \frac{1}{\sin A} - \frac{1}{\operatorname{cosec}A + \cot A}.$$

Ans : [Board Term-1 2011]

$$\frac{1}{\operatorname{cosec}A - \cot A} - \frac{1}{\sin A} = \frac{1}{\sin A} - \frac{1}{\operatorname{cosec}A + \cot A}$$

$$\frac{1}{\operatorname{cosec}A - \cot A} + \frac{1}{\operatorname{cosec}A + \cot A} = \frac{1}{\sin A} + \frac{1}{\sin A}$$

$$\frac{1}{\operatorname{cosec}A - \cot A} + \frac{1}{\operatorname{cosec}A + \cot A} = \frac{2}{\sin A}$$

$$\frac{\operatorname{cosec}A + \cot A + \operatorname{cosec}A - \cot A}{(\operatorname{cosec}A - \cot A)(\operatorname{cosec}A + \cot a)} = \frac{2}{\sin A}$$

h173

$$\frac{2\operatorname{cosec}A}{\operatorname{cosec}^2A - \cot^2A} = \frac{2}{\sin A}$$

$$\frac{2\frac{1}{\sin A}}{1} = \frac{2}{\sin A}$$

$$\frac{2}{\sin A} = \frac{2}{\sin A} \quad \text{Hence Proved.}$$

89. If $\sec\theta = x + \frac{1}{4x}$ prove that $\sec\theta + \tan\theta = 2x$ or, $\frac{1}{2x}$

Ans : [Board Term-1 2011]

We have $\sec\theta = x + \frac{1}{4x}$ (1)

h174

Squaring both side we have

$$\sec^2\theta = x^2 + 2x\frac{1}{4x} + \frac{1}{16x^2}$$

$$1 + \tan^2\theta = x^2 + \frac{1}{2} + \frac{1}{16x^2}$$

$$\tan^2\theta = x^2 + \frac{1}{2} + \frac{1}{16x^2} - 1$$

$$= x^2 - \frac{1}{2} + \frac{1}{16x^2}$$

$$= x^2 - 2x\frac{1}{4x} + \frac{1}{16x^2}$$

$$\tan^2\theta = \left(x - \frac{1}{4x}\right)^2$$

Taking square root both sides we obtain

$$\tan\theta = \pm\left(x - \frac{1}{4x}\right)$$

Now $\tan\theta = x - \frac{1}{4x}$ (2)

or $\tan\theta = -\left(x - \frac{1}{4x}\right) = -x + \frac{1}{4x}$ (3)

Adding (1) and (2) we have

$$\tan\theta + \sec\theta = 2x$$

Adding (1) and (3) we have

$$\sec\theta + \tan\theta = \frac{1}{4x} + \frac{1}{4x} = \frac{1}{2x}$$ Hence proved.

90. Prove that : $\dfrac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} + \dfrac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = \dfrac{2}{2\sin^2\theta - 1}$

Ans : [Board Term-1 2011]

$$\text{LHS} = \frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} + \frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta}$$

$$= \frac{(\sin\theta - \cos\theta)^2 + (\sin\theta + \cos\theta)^2}{\sin^2\theta - \cos^2\theta}$$

$$= \frac{(\sin^2\theta + \cos^2\theta) - 2\sin\theta\cos\theta + (\sin^2\theta + \cos^2\theta) + 2\sin\theta\cos\theta}{\sin^2\theta - (1 - \sin^2\theta)}$$

$$= \frac{1+1}{\sin^2\theta - 1 + \sin^2\theta}$$

$$= \frac{2}{2\sin^2\theta - 1} = \text{RHS}$$

Hence Proved.

91. If $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ and $x\sin\theta = y\cos\theta$, prove that $x^2 + y^2 = 1$.

Ans : [Board Term-1 2011]

We have $$x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta \quad (1)$$

and $$x\sin\theta = y\cos\theta$$

or, $$x = \frac{y\cos\theta}{\sin\theta} \quad (2)$$

Eliminating x from equation (1) and (2) we obtain,

$$\frac{y\cos\theta}{\sin\theta}\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$$

$$y\cos\theta\sin^2\theta + y\cos^3\theta = \sin\theta\cos\theta$$

$$y\cos\theta[\sin^2\theta + \cos^2\theta] = \sin\theta\cos\theta$$

$$y(\sin^2\theta + \cos^2\theta) = \sin\theta$$

$$y = \sin\theta \quad ...(3)$$

Substituting this value of y in equation (2) we have,

$$x = \cos\theta \quad (4)$$

Squaring and adding equation (3) and (4), we get

$$x^2 + y^2 = \cos^2\theta + \sin^2\theta = 1$$ Hence Proved.

92. Prove that $\dfrac{\cos^3\theta + \sin^3\theta}{\cos\theta + \sin\theta} + \dfrac{\cos^3\theta - \sin^3\theta}{\cos\theta - \sin\theta} = 2$

Ans : [Board Term-1 2011]

$$X = \frac{\cos^3\theta + \sin^3\theta}{\cos\theta + \sin\theta}$$

$$= \frac{(\cos\theta + \sin\theta)(\cos^2\theta + \sin^2\theta - \sin\theta\cos\theta)}{(\cos\theta + \sin\theta)}$$

$$= (1 - \sin\theta\cos\theta)$$

$$Y = \frac{\cos^3\theta - \sin^3\theta}{\cos\theta - \sin\theta}$$

$$= \frac{(\cos\theta - \sin\theta)(\cos^2\theta + \sin^2\theta + \sin\theta\cos\theta)}{(\cos\theta - \sin\theta)}$$

$$= (1 + \sin\theta\cos\theta)$$

Now given expression

$$X + Y = \frac{\cos^3\theta + \sin^3\theta}{\cos\theta + \sin\theta} + \frac{\cos^3\theta - \sin^3\theta}{\cos\theta - \sin\theta}$$

$$= (1 - \sin\theta\cos\theta) + (1 + \sin\theta\cos\theta)$$

$$= 2 - \sin\theta\cos\theta + \sin\theta\cos\theta$$

$$= 2 = \text{RHS}$$ Hence Proved.

93. Express : $\sin A, \tan A$ and $\operatorname{cosec} A$ in terms of $\sec A$.

Ans : [Board Term-1 2011]

(1) $$\sin^2 A + \cos^2 A = 1$$

$$\sin A = \sqrt{1 - \cos^2 A}$$

$$= \sqrt{1 - \frac{1}{\sec^2 A}}$$

$$= \sqrt{\frac{\sec^2 A - 1}{\sec^2 A}} = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$$

(2) $$\tan A = \frac{\sin A}{\cos A} = \sin A \sec A$$

$$= \frac{\sqrt{\sec^2 A - 1}}{\sec A} \times \sec A$$

$$= \sqrt{\sec^2 A - 1}$$

(3) $$\operatorname{cosec} A = \frac{1}{\sin A} = \frac{\sec A}{\sqrt{\sec^2 A - 1}}$$

94. If $\sin\theta + \cos\theta = \sqrt{2}$, then evaluate $\tan\theta + \cot\theta$.

Ans : [Board SQP 2018]

We have $$\sin\theta + \cos\theta = \sqrt{2}$$

Squaring both sides, we get

$$(\sin\theta + \cos\theta)^2 = (\sqrt{2})^2$$

$$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 2$$

$$1 + 2\sin\theta\cos\theta = 2$$

$$2\sin\theta\cos\theta \quad 2-1=1$$

$$\frac{1}{\sin\theta\cos\theta} = 2$$

Now, $\tan\theta+\cot\theta = \frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}$

$$= \frac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}$$

$$= \frac{1}{\cos\theta\sin\theta} = 2$$

FOUR MARKS QUESTIONS

95. If $\sin\theta+\cos\theta=\sqrt{3}$, then prove that $\tan\theta+\cot\theta=1$.

Ans : [Board 2020 Delhi Standard]

We have $\sin\theta+\cos\theta = \sqrt{3}$

Squaring both the sides, we get

$$(\sin\theta+\cos\theta)^2 = (\sqrt{3})^2$$

$$\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta = 3$$

$$1+2\sin\theta\cos\theta = 3$$

$$2\sin\theta\cos\theta = 3-1=2$$

$$\sin\theta\cos\theta = 1 \quad ...(1)$$

h297

Now $\tan\theta+\cot\theta = \frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}$

$$= \frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}$$

or $\tan\theta+\cot\theta = \frac{1}{\sin\theta\cos\theta}$

Substituting the value of $\sin\theta\cos\theta$ from equation (1) we have

$$\tan\theta+\cot\theta = \frac{1}{1} = 1$$

Hence, $\tan\theta+\cot\theta = 1$

96. If $\sec\theta = x+\frac{1}{4x}, x\neq 0$ find $(\sec\theta+\tan\theta)$.

Ans : [Board 2019 Delhi]

We have $\sec\theta = x+\frac{1}{4x}$ ...(1)

Since, $\tan^2\theta = \sec^2\theta-1$

h298

Substituting value of $\sec\theta$ we have

$$\tan^2\theta = \left(x+\frac{1}{4x}\right)^2-1$$

$$= x^2+\frac{2x}{4x}+\frac{1}{16x^2}-1$$

$$= x^2+\frac{1}{16x^2}-\frac{1}{2}$$

$$= \left(x-\frac{1}{4x}\right)^2$$

$$\tan\theta = \pm\left(x-\frac{1}{4x}\right)$$

When $\sec\theta = x+\frac{1}{4x}$ and $\tan\theta = x-\frac{1}{4x}$ we have

$$\sec\theta+\tan\theta = \left(x+\frac{1}{4x}\right)+\left(x-\frac{1}{4x}\right) = 2x$$

When $\sec\theta = x+\frac{1}{4x}$ and $\tan\theta = -\left(x-\frac{1}{4x}\right)$ we have

$$\sec\theta+\tan\theta = \left(x+\frac{1}{4x}\right)+\left\{-\left(x-\frac{1}{4x}\right)\right\}$$

$$= x+\frac{1}{4x}-x+\frac{1}{4x}$$

$$= \frac{2}{4x} = \frac{1}{2x}$$

97. If $\sin A = \frac{3}{4}$ calculate $\sec A$.

Ans : [Board 2019 OD]

We have $\sin A = \frac{3}{4}$

h299

Now $\cos^2 A = 1-\sin^2 A$

$$\cos^2 A = 1-\left(\frac{3}{4}\right)^2 = 1-\frac{9}{16} = \frac{7}{16}$$

$$\cos A = \frac{\sqrt{7}}{4}$$

Thus $\sec A = \frac{1}{\cos A} = \frac{4}{\sqrt{7}}$

98. Prove that: $\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta} = 1+\sec\theta\,\mathrm{cosec}\,\theta$

Ans : [Board 2019 OD]

$$\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta} = \frac{\tan\theta}{1-\frac{1}{\tan\theta}}+\frac{\frac{1}{\tan\theta}}{1-\tan\theta}$$

$$= \frac{\tan^2\theta}{\tan\theta-1}+\frac{1}{\tan\theta(1-\tan\theta)}$$

$$= \frac{\tan^2\theta}{\tan\theta-1}-\frac{1}{\tan\theta(\tan\theta-1)}$$

$$= \frac{\tan^3\theta-1}{\tan\theta(\tan\theta-1)}$$

$$= \frac{(\tan\theta-1)(\tan^2\theta+1+\tan\theta)}{\tan\theta(\tan\theta-1)}$$

$$= \frac{\tan^2\theta+1+\tan\theta}{\tan\theta}$$

$$= \tan\theta+\cot\theta+1$$

$$= \frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}+1$$

$$= \frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}+1$$

$$= \frac{1}{\sin\theta\cos\theta}+1$$

$$= \operatorname{cosec}\theta\sec\theta+1$$

$$= 1+\sec\theta\operatorname{cosec}\theta$$ Hence Proved

99. Prove that: $\frac{\sin\theta}{\cot\theta+\operatorname{cosec}\theta} = 2+\frac{\sin\theta}{\cot\theta-\operatorname{cosec}\theta}$

Ans : [Board 2019 OD]

$$\text{LHS} = \frac{\sin\theta}{\cot\theta+\operatorname{cosec}\theta}$$

$$= \frac{\sin\theta}{\frac{\cos\theta}{\sin\theta}+\frac{1}{\sin\theta}} = \frac{\sin^2\theta}{\cos\theta+1}$$

$$= \frac{1-\cos^2\theta}{\cos\theta+1} = \frac{(1-\cos\theta)(1+\cos\theta)}{\cos\theta+1}$$

$$= 1-\cos\theta \qquad ...(1)$$

Now, $$\text{RHS} = 2+\frac{\sin\theta}{\cot\theta-\operatorname{cosec}\theta}$$

$$= 2+\frac{\sin\theta}{\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}} = 2+\frac{\sin^2\theta}{\cos\theta-1}$$

$$2+\frac{1-\cos^2\theta}{\cos\theta-1} = 2-\frac{(\cos^2\theta-1)}{(\cos\theta-1)}$$

$$= 2-\frac{(\cos\theta-1)(\cos\theta+1)}{\cos\theta-1}$$

$$= 2-(\cos\theta+1) = 1-\cos\theta$$

$$= \text{LHS}$$ Hence Proved

100. Find A and B if $\sin(A+2B) = \frac{\sqrt{3}}{2}$ and $\cos(A+4B) = 0$, where A and B are acute angles.

Ans : [Board 2019 OD]

We have $$\sin(A+2B) = \frac{\sqrt{3}}{2}$$

$$\sin(A+2B) = \sin 60^\circ \quad \left(\sin 60^\circ = \tfrac{\sqrt{3}}{2}\right)$$

$$A+2B = 60^\circ \qquad ...(1)$$

Also, given $$\cos(A+4B) = 0$$

$$\cos(A+4B) = \cos 90^\circ \quad (\cos 90^\circ = 0)$$

$$A+4B = 90^\circ \qquad ...(2)$$

Subtracting equation (2) from equation (1) we get

$$-2B = -30^\circ \Rightarrow B = 15^\circ$$

From equation (1) we have

$$A+2(15^\circ) = 60^\circ$$

$$A = 60^\circ-30^\circ$$

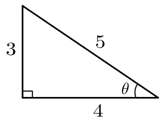
$= 30^\circ$

Hence angle $A = 30^\circ$ and angle $B = 15^\circ$.

101. If $4\tan\theta = 3$, evaluate $\left(\frac{4\sin\theta-\cos\theta+1}{4\sin\theta+\cos\theta-1}\right)$

Ans : [Board 2018]

We have $$4\tan\theta = 3 \Rightarrow \tan\theta = \frac{3}{4}$$



We know very well that if $\tan\theta = \frac{3}{4}$, then

$$\sin\theta = \frac{3}{5} \text{ and } \cos\theta = \frac{4}{5}$$

Substituting above values in given expression,

$$\frac{4\sin\theta-\cos\theta+1}{4\sin\theta+\cos\theta-1} = \frac{4\times\frac{3}{5}-\frac{4}{5}+1}{4\times\frac{3}{5}+\frac{4}{5}-1} = \frac{13}{11}$$

102. Evaluate :

$\tan^2 30^\circ \sin 30^\circ + \cos 60^\circ \sin^2 90^\circ \tan^2 60^\circ - 2\tan 45^\circ \cos^2 0^\circ \sin 90^\circ$

Ans : [Board Term-1 2015]

$\tan^2 30^\circ \sin 30^\circ + \cos 60^\circ \sin^2 90^\circ \tan^2 60^\circ - 2\tan 45^\circ \cos^2 0^\circ \sin 90^\circ$

$$= \left(\frac{1}{\sqrt{3}}\right)^2 \times \frac{1}{2} + \frac{1}{2}\times(1)^2\times(\sqrt{3})^2 - 2\times 1\times 1^2\times 1$$

$$= \frac{1}{3} \times \frac{1}{2} + \frac{1}{2} \times 3 - 2$$

$$= \frac{1}{6} + \frac{3}{2} - 2 = \frac{1+9-12}{6} = -\frac{2}{6} = -\frac{1}{3}$$

h144

103. Given that

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B},$$

find the values of $\tan 75^{\circ}$ and $\tan 90^{\circ}$ by taking suitable values of A and B.

Ans : [Board Term-1 2012, NCERT]

We have $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

h145

(i)

$$\tan 75^{\circ} = \tan(45^{\circ} + 30^{\circ})$$

$$= \frac{\tan 45^{\circ} + \tan 30^{\circ}}{1 - \tan 45^{\circ} \tan 30^{\circ}}$$

$$= \frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$$= \frac{(\sqrt{3}+1)(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$$

$$= \frac{3 + 2\sqrt{3} + 1}{(\sqrt{3})^2 - (1)^2} = \frac{4 + 2\sqrt{3}}{2}$$

$$= 2 + \sqrt{3}$$

Hence $\tan 75^{\circ} = 2 + \sqrt{3}$

(ii)

$$\tan 90^{\circ} = \tan(60^{\circ} + 30^{\circ})$$

$$= \frac{\tan 60^{\circ} + \tan 30^{\circ}}{1 - \tan 60^{\circ} \tan 30^{\circ}}$$

$$= \frac{\sqrt{3} + \frac{1}{\sqrt{3}}}{1 - \sqrt{3} \times \frac{1}{\sqrt{3}}} = \frac{\frac{3+1}{\sqrt{3}}}{0}$$

Hence, $\tan 90^{\circ} = \infty$

104. Evaluate :

$$\sin^2 30^{\circ} \cos^2 45^{\circ} + 4\tan^2 30^{\circ} + \frac{1}{2}\sin 90^{\circ} - 2\cos^2 90^{\circ} + \frac{1}{24}$$

Ans : [Board Term-1 2013]

h146

$$\sin^2 30^{\circ} \cos^2 45^{\circ} + 4\tan^2 30^{\circ} + \frac{1}{2}\sin 90^{\circ} - 2\cos^2 90^{\circ} + \frac{1}{24}$$

$$= \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{\sqrt{2}}\right)^2 + 4\left(\frac{1}{\sqrt{3}}\right)^2 + \frac{1}{2}(1)^2 - 2(0) + \frac{1}{24}$$

$$= \frac{1}{4}\left(\frac{1}{2}\right) + 4\left(\frac{1}{3}\right) + \frac{1}{2} + \frac{1}{24} = \frac{1}{8} + \frac{4}{3} + \frac{1}{2} + \frac{1}{24}$$

$$= \frac{3+32+12+1}{24} = \frac{48}{24} = 2$$

105. Evaluate : $4(\sin^4 30^{\circ} + \cos^4 60^{\circ}) - 3(\cos^2 45 - \sin^2 90^{\circ})$

Ans : [Board Term-1 2013]

$$4(\sin^4 30^{\circ} + \cos^4 60^{\circ}) - 3(\cos^2 45 - \sin^2 90^{\circ})$$

$$= 4\left[\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4\right] - 3\left[\left(\frac{1}{\sqrt{2}}\right)^2 - (1)^2\right]$$

$$= 4\left[\frac{1}{16} + \frac{1}{16}\right] - 3\left[\frac{1}{2} - 1\right]$$

h147

$$= 4\left(\frac{2}{16}\right) - 3\left(-\frac{1}{2}\right) = \frac{1}{2} + \frac{3}{2} = \frac{4}{2} = 2$$

106. If $15\tan^2\theta + 4\sec^2\theta = 23$, then find the value of $(\sec\theta + \operatorname{cosec}\theta)^2 - \sin^2\theta$.

Ans : [Board Term-1 2012]

We have

$$15\tan^2\theta + 4\sec^2\theta = 23$$

$$15\tan^2\theta + 4(\tan^2\theta + 1) = 23$$

h148

$$15\tan^2\theta + 4\tan^2\theta + 4 = 23$$

$$19\tan^2\theta = 19$$

$$\tan\theta = 1 = \tan 45^{\circ}$$

Thus $\theta = 45^{\circ}$

Now, $(\sec\theta + \operatorname{cosec}\theta)^2 - \sin^2\theta$

$$= (\sec 45^{\circ} + \operatorname{cosec} 45^{\circ})^2 - \sin^2 45^{\circ}$$

$$= (\sqrt{2} + \sqrt{2})^2 - \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= (2\sqrt{2})^2 - \frac{1}{2} = 8 - \frac{1}{2} = \frac{15}{2}$$

107. If $\sqrt{3}\cot^2\theta - 4\cot\theta + \sqrt{3} = 0$, then find the value of $\cot^2\theta + \tan^2\theta$.

Ans : [Board Term-1 2012]

We have $\sqrt{3}\cot^2\theta - 4\cot\theta + \sqrt{3} = 0$

Let $\cot\theta = x$, then we have

h149

$$\sqrt{3}x^2 - 4x + \sqrt{3} = 0$$

$$\sqrt{3}x^2 - 3x - x + \sqrt{3} = 0$$

$$(x - \sqrt{3})(\sqrt{3}x - 1) = 0$$

$$x = \sqrt{3} \text{ or } \frac{1}{\sqrt{3}}$$

Thus $\cot\theta = \sqrt{3}$ or $\cot\theta = \dfrac{1}{\sqrt{3}}$

Therefore $\theta = 30^{\circ}$ or $\theta = 60^{\circ}$

If $\theta = 30^{\circ}$, then

$$\cot^2 30^\circ + \tan^2 30^\circ = (\sqrt{3})^2 + \left(\frac{1}{\sqrt{3}}\right)^2$$

$$= 3 + \frac{1}{3} = \frac{10}{3}$$

If $\theta = 60^\circ$, then

$$\cot^2 60^\circ + \tan^2 60^\circ = \left(\frac{1}{\sqrt{3}}\right)^2 + (\sqrt{3})^2$$

$$= \frac{1}{3} + 3 = \frac{10}{3}.$$

108. Evaluate the following :

$$\frac{2\cos^2 60^\circ + 3\sec^2 30^\circ - 2\tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 45^\circ}$$

Ans : [Board Term-1 2012]

$$\frac{2\cos^2 60^\circ + 3\sec^2 30^\circ - 2\tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 45^\circ} = \frac{2(\frac{1}{2})^2 + 3(\frac{2}{\sqrt{3}})^2 - 2(1)^2}{(\frac{1}{2})^2 + (\frac{1}{\sqrt{2}})^2}$$

$$= \frac{2(\frac{1}{2})^2 + 3(\frac{2}{\sqrt{3}})^2 - 2(1)^2}{(\frac{1}{2})^2 + (\frac{1}{\sqrt{2}})^2}$$

$$= \frac{\frac{2}{4} + 4 - 2}{\frac{1}{4} + \frac{1}{2}} = \frac{10}{3}$$

h150

109. Prove that : $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \tan\theta + \cot\theta.$

Ans : [Board Term-1 2012]

$$\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = \frac{\tan\theta}{1-\frac{1}{\tan\theta}} + \frac{\frac{1}{\tan\theta}}{1-\tan\theta}$$

$$= \frac{\tan^2\theta}{\tan\theta - 1} + \frac{1}{(1-\tan\theta)\tan\theta}$$

$$= \frac{\tan^2\theta}{\tan\theta - 1} - \frac{1}{(\tan\theta - 1)\tan\theta}$$

$$= \frac{\tan^3\theta - 1}{(\tan\theta - 1)\tan\theta}$$

$$= \frac{(\tan\theta - 1)(\tan^2\theta + \tan\theta + 1)}{(\tan\theta - 1)(\tan\theta)}$$

$$= \frac{\tan^2\theta + \tan\theta + 1}{\tan\theta}$$

h152

$$= \tan\theta + 1 + \cot\theta$$

Hence Proved.

110. In an acute angled triangle ABC if $\sin(A+B-C) = \frac{1}{2}$ and $\cos(B+C-A) = \frac{1}{\sqrt{2}}$ find $\angle A, \angle B$ and $\angle C$.

Ans : [Board Term-1 2012]

We have $\sin(A+B-C) = \frac{1}{2} = \sin 30^\circ$

$$A + B - C = 30^\circ \quad ...(1)$$

and $\cos(B+C-A) = \frac{1}{\sqrt{2}} = \cos 45^\circ$

$$B + C - A = 45^\circ \quad ...(2)$$

Adding equation (1) and (2), we get

$$2B = 75^\circ \Rightarrow B = 37.5^\circ$$

Subtracting equation (2) from equation (1) we get,

$$2(A - C) = -15^\circ$$

$$A - C = -7.5^\circ \quad ...(3)$$

Now $A + B + C = 180^\circ$

$$A + C = 180^\circ - 37.5^\circ = 142.5^\circ \quad ...(4)$$

Adding equation (3) and (4), we have

$$2A = 135^\circ \Rightarrow A = 67.5^\circ$$

h153

and, $C = 75^\circ$

Hence, $\angle A = 67.5^\circ, \angle B = 37.5^\circ, \angle C = 75^\circ$

111. Prove that $b^2x^2 - a^2y^2 = a^2b^2$, if :

(1) $x = a\sec\theta, y = b\tan\theta$, or

(2) $x = a\,\mathrm{cosec}\,\theta, y = b\cot\theta$

Ans : [Board Term-1 2015]

(1) We have $x = a\sec\theta, y = b\tan\theta$,

$$\frac{x^2}{a^2} = \sec^2\theta, \; \frac{y^2}{b^2} = \tan^2\theta$$

h185

or, $\frac{x^2}{a^2} - \frac{y^2}{b^2} = \sec^2\theta - \tan^2\theta = 1$

Thus $b^2x^2 - a^2y^2 = a^2b^2$ Hence Proved

(ii) We have $x = a\,\mathrm{cosec}\,\theta, y = b\cot\theta$

$$\frac{x^2}{a^2} = \mathrm{cosec}^2\theta, \; \frac{y^2}{b^2} = \cot^2\theta$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = \mathrm{cosec}^2\theta - \cot\theta = 1$$

Thus $b^2x^2 - a^2y^2 = a^2b^2$ Hence Proved

112. If $\mathrm{cosec}\,\theta - \cot\theta = \sqrt{2}\cot\theta$, then prove that $\mathrm{cosec}\,\theta + \cot\theta = \sqrt{2}\,\mathrm{cosec}\,\theta$.

Ans : [Board Term-1 2015]

We have $\mathrm{cosec}\,\theta - \cot\theta = \sqrt{2}\cot\theta$

h186

Squaring both sides we have

$$\mathrm{cosec}^2\theta + \cot^2\theta - 2\,\mathrm{cosec}\,\theta\cot\theta = 2\cot^2\theta$$

$$\operatorname{cosec}^2\theta - \cot^2\theta = 2\operatorname{cosec}\theta\cot\theta$$

$$(\operatorname{cosec}\theta + \cot\theta)(\operatorname{cosec}\theta - \cot\theta) = 2\operatorname{cosec}\theta\cot\theta$$

$$(\operatorname{cosec}\theta - \cot\theta = \sqrt{2}\cot\theta)$$

$$(\operatorname{cosec}\theta + \cot\theta)\sqrt{2}\cot\theta = 2\operatorname{cosec}\theta\cot\theta$$

$$\operatorname{cosec}\theta + \cot\theta = \sqrt{2}\operatorname{cosec}\theta$$

Hence Proved.

113. Prove that :

$$\frac{\cot^3\theta\sin^3\theta}{(\cos\theta+\sin\theta)^2} + \frac{\tan^3\theta\cos^3\theta}{(\cos\theta+\sin\theta)^2} = \frac{\sec\theta\operatorname{cosec}\theta - 1}{\operatorname{cosec}\theta + \sec\theta}$$

Ans : [Board Term-1 2015]

h187

$$\frac{\cot^3\theta\sin^3\theta}{(\cos\theta+\sin\theta)^2} + \frac{\tan^3\theta\cos^3\theta}{(\cos\theta+\sin\theta)^2}$$

$$= \frac{\frac{\cos^3\theta}{\sin^3\theta}\times\sin^3\theta}{(\cos\theta+\sin\theta)^2} + \frac{\frac{\sin^3\theta}{\cos^3\theta}\times\cos^3\theta}{(\cos\theta+\sin\theta)^2}$$

$$= \frac{\cos^3\theta}{(\cos\theta+\sin\theta)^2} + \frac{\sin^3\theta}{(\cos\theta+\sin\theta)^2}$$

$$= \frac{(\cos\theta+\sin\theta)(\cos^2\theta+\sin^2\theta-\sin\theta\cos\theta)}{(\cos\theta+\sin\theta)^2}$$

$$= \frac{1-\sin\theta\cos\theta}{\cos\theta+\sin\theta} = \frac{\frac{1}{\cos\theta\sin\theta} - \frac{\sin\theta\cos\theta}{\cos\theta\sin\theta}}{\frac{\cos\theta}{\cos\theta\sin\theta} + \frac{\sin\theta}{\cos\theta\sin\theta}}$$

$$= \frac{\operatorname{cosec}\theta\sec\theta - 1}{\operatorname{cosec}\theta + \sec\theta}$$

Hence Proved

114. Prove that : $\sqrt{\frac{\sec\theta-1}{\sec\theta+1}} + \sqrt{\frac{\sec\theta+1}{\sec\theta-1}} = 2\operatorname{cosec}\theta.$

Ans : [Board Terim-1, 2012, Set-9]

$$\sqrt{\frac{\sec\theta-1}{\sec\theta+1}} + \sqrt{\frac{\sec\theta+1}{\sec\theta-1}} = \frac{(\sec\theta-1)+(\sec\theta+1)}{\sqrt{(\sec\theta+1)(\sec\theta-1)}}$$

$$= \frac{2\sec\theta}{\sqrt{\sec^2\theta-1}} = \frac{2\sec\theta}{\sqrt{\tan^2\theta}} = \frac{2\sec\theta}{\tan\theta}$$

h188

$$= 2\times\frac{1}{\cos\theta}\times\frac{\cos\theta}{\sin\theta}$$

$$= 2\times\frac{1}{\sin\theta}$$

$$= 2\operatorname{cosec}\theta$$

Hence Proved

115. Prove that : $\frac{\tan\theta+\sin\theta}{\tan\theta-\sin\theta} = \frac{\sec\theta+1}{\sec\theta-1}$.

Ans : [Board Term-1 2012]

We have $\frac{\tan\theta+\sin\theta}{\tan\theta-\sin\theta} = \frac{\frac{\sin\theta}{\cos\theta}+\sin\theta}{\frac{\sin\theta}{\cos\theta}-\sin\theta}$

h189

$$= \frac{\sin\theta(\frac{1}{\cos\theta}+1)}{\sin\theta(\frac{1}{\cos\theta}-1)}$$

$$= \frac{\sec\theta+1}{\sec\theta-1}$$

Hence Proved.

116. Prove that : $\frac{\operatorname{cosec}A}{\operatorname{cosec}A-1} + \frac{\operatorname{cosec}A}{\operatorname{cosec}A+1} = 2\sec^2 A$

Ans : [Board Term-1 2012]

$$\frac{\operatorname{cosec}A}{\operatorname{cosec}A-1} + \frac{\operatorname{cosec}A}{\operatorname{cosec}A+1}$$

$$= \frac{\operatorname{cosec}^2A + \operatorname{cosec}A + \operatorname{cosec}^2A - \operatorname{cosec}A}{(\operatorname{cosec}A-1)(\operatorname{cosec}A+1)}$$

$$= \frac{2\operatorname{cosec}^2A}{\operatorname{cosec}^2A-1} = \frac{2\operatorname{cosec}^2A}{\cot^2A}$$

h190

$$= \frac{\frac{2}{\sin^2A}}{\frac{\cos^2A}{\sin^2A}} = \frac{2}{\sin^2A}\times\frac{\sin^2A}{\cos^2A}$$

$$= \frac{2}{\cos^2A} = 2\sec^2A$$

Hence Proved.

117. If $\operatorname{cosec}\theta+\cot\theta = p$, then prove that $\cos\theta = \frac{p^2-1}{p^2+1}$.

Ans : [Board Term-1 2016

$$\frac{p^2-1}{p^2+1} = \frac{(\operatorname{cosec}\theta+\cot\theta)^2-1}{(\operatorname{cosec}\theta+\cot\theta)^2+1}$$

$$= \frac{\operatorname{cosec}^2\theta+\cot^2\theta+2\operatorname{cosec}\theta\cot\theta-1}{\operatorname{cosec}^2\theta+\cot^2\theta+2\operatorname{cosec}\theta\cot\theta+1}$$

$$= \frac{1+\cot^2\theta+\cot^2\theta+2\operatorname{cosec}\theta\cot\theta-1}{\operatorname{cosec}^2\theta+\operatorname{cosec}^2\theta-1+2\operatorname{cosec}\theta\cot\theta+1}$$

$$= \frac{2\cot\theta(\cot\theta+\operatorname{cosec}\theta)}{2\operatorname{cosec}\theta(\operatorname{cosec}\theta+\cot\theta)}$$

h191

$$= \frac{\cos\theta}{\sin\theta}\times\sin\theta = \cos\theta$$

118. If $a\cos\theta+b\sin\theta = m$ and $a\sin\theta-b\cos\theta = n$, prove that $m^2+n^2 = a^2+b^2$

Ans : [Board Term-1 2012]

We have

$$m^2 = a^2\cos^2\theta + 2ab\sin\theta\cos\theta + b^2\sin^2\theta \quad ...(1)$$

and, $n^2 = a^2\sin^2\theta - 2ab\sin\theta\cos\theta + b^2\cos^2\theta$ ...(2)

Adding equations (1) and (2) we get

$$m^2 + n^2 = a^2(\cos^2\theta + \sin^2\theta) + b^2(\cos^2\theta + \sin^2\theta)$$
$$= a^2(1) + b^2(1)$$
$$= a^2 + b^2$$

h192

119. Prove that : $\dfrac{\cos^2\theta}{1-\tan\theta} + \dfrac{\sin^3\theta}{\sin\theta - \cos\theta} = 1 + \sin\theta\cos\theta.$

Ans : [Board Term-1 2012]

$$\frac{\cos^2\theta}{1-\tan\theta} + \frac{\sin^3\theta}{\sin\theta-\cos\theta}$$
$$= \frac{\cos^2\theta}{1-\frac{\sin\theta}{\cos\theta}} + \frac{\sin^3\theta}{\sin\theta-\cos\theta}$$
$$= \frac{\cos^3\theta}{\cos\theta-\sin\theta} - \frac{\sin^3\theta}{\cos\theta-\sin\theta}$$
$$= \frac{\cos^3\theta-\sin^3\theta}{\cos\theta-\sin\theta}$$
$$= \frac{(\cos\theta-\sin\theta)(\cos^2\theta+\sin^2\theta+\sin\theta\cos\theta)}{(\cos\theta-\sin\theta)}$$
$$= 1+\sin\theta\cos\theta$$

Hence Proved

h193

120. If $\cos\theta + \sin\theta = p$ and $\sec\theta + \operatorname{cosec}\theta = q$, prove that $q(p^2-1) = 2p$

Ans : [Board Term-1 2012]

We have $\cos\theta + \sin\theta = p$ and $\sec\theta + \operatorname{cosec}\theta = q$

$$q(p^2-1) = (\sec\theta + \operatorname{cosec}\theta)[(\cos\theta+\sin\theta)^2 - 1]$$
$$= (\sec\theta + \operatorname{cosec}\theta)(\cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta - 1)$$
$$= (\sec\theta + \operatorname{cosec}\theta)[1 + 2\sin\theta\cos\theta - 1]$$
$$= \left(\frac{1}{\cos\theta} + \frac{1}{\sin\theta}\right)(2\sin\theta\cos\theta)$$
$$= \left(\frac{\sin\theta+\cos\theta}{\cos\theta\sin\theta}\right)2\sin\theta\cos\theta$$
$$= 2(\sin\theta+\cos\theta) = 2p$$

Hence Proved.

h194

121. If $x = r\sin A\cos C$, $y = r\sin A\sin C$ and $z = r\cos A$, then prove that $x^2 + y^2 + z^2 = r^2$

Ans : [Board Term-1 2012, Set-50]

Since, $x^2 = r^2\sin^2 A\cos^2 C$

$y^2 = r^2\sin^2 A\sin^2 C$

and $z^2 = r^2\cos^2 A$

h195

$$x^2 + y^2 + z^2 = r^2\sin^2 A\cos^2 C + r^2\sin^2 A\sin^2 C + r^2\cos^2 A$$
$$= r^2\sin^2 A(\cos^2 C + \sin^2 C) + r^2\cos^2 A$$
$$= r^2\sin^2 A + r^2\cos^2 A$$
$$= r^2(\sin^2 A + \cos^2 A)$$
$$= r^2$$

Hence Proved.

122. Prove that: $\sqrt{\dfrac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\dfrac{1-\sin\theta}{1+\sin\theta}} = 2\sec\theta.$

Ans : [Board Term-1 2012]

$$\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$$
$$= \sqrt{\frac{(1+\sin\theta)}{(1-\sin\theta)}\times\frac{(1+\sin\theta)}{(1+\sin\theta)}} + \sqrt{\frac{(1-\sin\theta)}{(1+\sin\theta)}\times\frac{(1-\sin\theta)}{(1-\sin\theta)}}$$
$$= \sqrt{\frac{(1+\sin\theta)^2}{(1-\sin^2\theta)}} + \sqrt{\frac{(1-\sin\theta)^2}{1-\sin^2\theta}}$$
$$= \sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}} + \sqrt{\frac{(1-\sin\theta)^2}{\cos^2\theta}}$$
$$= \frac{1+\sin\theta}{\cos\theta} + \frac{1-\sin\theta}{\cos\theta} = \frac{1+\sin\theta+1-\sin\theta}{\cos\theta}$$
$$= \frac{2}{\cos\theta} = 2\sec\theta$$

Hence Proved

h196

123. Prove that

$(1-\sin\theta+\cos\theta)^2 = 2(1+\cos\theta)(1-\sin\theta).$

Ans : [Board Term-1 2012]

$$(1-\sin\theta+\cos\theta)^2$$
$$= 1+\sin^2\theta+\cos^2\theta-2\sin\theta-2\sin\theta\cos\theta+2\cos\theta$$
$$= 1+1-2\sin\theta-2\sin\theta\cos\theta+2\cos\theta$$
$$= 2+2\cos\theta-2\sin\theta-2\sin\theta\cos\theta$$
$$= 2(1+\cos\theta)-2\sin\theta(1+\cos\theta)$$
$$= (1+\cos\theta)(2-2\sin\theta)$$
$$= 2(1+\cos\theta)(1-\sin\theta)$$

Hence Proved

h197

124. Prove that : $\dfrac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta-1} = \sec\theta+\tan\theta$

Ans : [Board Term-1 2012]

$$\frac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1}$$

h198

$$= \frac{(\tan\theta+\sec\theta)-(\sec^2\theta-\tan^2\theta)}{\tan\theta-\sec\theta+1}$$
$$= \frac{(\tan\theta+\sec\theta)-(\sec\theta-\tan\theta)(\sec\theta+\tan\theta)}{\tan\theta-\sec\theta+1}$$

$$= \frac{(\tan\theta + \sec\theta)(1 - \sec\theta + \tan\theta)}{\tan\theta - \sec\theta + 1}$$

$$= \tan\theta + \sec\theta \qquad \text{Hence Proved}$$

125. Prove that :

$(\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2 = 7 + \tan^2\theta + \cot^2\theta\cot^2\theta$

Ans : [Board Term-1 2012]

h199

$$(\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2$$

$$= \sin^2\theta + \operatorname{cosec}^2\theta + 2\sin\theta\operatorname{cosec}\theta + \cos^2\theta + \sec^2\theta + 2\cos\theta\sec\theta$$

$$= (\sin^2\theta + \cos^2\theta) + \operatorname{cosec}^2\theta + 2 + \sec^2\theta + 2$$

$$= 1 + (1 + \cot^2\theta) + 2 + (1 + \tan^2\theta) + 2$$

$$= 7 + \tan^2\theta + \cot^2\theta \qquad \text{Hence Proved}$$

126. If $\sin\theta = \frac{c}{\sqrt{c^2 + d^2}}$ and $d > 0$, find the value of $\cos\theta$ and $\tan\theta$.

Ans : [Board Term-1 2013]

We have $\sin\theta = \frac{c}{\sqrt{c^2 + d^2}}$

Now
$$\cos^2\theta = 1 - \sin^2\theta$$
$$= 1 - \left(\frac{c}{\sqrt{c^2 + d^2}}\right)^2$$
$$= 1 - \frac{c^2}{c^2 + d^2}$$
$$= \frac{c^2 + d^2 - c^2}{c^2 + d^2} = \frac{d^2}{c^2 + d^2}$$

h200

Thus $\cos\theta = \frac{d}{\sqrt{c^2 + d^2}}$

Again,
$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{c}{\sqrt{c^2 + d^2}}}{\frac{d}{\sqrt{c^2 + d^2}}} = \frac{c}{d}$$

Thus $\tan\theta = \frac{c}{d}$

127. If $\tan\theta = \frac{1}{\sqrt{5}}$,

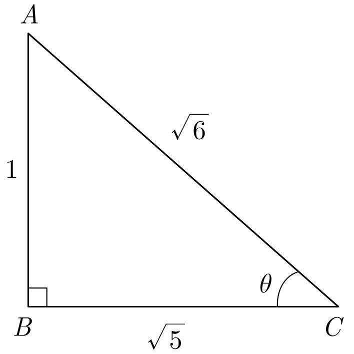
(1) Evaluate : $\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta}$

(2) Verify the identity : $\sin^2\theta + \cos^2\theta = 1$

Ans : [Board Term-1 2012]

We have $\tan\theta = \frac{1}{\sqrt{5}}$

We draw the triangle as shown below and write all dimensions.



Now
$$\cot\theta = \frac{1}{\tan\theta} = \sqrt{5}$$
$$\sin\theta = \frac{1}{\sqrt{6}}$$
$$\cos\theta = \frac{\sqrt{5}}{\sqrt{6}}$$

h201

(1)
$$\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta} = \frac{(1 + \cot^2\theta) - (1 + \tan^2\theta)}{(1 + \cot^2\theta) + (1 + \tan^2\theta)}$$
$$= \frac{\cot^2\theta - \tan^2\theta}{2 + \cot^2\theta + \tan^2\theta}$$
$$= \frac{(\sqrt{5})^2 - (\frac{1}{5})^2}{2 + (\sqrt{5})^2 + (\frac{1}{\sqrt{5}})^2}$$
$$= \frac{5 - \frac{1}{5}}{2 + 5 + \frac{1}{5}} = \frac{25 - 1}{35 + 1} = \frac{24}{36} = \frac{2}{3}$$

(2)
$$\sin^2\theta + \cos^2\theta = \left(\frac{1}{\sqrt{6}}\right)^2 + \left(\frac{\sqrt{5}}{\sqrt{6}}\right)^2$$
$$= \frac{1}{6} + \frac{5}{6} = \frac{6}{6}$$
$$= 1 \qquad \text{Hence proved.}$$

128. If $\sec\theta + \tan\theta = p$, show that $\sec\theta - \tan\theta = \frac{1}{p}$, Hence, find the values of $\cos\theta$ and $\sin\theta$.

Ans : [Board Term-1 2015]

We have $\sec\theta + \tan\theta = p \qquad (1)$

Now
$$\frac{1}{p} = \frac{1}{\sec\theta + \tan\theta} \times \frac{(\sec\theta - \tan\theta)}{(\sec\theta - \tan\theta)}$$
$$= \frac{\sec\theta - \tan\theta}{\sec^2\theta - \tan^2\theta} = \sec\theta - \tan\theta$$

h204

or $\qquad \dfrac{1}{p} = \sec\theta - \tan\theta \qquad (2)$

Solving $\sec\theta + \tan\theta = p$ and $\sec\theta - \tan\theta = \dfrac{1}{p}$,

$$\sec\theta = \frac{1}{2}\left(p + \frac{1}{p}\right) = \frac{p^2+1}{2p}$$

Thus $\qquad \cos\theta = \dfrac{2p}{p^2+1}$

and $\qquad \tan\theta = \dfrac{1}{2}\left(p - \dfrac{1}{p}\right) = \dfrac{p^2-1}{2p}$

and $\qquad \sin\theta = \tan\theta\cos\theta = \dfrac{p^2-1}{p^2+1}$

129. Prove that : $(\operatorname{cosec}\theta + \cot\theta)^2 = \dfrac{\sec\theta + 1}{\sec\theta - 1}$

Ans :

$$(\operatorname{cosec}\theta + \cot\theta)^2 = \operatorname{cosec}^2\theta + \cot^2\theta + 2\operatorname{cosec}\theta.\cot\theta$$

$$= \left(\frac{1}{\sin\theta}\right)^2 + \left(\frac{\cos\theta}{\sin\theta}\right)^2 + \frac{2\times 1}{\sin\theta}\times\frac{\cos\theta}{\sin\theta}$$

$$= \frac{1}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} + \frac{2\cos\theta}{\sin^2\theta}$$

$$= \frac{1+\cos^2\theta + 2\cos\theta}{\sin^2\theta} = \frac{(1+\cos\theta)^2}{1-\cos^2\theta}$$

$$= \frac{(1+\cos\theta)(1+\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$$

h205

$$= \frac{1+\cos\theta}{1-\cos\theta} = \frac{1+\frac{1}{\sec\theta}}{1-\frac{1}{\sec\theta}}$$

$$= \frac{\sec\theta+1}{\sec\theta-1}$$

Hence Prove.

130. Prove that :

$(\sin A + \sec A)^2 + (\cos A + \operatorname{cosec} A)^2 = (1 + \sec A\operatorname{cosec} A)^2$

Ans : [Board Term-1 2012]

$$\text{LHS} = (\sin A + \sec A)^2 + (\cos A + \operatorname{cosec} A)^2$$

$$= \left(\sin A + \frac{1}{\cos A}\right)^2 + \left(\cos A + \frac{1}{\sin A}\right)^2$$

$$= \sin^2 A + \frac{1}{\cos^2 A} + 2\frac{\sin A}{\cos A} + \cos^2 A + \frac{1}{\sin^2 A} + 2\frac{\cos A}{\sin A}$$

$$= \sin^2 A + \cos^2 A + \frac{1}{\sin^2 A} + \frac{1}{\cos^2 A} + 2\left(\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right)$$

$$= 1 + \frac{\sin^2 A + \cos^2 A}{\sin^2 A\cos^2 A} + 2\left(\frac{\sin^2 A + \cos^2 A}{\sin A\cos A}\right)$$

h206

$$= 1 + \frac{1}{\sin^2 A\cos^2 A} + \frac{2}{\sin A\cos A}$$

$$= \left(1 + \frac{1}{\sin A\cos A}\right)^2$$

$$= (1 + \sec A\operatorname{cosec} A)^2$$

Hence Proved

131. If $(\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C)$
$= (\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)$
Prove that each of the side is equal to ± 1.

Ans : [Board Term-1 2012]

We have

h207

$(\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C)$

$= (\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)$

Multiply both sides by

$(\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)$

or, $(\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C)\times$

$(\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)$

$= (\sec A - \tan A)^2(\sec B - \tan B)^2(\sec C - \tan C)^2$

or, $(\sec^2 A - \tan^2 A)(\sec^2 B - \tan^2 B)(\sec^2 C - \tan^2 C)$

$= (\sec A - \tan A)^2(\sec A - \tan B)^2(\sec C - \tan C)^2$

or, $1 = \left[(\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)\right]^2$

or, $(\sec A - \tan A)(\sec B - \tan B)(\sec C + \tan C) = \pm 1$

132. If $4\sin\theta = 3$, find the value of x if

$$\sqrt{\frac{\operatorname{cosec}^2\theta - \cot^2\theta}{\sec^2\theta - 1}} + 2\cot\theta = \frac{\sqrt{7}}{x} + \cos\theta$$

Ans : [Board Term-1 2012]

We have $\qquad \sin\theta = \dfrac{3}{4}$

h208

or, $\qquad \sin^2\theta = \dfrac{9}{16}$

Since $\sin^2\theta + \cos^2 = 1$, we have

$$\cos^2\theta = 1 - \sin^2\theta = 1 - \frac{9}{16} = \frac{7}{16}$$

$$\cos\theta = \frac{\sqrt{7}}{4}$$

and $\qquad \tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{\frac{3}{4}}{\frac{\sqrt{7}}{4}} = \dfrac{3}{\sqrt{7}}$

Thus $\qquad \sqrt{\dfrac{\operatorname{cosec}^2\theta - \cot^2\theta}{\sec^2\theta - 1}} + 2\cot\theta = \dfrac{\sqrt{7}}{x} + \cos\theta$

$$\sqrt{\frac{1}{\tan^2\theta}} + 2 \times \frac{\sqrt{7}}{3} = \frac{\sqrt{7}}{x} + \frac{\sqrt{7}}{4}$$

$$\frac{1}{\tan\theta} + \frac{2\sqrt{7}}{3} = \frac{\sqrt{7}}{x} + \frac{\sqrt{7}}{4}$$

$$\frac{\sqrt{7}}{3} + \frac{2\sqrt{7}}{3} - \frac{\sqrt{7}}{4} = \frac{\sqrt{7}}{x}$$

$$\frac{4\sqrt{7} - \sqrt{7}}{4} = \frac{\sqrt{7}}{x}$$

$$\frac{3\sqrt{7}}{4} = \frac{\sqrt{7}}{x}$$

Thus $x = \frac{4}{3}$

133. Prove that $\sec^2\theta + \csc^2\theta$ can never be less than 2.

Ans : [Board-Term 1 2011]

h209

Let $\sec^2\theta + \text{cosec}^2\theta = x$

$$1 + \tan^2\theta + 1 + \cot^2\theta = x$$

$$2 + \tan^2\theta + \cot^2\theta = x$$

$$2 + \tan^2\theta + \cot^2\theta = x$$

$\tan^2\theta \geq 0$ and $\cot^2\theta \geq 0$

Thus $x > 2$

Thus $\sec^2\theta + \text{cosec}^2\theta > 2$

Hence $\sec^2\theta + \text{cosec}^2\theta$ can never be less than 2.

134. (a) Solve for ϕ, if $\tan 5\phi = 1$

(b) Solve for ϕ, if $\frac{\sin\phi}{1+\cos\phi} + \frac{1+\cos\phi}{\sin\phi} = 4$

Ans :

h210

(a) $\tan 5\phi = 1$

$$\tan 5\phi = \tan 45^\circ$$

$$5\phi = 45^\circ$$

Thus $\phi = 9^\circ$

(b) $\frac{\sin\phi}{1+\cos\phi} + \frac{1+\cos\phi}{\sin\phi} = 4$

$$\frac{\sin^2\phi + (1+\cos\theta)^2}{\sin\phi(1+\cos\phi)} = 4$$

$$\frac{\sin^2\phi + 1 + 2\cos\phi + \cos^2\phi}{\sin\phi + \sin\phi\cos\phi} = 4$$

$$\frac{\sin^2\phi + \cos^2\phi + 1 + 2\cos\phi}{\sin\phi(1+\cos\phi)} = 4$$

$$\frac{2 + 2\cos\phi}{\sin\phi(1+\cos\phi)} = 4$$

$$\frac{2(1+\cos\phi)}{\sin\phi(1+\cos\phi)} = 4$$

$$\frac{2}{\sin\phi} = 4$$

$$\sin\phi = \frac{1}{2}$$

$$\sin\phi = \sin 30^\circ$$

Thus $\phi = 30^\circ$

135. If $\tan A + \sin A = m$ and $\tan A - \sin A = n$, show that $m^2 - n^2 = 4\sqrt{mn}$.

Ans : [Board-Term 1 2009

h211

We have $\tan A + \sin A = m$

and $\tan A - \sin A = n$

$$m^2 - n^2 = (\tan A + \sin A)^2 - (\tan A - \sin A)^2$$

$$= (\tan^2 A + \sin^2 A + 2\sin A\tan A)$$

$$-(\tan^2 A + \sin^2 A - 2\sin A\tan A)$$

$$= \tan^2 A + \sin^2 A + 2\sin A\tan A$$

$$-\tan^2 A - \sin^2 A + 2\sin A\tan A$$

$$= 4\sin A\tan A$$

$$4\sqrt{mn} = 4\sqrt{(\tan A + \sin A)(\tan A - \sin A)}$$

$$= 4\sqrt{\tan^2 A - \sin^2 A}$$

$$= 4\sqrt{\frac{\sin^2 A}{\cos^2 A} - \sin^2 A}$$

$$= 4\sqrt{\frac{\sin^2 A - \sin^2 A\cos^2 A}{\cos^2 A}}$$

$$= 4\sqrt{\frac{\sin^2 A(1-\cos^2 A)}{\cos^2 A}}$$

$$= 4\sqrt{\frac{\sin^2 A \times \sin^2 A}{\cos^2 A}}$$

$$= 4\frac{\sin A \times \sin A}{\cos A}$$

$$= 4\sin A \times \frac{\sin A}{\cos A}$$

$$= 4\sin A\tan A$$

Thus $m^2 - n^2 = 4\sqrt{mn}$ Hence Proved

136. If $\frac{\cos\alpha}{\cos\beta} = m$ and $\frac{\cos\alpha}{\sin\beta} = n$, show that

$(m^2+n^2)\cos^2\beta = n^2$.

Ans : [Board-Term 1 2010]

h212

We have $\dfrac{\cos\alpha}{\cos\beta} = m$ and $\dfrac{\cos\alpha}{\sin\beta} = n$

$$m^2 = \frac{\cos^2\alpha}{\cos^2\beta} \text{ and } n^2 = \frac{\cos^2\alpha}{\sin^2\beta}$$

$$(m^2+n^2)\cos^2\beta = \left[\frac{\cos^2\alpha}{\cos^2\beta} + \frac{\cos^2\alpha}{\sin^2\beta}\right]\cos^2\beta$$

$$= \cos^2\alpha\left[\frac{1}{\cos^2\beta} + \frac{1}{\sin^2\beta}\right]\cos^2\beta$$

$$= \cos^2\alpha\frac{\sin^2\beta + \cos^2\beta}{\cos^2\beta\sin^2\beta}\cos^2\beta$$

$$= \cos^2\alpha\left(\frac{1}{\cos^2\beta\sin^2\beta}\right)\cos^2\beta$$

$$= \frac{\cos^2\alpha}{\sin^2\beta}$$

$$= n^2$$ Hence Proved.

137. If $7\operatorname{cosec}\phi - 3\cot\phi = 7$, prove that $7\cot\phi - 3\operatorname{cosec}\phi = 3$.

Ans :

h213

We have $7\operatorname{cosec}\phi - 3\cot\phi = 7$

$$7\operatorname{cosec}\phi - 7 = 3\cot\phi$$

$$7(\operatorname{cosec}\phi - 1) = 3\cot\phi$$

$$7(\operatorname{cosec}\phi - 1)(\operatorname{cosec}\phi + 1) = 3\cot\phi(\operatorname{cosec}\phi + 1)$$

$$7(\operatorname{cosec}^2\phi - 1) = 3\cot\phi(\operatorname{cosec}\phi + 1)$$

$$7\cot^2\phi = 3\cot\phi(\operatorname{cosec}\phi + 1)$$

$$7\cot\phi = 3(\operatorname{cosec}\phi + 1)$$

$$7\cot\phi - 3\operatorname{cosec}\phi = 3$$ Hence Proved

138. Prove that : $\dfrac{\cos\theta - \sin\theta + 1}{\cos\theta + \sin\theta - 1} = \operatorname{cosec}\theta + \cot\theta$

Ans : [Board SQP 2018]

h214

$$\text{LHS} = \frac{\cos\theta - \sin\theta + 1}{\cos\theta + \sin\theta - 1}$$

$$= \frac{\sin\theta(\cos\theta - \sin\theta + 1)}{\sin\theta(\cos\theta + \sin\theta - 1)}$$

$$= \frac{\sin\theta\cos\theta - \sin^2\theta + \sin\theta}{\sin\theta(\cos\theta + \sin\theta - 1)}$$

$$= \frac{\sin\theta\cos\theta + \sin\theta - (1 - \cos^2\theta)}{\sin\theta(\cos\theta + \sin\theta - 1)}$$

$$= \frac{\sin\theta(\cos\theta + 1) - [(1 - \cos\theta)(1 + \cos\theta)]}{\sin\theta(\cos\theta + \sin\theta - 1)}$$

$$= \frac{(1 + \cos\theta)(\sin\theta - 1 + \cos\theta)}{\sin\theta(\cos\theta + \sin\theta - 1)}$$

$$= \frac{(1 + \cos\theta)(\cos\theta + \sin\theta - 1)}{\sin\theta(\cos\theta + \sin\theta - 1)}$$

$$= \frac{1 + \cos\theta}{\sin\theta}$$

$$= \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \operatorname{cosec}\theta + \cot\theta$$ Hence Proved

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Maths PART 1

Maths PART 2